

## Title: Nonparametric Testing of Hypothesis

### PROJECT 3.1 (Run Test):

In 30 toss of a coin the following sequence of heads (H) and tail (T) is obtained.

**H T T H T H H H T H H T T H T H T H H T H T T H T H H T H T**

Test at 0.05 level of significance level whether the sequence is random.

### PROCEDURE:

1. Enter the data
2. Select Analyze => Nonparametric Test => Legacy Dialogs => Runs
3. Click Options => Select Descriptive.
4. Click Continue => Ok.

### SOLUTION:

#### **STEP I: Null Hypothesis ( $H_0$ )**

The given sequence of sample observations is in random order.

#### **STEP II: Alternative Hypothesis ( $H_1$ )**

The given sequence of sample observations is not in random order.

#### **STEP III: Test Statistics**

$$Z = \frac{R - \mu_r}{\sigma_r}$$

where,

$$\mu_r = \frac{2n_1n_2}{n_1 + n_2} + 1$$

$$\sigma_r = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)}}$$

### Calculation (FROM SPSS):

<i>Descriptive Statistics</i>				
	<i>N</i>	<i>Mean</i>	<i>Std. Deviation</i>	<i>Std. Error Mean</i>
<b>Coin Toss</b>	30	.53	.507	0

<i>Runs Test</i>	
	<i>Coin Toss</i>
<i>Test Value<sup>a</sup></i>	1
<i>Cases &lt; Test Value</i>	14
<i>Cases &gt;= Test Value</i>	16
<i>Total Cases</i>	30
<i>Number of Runs</i>	22
<i>Z</i>	2.078
<i>Asymp. Sig. (2-tailed)</i>	.038

So, *P Value* = 0.038

***STEP IV : Critical Value***

$$\alpha = 5\% = 0.05$$

***STEP V : Decision***

Since  $P < \alpha$ , accept  $H_1$  and reject  $H_0$

**CONCLUSION:**

Hence, the given sequence of sample observations of coin toss is not in random order.

**PROJECT 3.2 (Binomial Test):**

Test whether the coin is unbiased from following observations.

***Tail Tail Head Head Tail Head Tail Head Head Tail Head Head Head Tail Head Head  
Head Head Head Head Tail Tail Tail Tail Head Tail Tail Tail Tail Tail Tail Head  
Tail Tail Tail Head Tail Tail Tail Tail Head Tail Tail Head Tail Head Tail Tail Tail***

**PROCEDURE:**

1. Enter the data
2. Select Analyze => Nonparametric Test => Legacy Dialogs => Binomial
3. Click Options => Select Descriptive.
4. Click Continue => Ok.

**SOLUTION:**

***STEP I: Null Hypothesis ( $H_0$ )***

$$P = \frac{1}{2}$$

i.e. The coin is unbiased.

***STEP II: Alternative Hypothesis ( $H_1$ )***

$$P \neq \frac{1}{2}$$

i.e. The coin is biased.

***STEP III: Test Statistics***

$$Z = \frac{X_0 - \mu_r}{\sigma_r}$$

where,

$$\mu_r = np$$

$$\sigma_r = \sqrt{npq}$$

***Since  $X_0$  is discrete, so that continuity correction is made as***

$$Z = \frac{(X_0 \pm 0.5) - \mu_r}{\sigma_r}$$

***Use +0.5 if  $X_0 < np$  else use -0.5***

**Calculation (From SPSS):**

<i>Descriptive Statistics</i>					
	<i>N</i>	<i>Mean</i>	<i>Std. Deviation</i>	<i>Minimum</i>	<i>Maximum</i>
<i>Coin</i>	50	0.40	0.495	0	1

<i>Binomial Test</i>						
		<i>Category</i>	<i>N</i>	<i>Observed Prop.</i>	<i>Test Prop.</i>	<i>Exact Sig. (2-tailed)</i>
<i>Coin</i>	<i>Group 1</i>	<i>Tail</i>	30	.60	.50	.203
	<i>Group 2</i>	<i>Head</i>	20	.40		
	<i>Total</i>		50	1.00		

So, *P Value* = 0.203

***STEP IV : Critical Value***

$$\alpha = 5\% = 0.05$$

***STEP V : Decision***

Since  $P > \alpha$  , accept  $H_0$  and reject  $H_1$

**CONCLUSION:**

Hence, the coin is unbiased.

