Section 1

a.

CustomerID	CustomerName	OrderID	OrderDate	ProductID	ProductName	Price	OrderedQuantity	TotalAmount	PersonID	PersonName
C001	Alice	R101	12-01-24	P123	Laptop	500	2	1000	E301	John
C002	David	R102	12-01-24	P124	Laptop Charger	25	1	25	E301	John
C003	Pearl	R103	13-01-24	P126	Printer	250	3	750	E305	Sam
C004	Samantha	R104	14-01-24	P127	Desktop	150	15	2250	E306	Jessica
C004	Samantha	R105	14-01-24	P128	CPU	75	15	1125	E310	Robin

b. CustomerID -> CustomerName

assumption :- by the help of the CustomerID we can figure out the name of the customer as CustomerID is an unique ID which functionally determines the CustomerName

OrderID -> OrderDate, CustomerID, PersonID

assumption :- the OrderID is an unique ID given to each order hence there will be only a specific date, customer for that order and the order handler which means that OrderDate, CustomerID and PersonID, all can be determined by OrderID

ProductID -> ProductName, Price

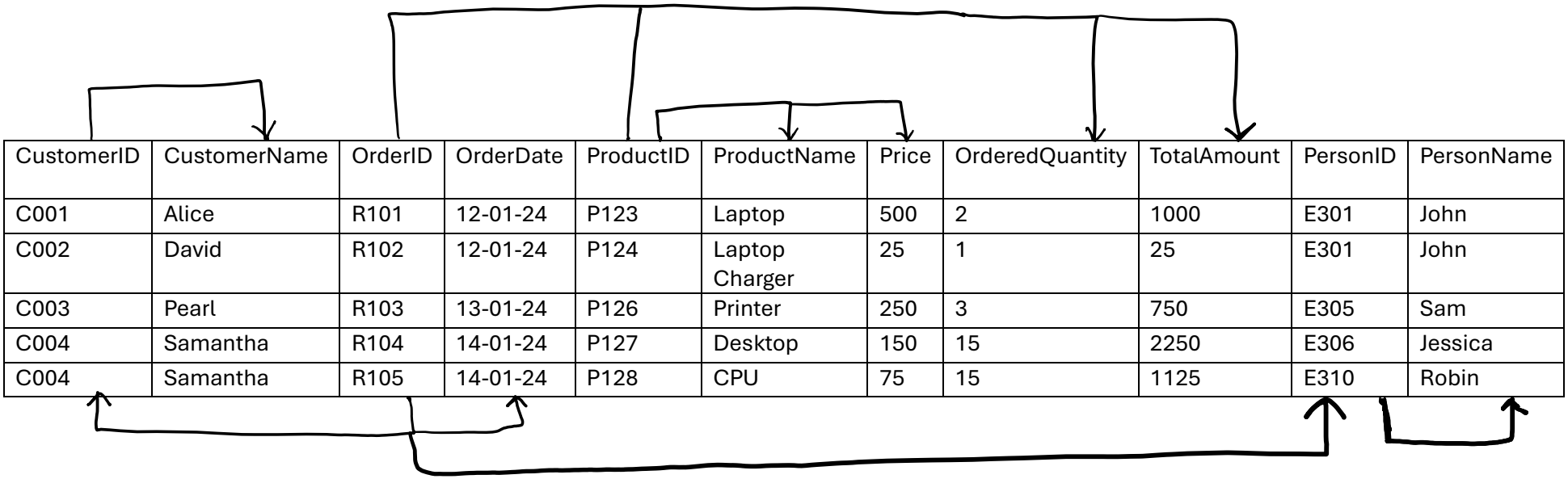
assumption :- by the help of the ProductID we can figure out the name and price of the product as ProductID is an unique ID which functionally determines the ProductName and Price

OrderID, ProductID -> OrderQuantity, TotalAmount

assumption :- every order can have different quantity ordered hence the combination of OrderID and ProductID can determine the OrderedQuantity and the TotalAmount for each order

