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Matrices

1

a) $C+D=\begin{bmatrix}2&1&1\\1&0&4\\5&1&0\end{bmatrix}+\begin{bmatrix}1&2&3\\-1&0&1\\2&-3&-4\end{bmatrix}=\begin{bmatrix}2+1&1+2&1+3\\1-1&0+0&4+1\\5+2&1-3&0-4\end{bmatrix}=$

$=\begin{bmatrix}3&3&4\\0&0&5\\7&-2&-4\end{bmatrix}$

b) $A-D=\begin{bmatrix}1&0&2\\3&2&1\\4&0&3\end{bmatrix}-\begin{bmatrix}1&2&3\\-1&0&1\\2&-3&-4\end{bmatrix}=\begin{bmatrix}1-1&0-2&2-3\\3+1&2-0&1-1\\4-2&0+3&3+4\end{bmatrix}=$

$=\begin{bmatrix}0&-2&-1\\4&2&0\\2&3&7\end{bmatrix}$

c) $A+A^T=\begin{bmatrix}1&0&2\\3&2&1\\4&0&3\end{bmatrix}+\begin{bmatrix}1&3&4\\0&2&0\\2&1&3\end{bmatrix}=\begin{bmatrix}1+1&0+3&2+4\\3+0&2+2&1+0\\4+2&0+1&3+3\end{bmatrix}=$

$=\begin{bmatrix}2&3&6\\3&4&1\\6&1&6\end{bmatrix}$

d) $3B=3\cdot\begin{bmatrix}1&1&3&0\\4&2&1&1\\3&2&0&1\end{bmatrix}=\begin{bmatrix}3\cdot1&3\cdot1&3\cdot3&3\cdot0\\3\cdot4&3\cdot2&3\cdot1&3\cdot1\\3\cdot3&3\cdot2&3\cdot0&3\cdot1\end{bmatrix}=\begin{bmatrix}3&3&9&0\\12&6&3&3\\9&6&0&3\end{bmatrix}$

e) $4A + 3C = 4 \cdot \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix} + 3 \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix} =$

$= \begin{bmatrix} 4 & 0 & 8 \\ 12 & 8 & 4 \\ 16 & 0 & 12 \end{bmatrix} + \begin{bmatrix} 6 & 3 & 3 \\ 3 & 0 & 12 \\ 15 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 3 & 11 \\ 15 & 8 & 16 \\ 31 & 3 & 12 \end{bmatrix}$

f) $2A - 3B$ = Not possible.
Matrices must have same dimensions for addition/subtraction.

g) $3D - 2A = 3 \cdot \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 2 & -3 & -4 \end{bmatrix} - 2 \cdot \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix} =$

$= \begin{bmatrix} 3 & 6 & 9 \\ -3 & 0 & 3 \\ 6 & -9 & -12 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 4 \\ 6 & 4 & 2 \\ 8 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 6 & 5 \\ -9 & -4 & 1 \\ -2 & -9 & -18 \end{bmatrix}$

h) $C + D^T = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & -3 \\ 3 & 1 & -4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 3 & 0 & 1 \\ 8 & 2 & -4 \end{bmatrix}$

i) $A - C^T = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 5 \\ 1 & 0 & 1 \\ 1 & 4 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -3 \\ 2 & 2 & 0 \\ 3 & -4 & 3 \end{bmatrix}$

j) $3A - 2C^T = 3 \cdot \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix} - 2 \cdot \begin{bmatrix} 2 & 1 & 5 \\ 1 & 0 & 1 \\ 1 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 6 \\ 9 & 6 & 3 \\ 12 & 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 10 \\ 2 & 0 & 2 \\ 2 & 8 & 0 \end{bmatrix} =$

$= \begin{bmatrix} -1 & -2 & -4 \\ 7 & 6 & 1 \\ 10 & -8 & 9 \end{bmatrix}$

(2)

a) $\begin{bmatrix} x & 3 & (2x-1) \\ y & 4 & 4y \end{bmatrix} = \begin{bmatrix} 2x-4 & z & 7 \\ 1 & w+1 & 3y+1 \end{bmatrix}$

For two matrices to be equal, all corresponding elements must be equal.

