# Robust Weighting and Matching Techniques for Causal Inference in Observational Studies with Continuous Treatment

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# Contents

# Chapter 1

# Chapter One Title

hello $\mathbb{R}$

# Chapter 2

# Convex Analysis

We begin by defining convex sets

**Definition 1** *A subset $\Omega \subseteq \mathbb{R}^n$ is called CONVEX if we have $\lambda x + (1-\lambda) y \in \Omega$ for all $x, y \in \Omega$ and $\lambda \in (0,1)$.*

Clearly, the line segment $[a,b] := \{\lambda a + (1-\lambda) b \mid \lambda \in [0,1]\}$ is contained in $\Omega$ for all $a, b \in \Omega$ if and only if $\Omega$ is a convex set.

Next we define convex functions.

The concept of convex functions is closely related to convex sets.

The line segment between two points on the graph of a convex function lies on or above and does not intersect the graph.

In other words: The area above the graph of a convex function $f$ is a convex set, i.e. the *epigraph* $\mathrm{epi}(f) := \{(x, \alpha) \in \mathbb{R}^n \times \mathbb{R} \mid f(x) \leq \alpha\}$ is a convex set in $\mathbb{R}^{n+1}$.

Often an equivalent characterisation of convex functions is more useful.

**Theorem 1** *The convexity of a function $f : \mathbb{R}^n \to \overline{\mathbb{R}}$ on $\mathbb{R}^n$ is equivalent to the following statement:*

*For all $x, y \in \mathbb{R}^n$ and $\lambda \in (0,1)$ we have*

$$f(\lambda x + (1-\lambda) y) \leq \lambda f(x) + (1-\lambda) f(y). \tag{2.1}$$

# Chapter 3

# Random Matrix Inequality

**Theorem 2** *Let* $(A_k)_{1\leq k\leq n} \subseteq \mathbb{R}^{d_1\times d_2}$ *be a finite sequence of independent, random matrices. Assume that*

$$\mathbb{E}(A_k) = 0 \quad \textit{and} \quad \|A_k\| \leq L \quad \textit{for each} \quad k \in \{1, \ldots, n\}. \tag{3.1}$$

*Introduce the random matrix*

$$S := \sum_{k=1}^{n} A_k. \tag{3.2}$$

*Let* $v(S)$ *be the matrix variance statistic of the sum:*

$$v(S) : = \max\left\{\left\|\mathbb{E}(SS^T)\right\|, \left\|\mathbb{E}(S^T S)\right\|\right\} \tag{3.3}$$

$$= \max\left\{\left\|\sum_{k=1}^{n}\mathbb{E}(A_k A_k^T)\right\|, \left\|\sum_{k=1}^{n}\mathbb{E}(A_k^T A_k)\right\|\right\}. \tag{3.4}$$

*Then*

$$\mathbb{E}\,\|S\| \leq \sqrt{2v(S)\log(d_1+d_2)} + \frac{1}{3}L\log(d_1+d_2). \tag{3.5}$$

*Furthermore, for all* $t \geq 0$,

$$\mathbb{P}(\|S\| \geq t) \geq (d_1+d_2)\exp\left(\frac{-t^2/2}{v(S)+Lt/3}\right). \tag{3.6}$$