CV Assignment 1

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H 1.1 Edge Detection.

- Given 2D signal (5 × 5)

$$
\begin{array}{cccccc}
 & & 1 & 2 & 3 & \\
0 & & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
5 & 2 & 5 & 0 & 0 & 0 \\
5 & 3 & 5 & 0 & 0 & 0 \\
5 & & 5 & 0 & 0 & 0
\end{array}
$$

(a) $$[f_x]_{i,j} = \frac{f_{i+1,j} - f_{i-1,j}}{2}$$

$$[f_y]_{i,j} = \frac{f_{i,j+1} - f_{i,j-1}}{2}$$

$$\therefore \quad \partial_x = \frac{1}{2}\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \longrightarrow (1)$$

$$\partial_y = \frac{1}{2}\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \longrightarrow (2)$$

(∴) we apply the above equations ^{1} to 2D signal

For $i = 1$ $j = 1$.

$$[f_a]_{1,1} = \frac{1}{2}[-1 \times 0 + 0 \times 0 + 1 \times 0]$$

$$= 0.$$

<u>Similarly</u>:

For $i = 1$ $j = 2$ $\quad [f_a]_{1,2} = 0.$

For $i = 1$ $j = 3$ $\quad [f_a]_{1,3} = 0.$

For $i = 2$ $j = 3$. $\quad [f_a]_{2,3} = 0$

For $i = 3$ $j = 3$ $\quad [f_a]_{3,3} = 0.$

For $i = 2$ $j = 1$.

$$[f_a]_{2,1} = \frac{1}{2}[-1 \times 5 + 0 \times 5 + 1 \times 0]$$

$$= -2.5.$$

Similarly for $i = 3$ $j = 1$,

$$[f_a]_{3,1} = -2.5.$$

For $i = 2$ $j = 2$,

$$[f_a]_{2,2} = \frac{1}{2}[-1 \times 5 + 0 \times 5 + 1 \times 0]$$

$$= -2.5.$$

Similarly for $i = 3$, $j = 2$,

$$\therefore [f_a]_{i,j} = \begin{bmatrix} 0 & 0 & 0 \\ -2.5 & -2.5 & 0 \\ -2.5 & -2.5 & 0 \end{bmatrix}$$

Now apply equation (2) also 2D signal:

For $i=1$ $j=1$

$$[f_y]_{1,1} = \frac{-1\times 0 + 0\times 0 + 1\times 5}{2}$$

$$\Rightarrow 2.5.$$

For $i=2$ $j=1$.

$$[f_y]_{2,1} = \frac{1}{2}[-1\times 0 + 0\times 5 + 1\times 5]$$

$$= 2.5$$

For $i=3$ $j=1$

$$[f_y]_{3,1} = \frac{1}{2}[-1\times 5 + 0\times 5 + 1\times 5]$$

$$= 0.$$

For $i=1$ $j=2$

$$[f_y]_{1,2} = \frac{1}{2}[-1\times 0 + 0\times 0 + 1\times 0] \Rightarrow 0$$

Similarly

for $i=2$ $j=2$ $[f_y]_{2,2} = 0$

for $i=3$ $j=2$ $[f_y]_{3,2} = 0$

for $i=1$ $j=3$ $[f_y]_{1,3} = 0$

for $i=2$ $j=3$ $[f_y]_{2,3} = 0$

for $i=3$ $j=3$ $[f_y]_{3,3} = 0$

$$[f_y]_{i,j} = \begin{bmatrix} 2.5 & 0 & 0 \\ 2.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[f_x]_{i,j} = \begin{bmatrix} 0 & 0 & 0 \\ -2.5 & -2.5 & 0 \\ -2.5 & -2.5 & 0 \end{bmatrix}$$

$$[f_y]_{i,j} = \begin{bmatrix} 2.5 & 0 & 0 \\ 2.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This allows us to compute the gradient $[\nabla f]_{ij} = ([f_x]_{ij}, [f_y]_{i,j})^T$.