PersonID -> PersonName

assumption :- each order handled has a person with an unique ID which will PersonID that determines here their name, called PersonName



CustomerID	CustomerName	OrderID	OrderDate	ProductID	ProductName	Price	OrderedQuantity	TotalAmount	PersonID	PersonName
C001	Alice	R101	12-01-24	P123	Laptop	500	2	1000	E301	John
C002	David	R102	12-01-24	P124	Laptop Charger	25	1	25	E301	John
C003	Pearl	R103	13-01-24	P126	Printer	250	3	750	E305	Sam
C004	Samantha	R104	14-01-24	P127	Desktop	150	15	2250	E306	Jessica
C004	Samantha	R105	14-01-24	P128	CPU	75	15	1125	E310	Robin

Section 2

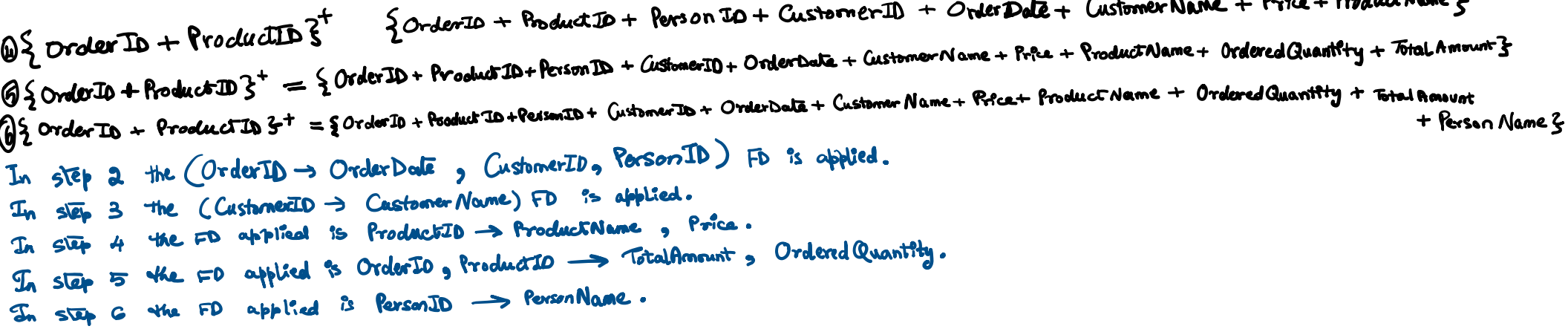
a. Primary key is the unique key identifying each record. The closure (+) is essentially the full set of attributes that can be determined from a set of known attributes, for a given database, using its functional dependencies. For closure I have chosen OrderID and ProductID. As noticed even the CustomerID is a primary key but the reason it was not chosen for closure is that if we can identify OrderID, then we can also determine CustomerID with the help of OrderID.

OrderID + ProductID :

① {OrderID + ProductID}$^+$ = {OrderID + ProductID}

② {OrderID + ProductID}$^+$ = {OrderID + ProductID + PersonID + CustomerID + OrderDate}

③ {OrderID + ProductID}$^+$ = {OrderID + ProductID + PersonID + CustomerID + OrderDate + CustomerName}



This shows they are the minimal super key hence the primary key.

b. Since the primary keys are OrderID and ProductID, the primary attributes are OrderID and ProductID. The non – prime attributes will PersonID, PersonName, CustomerID, OrderDate, ProductName, OrderedQuantity, TotalAmount, Price and CustomerName.

Section 3

Full key FD is a functional dependency what occurs when a primary key functionally determines the column of a relation and no separate component of the primary key partially determines the same column.

Full key FD: OrderID, ProductID -> TotalAmount, OrderedQuantity

As TotalAmount and OrderedQuantity can not determined solely just by OrderID or ProductID, this makes them fully dependent on both OrderID and ProductID.

Partial functional dependency is what occurs when a proper subset of the primary key in a relation determines non-prime attributes.

Partial functional dependency: OrderID -> OrderDate, CustomerID, PersonID

It has a prime attribute determining a non prime attribute.