Position [1; 1]: $x = 2x-4 \Rightarrow -x = -4 \Rightarrow x = 4$ ✓

Position [1; 2]: $3 = z \Rightarrow z = 3$

Position [1; 3]: $2x-1 = 7 \Rightarrow 2x = 8 \Rightarrow x = 4$ ✓

Position [2; 1]: $y = 1 \Rightarrow y = 1$ ✓

Position [2; 2]: $4 = w+1 \Rightarrow 3 = w \Rightarrow w = 3$

Position [2; 3]: $4y = 3y+1 \Rightarrow y = 1$ ✓

Answer: $x = 4$; $y = 1$; $z = 3$; $w = 3$

b) Calculate left side of equation first

$$3 \cdot \begin{bmatrix} x & 4 \\ 4y & w \end{bmatrix} - 2 \begin{bmatrix} 4x & 2z \\ -3 & -2w \end{bmatrix} = \begin{bmatrix} 3x & 12 \\ 12y & 3w \end{bmatrix} - \begin{bmatrix} 8x & 4z \\ -6 & -4w \end{bmatrix} =$$

$$= \begin{bmatrix} 3x-8x & 12-4z \\ 12y+6 & 3w+4w \end{bmatrix} = \begin{bmatrix} -5x & 12-4z \\ 12y+6 & 7w \end{bmatrix}$$

$$\begin{bmatrix} -5x & 12-4z \\ 12y+6 & 7w \end{bmatrix} = \begin{bmatrix} 20 & 20 \\ 6 & 14 \end{bmatrix}$$

Pos [1; 1]: $-5x = 20 \Rightarrow x = -4$

Pos [1; 2]: $12 - 4z = 20 \Rightarrow 4z = -8 \Rightarrow z = -2$

Pos [2; 1]: $12y + 6 = 6 \Rightarrow 12y = 0 \Rightarrow y = 0$

Pos [2; 2]: $7w = 14 \Rightarrow w = 2$

Answer: $x = -4$; $y = 0$; $z = -2$; $w = 2$

(3)

a) $[1\ 2\ 3]\cdot\begin{bmatrix}4\\5\\6\end{bmatrix}=[1\cdot4+2\cdot5+3\cdot6]=[4+10+18]=32$

b) $[2\ 0\ 3]\cdot\begin{bmatrix}0\\1\\-3\end{bmatrix}=[2\cdot0+0\cdot1-9]=-9$

c) $[1\ 2]\cdot\begin{bmatrix}3&5\\4&6\end{bmatrix}=[1\cdot3+2\cdot4\quad 1\cdot5+2\cdot6]=[11\ \ 17]$

d) $[3\ 0]\cdot\begin{bmatrix}1&2\\4&5\end{bmatrix}=[3\cdot1+0\cdot4\quad 3\cdot2+0\cdot5]=[3\ \ 6]$

(4)

a) AB $3\times3\cdot3\times4$ => Result 3×4

Row 1: $[1\cdot1+0\cdot4+2\cdot3,\ 1\cdot1+0\cdot2+2\cdot2,\ 1\cdot3+0\cdot1+2\cdot0,\ 1\cdot0+0\cdot1+2\cdot1]=$
$=[7\ 5\ 3\ 2]$

Row 2: $[3\cdot1+2\cdot4+1\cdot3,\ 3\cdot1+2\cdot2+1\cdot2,\ 3\cdot3+2\cdot1+1\cdot0,\ 3\cdot0+2\cdot1+1\cdot1]=$
$=[14\ 9\ 11\ 3]$

Row 3: $[4\cdot1+0\cdot4+3\cdot3,\ 4\cdot1+0\cdot2+3\cdot2,\ 4\cdot3+0\cdot1+3\cdot0,\ 4\cdot0+0\cdot1+3\cdot1]=$