(b) Given $\rho = 0$: Structure Tensor.

$$[J_0]_{i,j} = [\nabla f]_{ij} \cdot [\nabla f]_{ij}^T$$

$$= ([f_x]_{ij}, [f_y]_{i,j})^T \cdot [([f_x]_{ij}, [f_y]_{i,j})^T]^T$$

$$[J_0]_{i,j} = \begin{bmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 6.25 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 6.25 & -6.25 \\ -6.25 & 6.25 \end{pmatrix} & \begin{pmatrix} 6.25 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 6.25 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 6.25 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{bmatrix}$$

(c). The classification is based on following :-

$\lambda_1 \approx 0 \; ; \; \lambda_2 \approx 0 \Rightarrow$ flat.

$\lambda_1 >> 0 \; ; \; \lambda_2 \approx 0 \Rightarrow$ edge.

$\lambda_1 >> 0 \; ; \; \lambda_2 >> 0 \Rightarrow$ corner.

$\therefore$ Eigenvalue analysis :-

(i). $\begin{pmatrix} 0 & 0 \\ 0 & 6.25 \end{pmatrix} \Rightarrow \vec{v_1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ & $\vec{v_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

since $\lambda_1 > \lambda_2$ always, we have :-

$\lambda_1 = 6.25 \; ; \; \lambda_2 = 0 \Rightarrow$ Edge.

(ii). $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \lambda_1 = 0 \; ; \; \lambda_2 = 0 \Rightarrow$ flat.

(iii). $\begin{pmatrix} 6.25 & -6.25 \\ -6.25 & 6.25 \end{pmatrix} \Rightarrow \vec{v_1} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ & $\vec{v_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow$

$\therefore \lambda_1 = 12.5$ & $\lambda_2 = 0 \Rightarrow$ Edge.

(iv). $\begin{pmatrix} 6.25 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \lambda_1 = 6.25 \; ; \; \lambda_2 = 0 \Rightarrow$ Edge.

$\therefore$ The classification is as follows :-

Edge	Flat	Flat
Edge	Edge	Flat
Edge	Edge	Flat

(d). Given the kernel :-

$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

The convolution operation then becomes :-

$$J_p = \frac{1}{16}\begin{pmatrix} 0 & 0 \\ 0 & 6.25 \end{pmatrix} + \frac{1}{8}\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{16}\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$+ \frac{1}{8}\begin{pmatrix} 6.25 & -6.25 \\ -6.25 & 6.25 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 6.25 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{8}\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$+ \frac{1}{16}\begin{pmatrix} 6.25 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{8}\begin{pmatrix} 6.25 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{16}\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

$$= \begin{pmatrix} 6.25(1/8 + 1/4 + 1/16 + 1/8) & 6.25(-1/8) \\ 6.25(-1/8) & 6.25(1/16 + 1/8) \end{pmatrix}$$

$$= \frac{6.25}{16}\begin{pmatrix} 9 & -2 \\ -2 & 3 \end{pmatrix}$$

Now, $\det\left[\frac{6.25}{16}\begin{pmatrix} 9 & -2 \\ -2 & 3 \end{pmatrix}\right] = \frac{6.25}{16}(27-4) = 8.98 > 0$

also, $\mathrm{tr}\left[\frac{6.25}{16}\begin{pmatrix} 9 & -2 \\ -2 & 3 \end{pmatrix}\right] = \frac{6.25}{16}(9+3) = 4.68 > 0.$

since determinant & trace is greater than zero, there exists a corner.

Without convolution, this wasn't detected. //

(e). $|\nabla f_R + \nabla f_G + \nabla f_B| = \left|\begin{pmatrix} -2.5 \\ 0 \end{pmatrix} + \begin{pmatrix} 2.5 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right| = \left|\begin{pmatrix} 0 \\ 0 \end{pmatrix}\right| = 0$

since the norm of sum of gradients $= 0$, we cannot detect an edge. (same direction, but opposite orientation).