A transitive functional dependency occurs a non prime attribute depends on another non prime attribute.

Transitive functional dependency: CustomerID -> CustomerName AND PersonID -> PersonName

They are TFD because CustomerName is indirectly dependent on OrderID through CustomerID which applies to PersonID and PersonName as they also can be determined by OrderID.

Section 4

a. Table 1: Order

CustomerID	CustomerName	OrderID	PersonID	PersonName	OrderDate
C001	Alice	R101	E301	John	12-01-24
C002	David	R102	E301	John	12-01-24
C003	Pearl	R103	E305	Sam	13-01-24
C004	Samantha	R104	E306	Jessica	14-01-24
C004	Samantha	R105	E310	Robin	14-01-24

Primary key: CustomerID, OrderID, PersonID

Table 2: Product

ProductID	ProductName	Price
P123	Laptop	500
P124	Laptop Charger	25
P126	Printer	250
P127	Desktop	150
P128	CPU	75

Primary key: ProductID

Table 3: Total Amount

OrderID	ProductID	OrderedQuantity	TotalAmount
R101	P123	2	1000
R102	P124	1	25
R103	P126	3	750
R104	P127	15	2250
R105	P128	15	1125

Primary key: OrderID, ProductID

Foreign key: OrderID, ProductID

b. To verify this is lossless or not, we can see that all the tables become into one without losing any data. The dependencies are preserved with the help of foreign keys.

CustomerID	CustomerName	OrderID	OrderDate	ProductID	ProductName	Price	OrderedQuantity	TotalAmount	PersonID	PersonName
C001	Alice	R101	12-01-24	P123	Laptop	500	2	1000	E301	John
C002	David	R102	12-01-24	P124	Laptop Charger	25	1	25	E301	John
C003	Pearl	R103	13-01-24	P126	Printer	250	3	750	E305	Sam
C004	Samantha	R104	14-01-24	P127	Desktop	150	15	2250	E306	Jessica
C004	Samantha	R105	14-01-24	P128	CPU	75	15	1125	E310	Robin
A	B	C	D	E	F	G	H	I	J	K

$R = \{A, B, C, D, E, F, G, H, I, J, K\}$

$R_1 = A, B, C, D, J, K$

$R_2 = E, F, G$

$R_3 = C, E, H, I$

3 conditions need to be fulfilled to determine whether this is lossy or lossless:

a) Union of attributes: all relations should equal to R

$Union = R_1 \cup R_2$

$= \{A, B, C, D, J, K\} \cup \{E, F, G\}$

$R_a = \{A, B, C, D, J, K, E, F, G\}$

$= R_a \cup R_3 = \{A, B, C, D, J, K, E, F, G\} \cup \{C, E, H, I\}$

$= \{A, B, C, D, E, F, G, H, I, J, K\} = R$

b) Intersection of attributes: no intersection should lead to null

$Intersection = R_1 \cap R_3$

$= \{A, B, C, D, J, K\} \cap \{C, E, H, I\}$

$= \{C\}$

$= R_2 \cap R_3$

$= \{E, F, G\} \cap \{C, E, H, I\}$

$= \{E\}$

c) A common attribute: there should be something common between all the relations

The common attributes are checked between 2 relations at a time.
Hence, R_1 and R_3 have {C} as a common attribute and R_2 and R_3 have attribute {E} in common.
This means the condition is satisfied.
Based on all conditions, it is lossless since all conditions are satisfied.