$=[13\ 10\ 12\ 3]$

$$AB=\begin{bmatrix}7&5&3&2\\14&9&11&3\\13&10&12&3\end{bmatrix}$$

b) BA $3\times4\cdot3\times3$ Not possible

Columns of B (4) $\neq$ Rows of A (3)

c) DB^T $B^T = \begin{bmatrix} 1 & 4 & 3 \\ 1 & 2 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ $D = 2\times4$ $B^T = 4\times3$

Result 2×3

$$DB^T = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 2 & -1 & 3 & -4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 & 3 \\ 1 & 2 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Row 1: $[1\cdot1+0\cdot1+(-1)\cdot3+3\cdot0,\ 1\cdot4+0\cdot2+(-1)\cdot1+3\cdot1,\ 1\cdot3+0\cdot2+(-1)\cdot0+3\cdot1] =$

$= [-2 \quad 6 \quad 6]$

Row 2: $[2\cdot1+(-1)\cdot1+3\cdot3+(-4)\cdot0,\ 2\cdot4+(-1)\cdot2+3\cdot1+(-4)\cdot1,\ 2\cdot3+(-1)\cdot2+3\cdot0+(-4)\cdot1] =$

$= [10 \quad 5 \quad 0]$

$$DB^T = \begin{bmatrix} -2 & 6 & 6 \\ 10 & 5 & 0 \end{bmatrix}$$

d) CB $2\times3 \cdot 3\times4 \Rightarrow$ Result 2×4

$$\begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 2 & 1 & 1 \\ 3 & 2 & 0 & 1 \end{bmatrix}$$

Row 1: $[1+0+12,\ 1+0+8,\ 3+0+0,\ 0+0+4] = [13 \quad 9 \quad 3 \quad 4]$

Row 2: $[5+4+0,\ 5+2+0,\ 15+1+0,\ 0+1+0] = [9 \quad 7 \quad 16 \quad 1]$

$$CB = \begin{bmatrix} 13 & 9 & 3 & 4 \\ 9 & 7 & 16 & 1 \end{bmatrix}$$

e) BC $3\times4 \cdot 2\times3$ Not possible

Columns of B (4) $\neq$ Rows of C (2)

f) $C \cdot A^T$ $2\times3 \cdot 3\times3$ Result: 2×3

$$A^T = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 0 \\ 2 & 1 & 3 \end{bmatrix}$$

$$C \cdot A^T = \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 0 \\ 2 & 1 & 3 \end{bmatrix}$$

Row 1: $[1+0+8,\ 3+0+4,\ 4+0+12] = [9 \quad 7 \quad 16]$

Row 2: $[5+0+0,\ 15+2+0,\ 20+0+0] = [5 \quad 17 \quad 20]$

$$C \cdot A^T = \begin{bmatrix} 9 & 7 & 16 \\ 5 & 17 & 20 \end{bmatrix}$$

g) AC^T $3\times3 \cdot 3\times2$ Result: 3×2

$$C^T = \begin{bmatrix} 1 & 5 \\ 0 & 1 \\ 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 \\ 0 & 1 \\ 4 & 0 \end{bmatrix}$$

Row 1: $[1+0+8,\ 5+0+0] = [9 \quad 5]$

Row 2: $[3+0+4,\ 15+2+0] = [7 \quad 17]$

Row 3: $[4+0+12,\ 20+0+0] = [16 \quad 20]$

$$AC^T = \begin{bmatrix} 9 & 5 \\ 7 & 17 \\ 16 & 20 \end{bmatrix}$$

h) $C^T D$ $\quad 3\times2 \cdot 2\times4$ Result 3×4

$$\begin{bmatrix} 1 & 5 \\ 0 & 1 \\ 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -1 & 3 \\ 2 & -1 & 3 & -4 \end{bmatrix}$$

