

# Oct 30 update

## Renyi divergence calculation (fixed code)

- Rényi divergence for 2D continuous distributions

The formula is given by

$$D_\alpha(P\|Q) = \frac{1}{\alpha - 1} \log \int_{\mathbb{R}^2} p(x, y)^\alpha q(x, y)^{1-\alpha} dx dy$$

For Monte Carlo estimation:

$$D_\alpha(P\|Q) \approx \frac{1}{\alpha - 1} \log \frac{1}{n} \sum_{i=1}^n \frac{p(x_i, y_i)^{\alpha-1}}{q(x_i, y_i)^{\alpha-1}}, \quad (x_i, y_i) \sim P$$

- If  $y$  is binary, then the joint distribution  $p(x, y)$  can be decomposed as  $p(y|x)p(x)$ , and we have

$$D_\alpha(P\|Q) = \frac{1}{\alpha - 1} \log \sum_y p(y)^\alpha q(y)^{1-\alpha} \int p(x \mid y)^\alpha q(x \mid y)^{1-\alpha} dx.$$

- In implementation, we use `gaussian_kde` from `scipy` to estimate the density functions and sample from the estimated distributions to compute the divergence.