Section 5

a. Table 1: Customers

CustomerID	CustomerName
C001	Alice
C002	David
C003	Pearl
C004	Samantha

Primary key: CustomerID

Table 2: Orders

OrderID	OrderDate	CustomerID
R101	12-01-24	C001
R102	12-01-24	C002
R103	13-01-24	C003
R104	14-01-24	C004
R105	14-01-24	C004

Primary key: OrderID

Foreign key: CustomerID

Table 3: Products

ProductID	ProductName	Price
P123	Laptop	500
P124	Laptop Charger	25
P126	Printer	250
P127	Desktop	150
P128	CPU	75

Primary key: ProductID

Table 4: Total Amount

OrderID	ProductID	OrderedQuantity	TotalAmount
R101	P123	2	1000
R102	P124	1	25
R103	P126	3	750
R104	P127	15	2250
R105	P128	15	1125

Primary key: OrderID, ProductID

Foreign key: ProductID, OrderID

Table 5: Person

PersonID	PersonName
E301	John
E305	Sam
E306	Jessica
E310	Robin

Primary key: PersonID

b. To verify this is lossless or not, we can see that all the tables become into one without losing any data. The dependencies are preserved with the help of foreign keys.

CustomerID	CustomerName	OrderID	OrderDate	ProductID	ProductName	Price	OrderedQuantity	TotalAmount	PersonID	PersonName
C001	Alice	R101	12-01-24	P123	Laptop	500	2	1000	E301	John
C002	David	R102	12-01-24	P124	Laptop Charger	25	1	25	E301	John
C003	Pearl	R103	13-01-24	P126	Printer	250	3	750	E305	Sam
C004	Samantha	R104	14-01-24	P127	Desktop	150	15	2250	E306	Jessica
C004	Samantha	R105	14-01-24	P128	CPU	75	15	1125	E310	Robin
A	B	C	D	E	F	G	H	I	J	K

$R = \{A, B, C, D, E, F, G, H, I, J, K\}$

$A = B$
$J = K$
$E = H, I$
$E = F, G$
$C = A, D$

$R_1 = \{A, B\}$
$R_2 = \{J, K\}$
$R_3= \{C, E, H, I\}$
$R_4 = \{E, F, G\}$
$R_5 = \{C, A, D\}$

3 conditions need to be fulfilled to determine whether this is lossy or lossless:

a) Union of attributes: all relations should equal to R

$$\text{Union} = R_1 \cup R_2$$
$$= \{A, B\} \cup \{J, K\} \qquad R_a = \{A, B, J, K\}$$
$$= R_a \cup R_3 = \{A, B, J, K\} \cup \{C, E, H, I\} \qquad R_b = \{A, B, J, K, C, E, H, I\}$$
$$= R_b \cup R_4 = \{A, B, J, K, C, E, H, I\} \cup \{E, F, G\} \qquad R_c = \{A, B, J, K, C, E, H, I, F, G\}$$
$$= R_c \cup R_5 = \{A, B, J, K, C, E, H, I, F, G\} \cup \{C, A, D\}$$
$$= \{A, B, C, D, E, F, G, H, I, J, K\}$$
$$= R$$

b) Intersection of attributes: no intersection should lead to null

$$= R_1 \cap R_5$$
$$= \{A, B\} \cap \{C, A, D\}$$
$$= \{A\}$$
$$= R_3 \cap R_5$$
$$= \{C, E, H, I\} \cap \{C, AD\}$$
$$= \{C\}$$
$$= R_3 \cap R_4$$
$$= \{C, E, H, I\} \cap \{E, F, G\}$$
$$= \{E\}$$

R2 is not used because it does not interact with any of the other tables because it does not share any common attributes with other relations. Therefore, it does not relevant for the lossless join check.

c) A common attribute: there should be something common between all the relations

Keeping R_2 aside between all other relations there are {A}, {C} and {E} attributes.

Based all the conditions, it is lossless as all the conditions are satisfied.

c. Find the total quantity of each product sold

ProductID	ProductName	SUM(OrderedQuantity)
P123	Laptop	2
P124	Laptop Charger	1
P126	Printer	3
P127	Desktop	15
P128	CPU	15

Retrieve all customer orders along with the total price

OrderID	CustomerID	CustomerName	Total
R101	C001	Alice	1000.00
R102	C002	David	25.00
R103	C003	Pearl	750.00
R104	C004	Samantha	2250.00
R105	C004	Samantha	1125.00

Find the top 3 customers who spent the most

CustomerID	CustomerName	Total
C004	Samantha	3375.00
C001	Alice	1000.00
C003	Pearl	750.00