Row 1: $[1+10, 0-5, -1+15, 3-20] = [11 \quad -5 \quad 14 \quad -17]$

Row 2: $[0+2, 0-1, 0+3, 0-4] = [2 \quad -1 \quad 3 \quad -4]$

Row 3: $[4+0, 0+0, -4+0, 12+0] = [4 \quad 0 \quad -4 \quad 12]$

$$C^T D = \begin{bmatrix} 11 & -5 & 14 & -17 \\ 2 & -1 & 3 & -4 \\ 4 & 0 & -4 & 12 \end{bmatrix}$$

i) BD^T $\quad 3\times4 \cdot 4\times2$ Result: 3×2

$$\begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 2 & 1 & 1 \\ 3 & 2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & 3 \\ 3 & -4 \end{bmatrix}$$

Row 1: $[1+0-3+0, 2-1+9+0] = [-2 \quad 10]$

Row 2: $[4+0-1+3, 8-2+3-4] = [6 \quad 5]$

Row 3: $[3+0+0+3, 6-2+0-4] = [6 \quad 0]$

$$BD^T = \begin{bmatrix} -2 & 10 \\ 6 & 5 \\ 6 & 0 \end{bmatrix}$$

j) D^2 2×4 2×4 Not possible $4 \neq 2$

k) B^TA $4\times3 \cdot 3\times3$ Result 4×3

$$\begin{bmatrix} 1 & 4 & 3 \\ 1 & 2 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix}$$

Row1: $[1+12+12, 0+8+0, 2+4+9] = [25 \quad 8 \quad 15]$

Row2: $[1+6+8, 0+4+0, 2+2+6] = [15 \quad 4 \quad 10]$

Row3: $[3+3+0, 0+2+0, 6+1+0] = [6 \quad 2 \quad 7]$

Row4: $[0+3+4, 0+2+0, 0+1+3] = [7 \quad 2 \quad 4]$

$$B^TA = \begin{bmatrix} 25 & 8 & 15 \\ 15 & 4 & 10 \\ 6 & 2 & 7 \\ 7 & 2 & 4 \end{bmatrix}$$

l) CA $2\times3 \cdot 3\times3$ Result 2×3

$$\begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix}$$

Row1: $[1+0+16, 0+0+0, 2+0+12] = [17 \quad 0 \quad 14]$

Row2: $[5+3+0, 0+2+0, 10+1+0] = [8 \quad 2 \quad 11]$

$$CA = \begin{bmatrix} 17 & 0 & 14 \\ 8 & 2 & 11 \end{bmatrix}$$

m) $A^2 = A \cdot A \quad 3\times3 \cdot 3\times3$ Result: 3×3

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix}$$

Row1: $[1+0+8,\ 0+0+0,\ 2+0+6] = [9 \ \ 0 \ \ 8]$

Row2: $[3+6+4,\ 0+4+0,\ 6+2+3] = [13 \ \ 4 \ \ 11]$

Row3: $[4+0+12,\ 0+0+0,\ 8+0+9] = [16 \ \ 0 \ \ 17]$

$$A^2 = \begin{bmatrix} 9 & 0 & 8 \\ 13 & 4 & 11 \\ 16 & 0 & 17 \end{bmatrix}$$

n) $A^3 = A^2 \cdot A \quad 3\times3 \cdot 3\times3$ Result: 3×3

$$\begin{bmatrix} 9 & 0 & 8 \\ 13 & 4 & 11 \\ 16 & 0 & 17 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix}$$

Row 1: $[9+0+32,\ 0+0+0,\ 18+0+24] = [41 \ \ 0 \ \ 42]$

Row 2: $[13+12+44,\ 0+8+0,\ 26+4+33] = [69 \ \ 8 \ \ 63]$

Row 3: $[16+0+68,\ 0+0+0,\ 32+0+51] = [84 \ \ 0 \ \ 83]$

$$A^3 = \begin{bmatrix} 41 & 0 & 42 \\ 69 & 8 & 63 \\ 84 & 0 & 83 \end{bmatrix}$$

(5) Find product AB, give observations.

$$\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \cdot \begin{bmatrix} 2 & -6 \\ -1 & 3 \end{bmatrix}$$