**PROJECT 3.3 (One Sample Kolmogorov Smirnov Test):**

The number of disease infected tomato plants in 10 different plots of equal size are given below. Test whether the disease infected plants are uniformly distributed over the entire area. Use Kolmogorov Smirnov Test.

<b><i>Plot no.</i></b>	1	2	3	4	5	6	7	8	9	10
<b><i>No. of infected plants</i></b>	8	10	9	12	15	7	5	12	13	9

**PROCEDURE:**

1. Enter the data
2. Select Analyze => Nonparametric Test => Legacy Dialogs => 1 Sample K-S
3. Click Options => Select Descriptive.
4. Click Continue => Ok.

**SOLUTION:**

***STEP I: Null Hypothesis ( $H_0$ )***

The disease infected plants are uniformly distributed over the entire area.

***STEP II: Alternative Hypothesis ( $H_1$ )***

The disease infected plants are not uniformly distributed over the entire area.

***STEP III: Test Statistics***

$$D_0 = \text{Max } |F_e - F_o|$$

**Calculation (From SPSS):**

<b><i>Descriptive Statistics</i></b>					
	<b><i>N</i></b>	<b><i>Mean</i></b>	<b><i>Std. Deviation</i></b>	<b><i>Minimum</i></b>	<b><i>Maximum</i></b>
<b><i>No. of infected plants</i></b>	10	10.00	3.018	5	15

<b><i>One-Sample Kolmogorov-Smirnov Test</i></b>		
		<b><i>No. of infected plants</i></b>
<b><i>N</i></b>		10
<b><i>Normal Parameters<sup>a,b</sup></i></b>	<b><i>Mean</i></b>	10.00
	<b><i>Std. Deviation</i></b>	3.018
<b><i>Most</i></b>	<b><i>Absolute</i></b>	.146
	<b><i>Positive</i></b>	.130

<i>Extreme Differences</i>	<i>Negative</i>	-0.146
<i>Test Statistic</i>		0.146
<i>Asymp. Sig. (2-tailed)</i>		0.200 <sup>c,d</sup>

*a. Test distribution is Normal.*

*b. Calculated from data.*

*c. Lilliefors Significance Correction.*

*d. This is a lower bound of the true significance.*

So, *P Value* = 0.200

**STEP IV : Critical Value**

$$\alpha = 5\% = 0.05$$

**STEP V : Decision**

Since  $P > \alpha$ , accept  $H_0$  and reject  $H_1$

**CONCLUSION:**

Hence, the disease infected plants are uniformly distributed over the entire area.

**PROJECT 3.4 (Mann Whitney U Test):**

Test the hypothesis of no difference between the ages of male and female employees of a certain company, using the Mann-Whitney U test for the samples data below.

<i>Male</i>	35	43	26	44	40	42	33	38	25	26
<i>Female</i>	30	41	34	31	36	32	25	47	28	24

**PROCEDURE:**

1. Enter the data
2. Select Analyze => Nonparametric Test => Legacy Dialogs => 2 independent sample.
3. Click Options => Select Descriptive.
4. Click Continue => Ok.

**SOLUTION:**

***STEP I: Null Hypothesis ( $H_0$ )***

$$Md_1 = Md_2$$

i.e. There is no significant difference between age of male and female employees of the company.

***STEP II: Alternative Hypothesis ( $H_1$ )***

$$Md_1 \neq Md_2$$

i.e. There is significant difference between age of male and female employees of the company.

***STEP III: Test Statistics***

$$U_0 = \min\{U_1, U_2\}$$

where,

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - \Sigma R_1$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - \Sigma R_2$$

**Calculation (From SPSS):**

<i>Descriptive Statistics</i>					
	<i>N</i>	<i>Mean</i>	<i>Std. Deviation</i>	<i>Minimum</i>	<i>Maximum</i>
<i>Age</i>	20	34.00	7.167	24	47
<i>Gender</i>	20	1.50	0.513	1	2

<i>Test Statistics<sup>a</sup></i>	
	<i>Age of Employee</i>
<i>Mann-Whitney U</i>	38.500
<i>Wilcoxon W</i>	93.500
<i>Z</i>	-0.870
<i>Asymp. Sig. (2-tailed)</i>	0.384
<i>Exact Sig. [2*(1-tailed Sig.)]</i>	0.393 <sup>b</sup>

*a. Grouping Variable : Gender*

*b. Not corrected for ties*

So, *P Value* = 0.384

**STEP IV : Critical Value**

$$\alpha = 5\% = 0.05$$

**STEP V : Decision**

Since  $2P > \alpha$  , accept  $H_0$  and reject  $H_1$

**CONCLUSION:**

Hence, there is no significant difference between age of male and female employees of the company.