(f). $\left|\left(|\nabla f_R|, |\nabla f_G|, |\nabla f_B|\right)^T\right| = \left|\begin{pmatrix} 2.5 \\ 2.5 \\ 0 \end{pmatrix}\right| = \sqrt{12.5}$.

Here, we eliminate orientation & hence we detect an edge.

(g). $\nabla f_R \nabla f_R^T + \nabla f_G \nabla f_G^T + \nabla f_B \nabla f_B^T$

$$= \begin{pmatrix} 6.25 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 6.25 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 12.5 & 0 \\ 0 & 0 \end{pmatrix}.$$

On eigenvalue analysis, $\lambda_1 = 12.5$ & $\lambda_2 = 0$.

$\therefore$ We detect an edge. //

Therefore, expression (e) doesn't give info., whereas, (f) & (g) are invariant from orientation and hence provide info. on edge detection. //

H 1.2 a) Given image :-

$$\begin{bmatrix} 0 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \\ 0 & 0 & 2 & 1 \\ 3 & 0 & 1 & 0 \end{bmatrix} \qquad d = (-1,-1)^T$$

j

i

<u>Concurrence matrix :-</u>

$$\frac{1}{9} \times \begin{array}{c} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \\ \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \end{array}$$

$\downarrow$ i (reference pixel)

$\rightarrow$ j (neighbour pixel)

<u>Largest probability :-</u>

With $d = (-1,-1)^T$, the configuration $(1,0)$ was most frequent

$$\boxed{P_{1,0} = \frac{2}{9}}$$

<u>Contrast :-</u>

$$\text{Contrast} = \sum_{i,j} (i-j)^2 \, P_{i,j}$$

$$= \Big[\underset{00}{0} + \underset{01}{1\left(\tfrac{1}{9}\right)} + \underset{02}{4\left(\tfrac{1}{9}\right)} + \underset{03}{0} +$$

$$\underset{10}{1\left(\tfrac{2}{9}\right)} + \underset{11}{0} + \underset{12}{0} + \underset{13}{4\left(\tfrac{1}{9}\right)} +$$

$$\underset{20}{0} + \underset{21}{1\left(\tfrac{1}{9}\right)} + \underset{22}{0} + \underset{23}{0} +$$

$$\underset{30}{0} + \underset{31}{0} + \underset{32}{0} + \underset{33}{0} \Big]$$

$$= \frac{1}{9} + \frac{4}{9} + \frac{2}{9} + \frac{4}{9} + \frac{1}{9} = \frac{12}{9}$$

$$\text{Contrast} = \frac{4}{3} = 1.33$$

2.b) when $d = (1,1)^T$

Concurrence matrix :-

$$\frac{1}{9} \times \begin{array}{c|c} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} & \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \\ \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} & \end{array}$$

$\downarrow$ i (reference pixel)

$\overrightarrow{j}$ (neighbour pixel)

Largest probability :-

with $d = (1,1)^T$, the configuration $(0,1)$ was most frequent

$$P_{0,1} = \frac{2}{9}$$

(same as that of d with $(-1,-1)^T$.

Contrast :-

$$\sum_{i,j} (i-j)^2 P_{i,j} = \Big(0 + 1\left(\frac{2}{9}\right) + 0 + 0 + 1\left(\frac{1}{9}\right) + 0 + 1\left(\frac{1}{9}\right) + 0 + 4\left(\frac{1}{9}\right) + 0 + 0 + 0 + 0 + 4\left(\frac{1}{9}\right) + 0 + 0 \Big)$$

$$= \frac{2}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{4}{9}$$

$$= \frac{12}{9}$$

$$= \frac{4}{3}$$

Contrast $= 1.33$

$\Rightarrow$ Same as previous case with $d = (-1,-1)^T$.