Row 1: $[4-4, -12+12] = [0,0]$

Row 2: $[6-6, -18+18] = [0,0]$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{Zero Matrix}$$

$AB = 0$ even though $A \neq 0$ and $B \neq 0$

In regular arithmetic $ab = 0 \Rightarrow a = 0$ or $b = 0$ (or both)

In matrix algebra $AB = 0$ does NOT imply $A = 0$ or $B = 0$

(6.) Verify transpose properties

$$A = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 3 & 2 & 4 & 1 \\ 4 & 0 & 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 2 & 1 & 1 \\ 3 & 2 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 0 \\ 2 & 3 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

Property 1: $(A^T)^T = A$

$$A^T = \text{rows} \rightarrow \text{columns} = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 0 \\ 0 & 4 & 3 \\ 2 & 1 & 2 \end{bmatrix}$$

$$(A^T)^T = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 3 & 2 & 4 & 1 \\ 4 & 0 & 3 & 2 \end{bmatrix} = A \quad \text{(Back to original)}$$

Property 2: $(A+B)^T = A^T + B^T$

$$A+B = \begin{bmatrix} 1+1 & 3+1 & 0+3 & 2+0 \\ 3+4 & 2+2 & 4+1 & 1+1 \\ 4+3 & 0+2 & 3+0 & 2+1 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 3 & 2 \\ 7 & 4 & 5 & 2 \\ 7 & 2 & 3 & 3 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 2 & 7 & 7 \\ 4 & 4 & 2 \\ 3 & 5 & 3 \\ 2 & 2 & 3 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 0 \\ 0 & 4 & 3 \\ 2 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 3 \\ 1 & 2 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 7 & 7 \\ 4 & 4 & 2 \\ 3 & 5 & 3 \\ 2 & 2 & 3 \end{bmatrix} = (A+B)^T$$

Property 3: $(AC)^T = C^T A^T$ (ORDER REVERSES!)

$$AC = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 3 & 2 & 4 & 1 \\ 4 & 0 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 0 \\ 2 & 3 & -1 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 22 & 7 & 6 \\ 24 & 16 & 9 \\ 16 & 13 & 15 \end{bmatrix}$$

$$(AC)^T = \begin{bmatrix} 22 & 24 & 16 \\ 7 & 16 & 13 \\ 6 & 9 & 15 \end{bmatrix}$$

$$C^T \cdot A^T = \begin{bmatrix} 1 & 5 & 2 & 3 \\ 0 & 1 & 3 & 2 \\ 4 & 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 0 \\ 0 & 4 & 3 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 22 & 24 & 16 \\ 7 & 16 & 13 \\ 6 & 9 & 15 \end{bmatrix} = (AC)^T$$

Property 4: $(cA)^T = c \cdot A^T$

$$c \cdot A = \begin{bmatrix} c & 3c & 0 & 2c \\ 3c & 2c & 4c & c \\ 4c & 0 & 3c & 2c \end{bmatrix} \quad (c \cdot A)^T = \begin{bmatrix} c & 3c & 4c \\ 3c & 2c & 0 \\ 0 & 4c & 3c \\ 2c & c & 2c \end{bmatrix}$$

$$c \cdot A^T = c \cdot \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 0 \\ 0 & 4 & 3 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} c & 3c & 4c \\ 3c & 2c & 0 \\ 0 & 4c & 3c \\ 2c & c & 2c \end{bmatrix} = (c \cdot A)^T$$

Property 5: AA^T is a symmetric matrix.

$$A \cdot A^T = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 3 & 2 & 4 & 1 \\ 4 & 0 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & 0 \\ 0 & 4 & 3 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 11 & 8 \\ 11 & 30 & 26 \\ 8 & 26 & 29 \end{bmatrix}$$

$$(A \cdot A^T)^T = \begin{bmatrix} 14 & 11 & 8 \\ 11 & 30 & 26 \\ 8 & 26 & 29 \end{bmatrix} = A \cdot A^T$$

ALWAYS TRUE FOR ANY MATRIX A