**PROJECT 3.5 (Median Test):**

An IQ test was given to a randomly selected 15 male and 20 female students of a university. Their scores were recorded as follows:

**Male: 56, 66, 62, 81, 75, 73, 83, 68, 48, 70, 60, 77, 86, 44, 72**

**Female: 63, 77, 65, 71, 74, 60, 76, 61, 67, 72, 64, 65, 55, 89, 45, 53, 68, 73, 50, 81**

Use median test to determine whether IQ of male and female students is same in the university.

**PROCEDURE:**

1. Enter the data
2. Select Analyze => Nonparametric Test => Legacy Dialogs => K independent sample.
3. Click Options => Select Descriptive.
4. Define Groups {Gender(1,2)}.
5. Click Continue => Ok.

**SOLUTION:**

**STEP I: Null Hypothesis ( $H_0$ )**

$$Md_1 = Md_2$$

i.e. The IQ of male and female students is same in the university.

**STEP II: Alternative Hypothesis ( $H_1$ )**

$$Md_1 \neq Md_2$$

i.e. The IQ of male and female students is not same in the university.

**STEP III: Test Statistics**

$$\chi^2 = \frac{N(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)}$$

**Calculation (From SPSS):**

<i>Descriptive Statistics</i>					
	<i>N</i>	<i>Mean</i>	<i>Std. Deviation</i>	<i>Minimum</i>	<i>Maximum</i>
<b><i>IQ</i></b>	35	67.1429	11.314	44	89
<b><i>Gender</i></b>	35	1.57	0.502	1	2

<i>Test Statistics<sup>a</sup></i>		
		<b><i>IQ</i></b>
<i>N</i>		35
<i>Median</i>		68.00
<i>Chi-Square</i>		0.614
<i>df</i>		1
<i>Asymp. Sig.</i>		0.433
<i>Yates' Continuity Correction</i>	<i>Chi-Square</i>	0.194
	<i>df</i>	1
	<i>Asymp. Sig.</i>	0.659

*a. Grouping Variable : Gender*

So, *P Value* = 0.659

**STEP IV : Critical Value**

$$\alpha = 5\% = 0.05$$

**STEP V : Decision**

Since  $P > \alpha$ , accept  $H_0$  and reject  $H_1$

**CONCLUSION:**

Hence, The IQ of male and female students is same in the university.

**PROJECT 3.6 (Wilcoxon Matched Pair Signed Rank Test):**

Use Wilcoxon matched pair signed rank test to determine the equality of effectiveness of two types of drugs in suppressing pain from following data.

<i>Patient No.</i>	<i>Drug A</i>	<i>Drug B</i>	<i>Patient No.</i>	<i>Drug A</i>	<i>Drug B</i>
<b>1</b>	6.5	3.5	<b>11</b>	5.4	5.5
<b>2</b>	3.7	3.7	<b>12</b>	4.0	4.1
<b>3</b>	3.9	4.7	<b>13</b>	5.7	4.1
<b>4</b>	6.7	5.0	<b>14</b>	3.9	4.2
<b>5</b>	6.2	5.6	<b>15</b>	3.6	3.7
<b>6</b>	6.7	4.3	<b>16</b>	4.9	4.1
<b>7</b>	6.1	5.4	<b>17</b>	3.9	5.4
<b>8</b>	4.3	5.8	<b>18</b>	5.8	3.7
<b>9</b>	5.5	4.3	<b>19</b>	4.9	4.1
<b>10</b>	6.8	4.3	<b>20</b>	3.9	4.1

**PROCEDURE:**

1. Enter the data
2. Select Analyze => Nonparametric Test => Legacy Dialogs => 2 Related sample.
3. Click Options => Select Drug A and Drug B (in test pair variable 1 - drug A and variable 2 - drug B)
4. Click Continue => Ok.

**SOLUTION:**

***STEP I: Null Hypothesis ( $H_0$ )***

$$Md_1 = Md_2$$

i.e. Both Drug A and Drug B are equally effective in suppressing the pain.

***STEP II: Alternative Hypothesis ( $H_1$ )***

$$Md_1 \neq Md_2$$

i.e. Both Drug A and Drug B are not equally effective in suppressing the pain.

***STEP III: Test Statistics***

$$T = \min\{S(+), S(-)\}$$

where,

$S(+)$  = Sum of ranks of + sign

$S(-)$  = Sum of ranks of - sign

**Calculation (From SPSS):**

<i>Descriptive Statistics</i>					
	<i>N</i>	<i>Mean</i>	<i>Std. Deviation</i>	<i>Minimum</i>	<i>Maximum</i>
<i>Drug A</i>	20	5.170	1.1164	3.6	6.8
<i>Drug B</i>	20	4.480	0.7157	3.5	5.8

<i>Test Statistics<sup>a</sup></i>	
	<i>Drug B – Drug A</i>
<i>Z</i>	-2.076 <sup>b</sup>
<i>Asymp. Sig. (2-tailed)</i>	.038

*a. Grouping Variable : Gender*

*b. Based on positive ranks*

So, *P Value* = 0.038

**STEP IV : Critical Value**

$$\alpha = 5\% = 0.05$$

**STEP V : Decision**

Since  $P > \alpha$  , accept  $H_0$  and reject  $H_1$

**CONCLUSION:**

Hence, Both Drug A and Drug B are equally effective in suppressing the pain.

**PROJECT 3.7 (Friedman F Test):**

A survey was conducted in four hospitals in a particular city to obtain the number of babies born over a 12 months' period. This time period was divided into four seasons to test the hypothesis that the birth rate is constant over all the four seasons. The results of the survey were as follows:

	<i>No. of Births</i>			
<i>Hospital</i>	<i>Winter</i>	<i>Spring</i>	<i>Summer</i>	<i>Fall</i>
<i>A</i>	92	72	94	77
<i>B</i>	15	16	10	17
<i>C</i>	58	71	51	62
<i>D</i>	19	26	20	18

Analyze the data using Friedman two ANOVA test.

**PROCEDURE:**

1. Enter the data
2. Select Analyze => Nonparametric Test => Legacy Dialogs => K Related sample.
3. Click Test Variable => Select Friedman.
4. Click Continue => Ok.

**SOLUTION:**

***STEP I: Null Hypothesis ( $H_0$ )***

$$Md_1 = Md_2 = Md_3 = Md_4$$

i.e. The birth rate is constant over all four seasons.

***STEP II: Alternative Hypothesis ( $H_1$ )***

$$Md_1 \neq Md_2 = Md_3 = Md_4$$

i.e. The birth rate isn't constant over all four seasons.

***STEP III: Test Statistics***

$$F_r = \frac{12}{nk(k+1)} \sum_{i=1}^k R_i^2 - 3n(k+1)$$

***If tie occurs, then test statistics is***

$$F_r = \frac{\frac{12}{nk(k+1)} \sum_{i=1}^k R_i^2 - 3n(k+1)}{1 - \sum \frac{(t_i^3 - t_i)}{n(k^3 - k)}}$$

**Calculation (From SPSS):**

<i>Test Statistics<sup>a</sup></i>			
<i>N</i>			4
<i>Chi-Square</i>			.900
<i>df</i>			3
<i>Asymp. Sig.</i>			.825
<i>Monte Carlo Sig.</i>	<i>Sig.</i>		.750
	<i>95% Confidence Interval</i>	<i>Lower Bound</i>	.326
		<i>Upper Bound</i>	1.000

***a. Friedman Test***

***So, P Value = 0.825***

***STEP IV : Critical Value***

$$\alpha = 5\% = 0.05$$

***STEP V : Decision***

Since  $P > \alpha$ , accept  $H_0$  and reject  $H_1$

**CONCLUSION:**

Hence, the birth rate is constant over all four seasons.