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DETERMINANTS

(1) $A=\begin{bmatrix} 2 & 4 \\ -1 & -3 \end{bmatrix}$ $|A| = 2\cdot(-3) - 4\cdot(-1) = -6+4 = -2$.

$B=\begin{bmatrix} -5 & 7 \\ -4 & -12 \end{bmatrix}$ $|B| = (-5)\cdot(-12) - 7\cdot(-4) = 60+28 = 88$

$C=\begin{bmatrix} -3 & -2 \\ 6 & 5 \end{bmatrix}$ $|C| = (-3)\cdot 5 - (-2)\cdot 6 = -15+12 = -3$

$D=\begin{bmatrix} -1 & 3 \\ -4 & 3 \end{bmatrix}$ $|D| = (-1)\cdot 3 - 3\cdot(-4) = -3+12 = 9$

$E=\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ $|E| = 3\cdot 0 - 2\cdot(-1) = +2 = 2$

$F=\begin{bmatrix} -1 & 5 \\ -3 & -4 \end{bmatrix}$ $|F| = (-1)\cdot(-4) - 5\cdot(-3) = 4+15 = 19$

$G=\begin{bmatrix} \sqrt{3} & 3 \\ 1 & -\sqrt{3} \end{bmatrix}$ $|G| = \sqrt{3}(-\sqrt{3}) - 3 = -3-3 = -6$

$H=\begin{bmatrix} \pi & -2 \\ -\pi & 1 \end{bmatrix}$ $|H| = \pi - (-2)\cdot(-\pi) = \pi - 2\pi = -\pi$

(2) $A=\begin{bmatrix} 3 & 1 & 2 \\ -3 & 4 & 3 \\ 4 & 0 & -2 \end{bmatrix}$

$|A| = 3\cdot 4\cdot(-2) + 1\cdot 3\cdot 4 + (-3)\cdot 0\cdot 2 -$
$- 2\cdot 4\cdot 4 - 3\cdot 0\cdot 3 - (-3)\cdot 1\cdot(-2) =$
$= -24+12-32-6 = -30-32+12 = -50$

$B=\begin{bmatrix} -3 & -2 & 4 \\ -1 & 2 & 1 \\ -4 & 3 & -1 \end{bmatrix}$

$|B| = (-3)\cdot 2\cdot(-1) + (-2)\cdot 1\cdot(-4) + (-1)\cdot 3\cdot 4 -$
$- 4\cdot 2\cdot(-4) - 1\cdot 3\cdot(-3) - (-1)\cdot(-2)\cdot(-1) =$
$= \frac{6+8}{2} + 12 + 32 + 9 + 2 = 34+11 = 45$

$$C=\begin{bmatrix}6 & -1 & 2\\ -4 & -3 & 7\\ -5 & 6 & 0\end{bmatrix}$$

$|C| = 6(-3)\cdot 0 + (-1)\cdot 7\cdot(-5) + (-4)\cdot 6\cdot 2 - 2\cdot(-3)(-5) - 7\cdot 6\cdot 6 - (-1)(-4)\cdot 0 =$
$= 35 - 48 - 30 - 252 = 5 - 300 = -295$

$$D=\begin{bmatrix}-3 & -1 & 2\\ 4 & -5 & 1\\ 0 & \pi & 0\end{bmatrix}$$

$|D| = (-3)(-5)\cdot 0 + 4\cdot\pi\cdot 2 + (-1)\cdot 1\cdot 0 - 2(-5)\cdot 0 - \pi\cdot(-3) - 4\cdot(-1)\cdot 0 =$
$= 8\pi + 3\pi = 11\pi$

$$E=\begin{bmatrix}5 & -7 & 2\\ 8 & -1 & 3\\ 4 & 0 & -5\end{bmatrix}$$

$|E| = 5\cdot(-1)\cdot(-5) + 8\cdot 0\cdot 2 + (-7)\cdot 3\cdot 4 - 2(-1)\cdot 4 - 3\cdot 0\cdot 5 - (-7)\cdot 8\cdot(-5) =$
$= 25 - 84 + 8 - 280 = -331$

$$F=\begin{bmatrix}3 & -1 & 0\\ -1 & 2 & -3\\ 0 & -3 & -6\end{bmatrix}$$

$|F| = 3\cdot 2(-6) + 0 + 0 - 0 - (-3)(-3)\cdot 3 - (-1)(-1)(-6) = -36 - 27 + 6 = -57$

$$G=\begin{bmatrix}1 & 0 & 0\\ 0 & \sqrt{3} & \sqrt{6}\\ 0 & \sqrt{2} & \sqrt{5}\end{bmatrix}$$

$|G| = \sqrt{3}\cdot\sqrt{5} - \sqrt{2}\cdot\sqrt{6} = \sqrt{15} - \sqrt{12}$

$$H=\begin{bmatrix}-1 & 0 & \sqrt{2}\\ -\sqrt{3} & 0 & \pi\\ 0 & \pi & 0\end{bmatrix}$$

$|H| = -\sqrt{3}\cdot\pi\cdot\sqrt{2} - \pi^2\cdot(-1) =$
$= -\pi\sqrt{6} + \pi^2 = \pi^2 - \pi\cdot\sqrt{6}$

(3) Find inverse matrices

$$A=\begin{bmatrix}3 & 1 & 2\\ -3 & 4 & 3\\ 4 & 0 & -2\end{bmatrix}$$

$|A| = -50$ - Invertible ($\neq 0$)

$$A^{-1} = \frac{1}{|A|}\cdot\begin{bmatrix}C_{11} & C_{21} & C_{31}\\ C_{12} & C_{22} & C_{32}\\ C_{13} & C_{23} & C_{33}\end{bmatrix}$$

$$C_{11} = +\begin{vmatrix}4 & 3\\ 0 & -2\end{vmatrix} = 4\cdot(-2) = -8$$

$$C_{12} = -\begin{vmatrix}-3 & 3\\ 4 & -2\end{vmatrix} = -\left((-3)\cdot(-2) - 3\cdot 4\right) = -(6-12) = -(-6) = 6$$

$$C_{13} = +\begin{vmatrix}-3 & 4\\ 4 & 0\end{vmatrix} = +(-16) = -16$$

$$\begin{bmatrix}+ & - & +\\ - & + & -\\ + & - & +\end{bmatrix}$$

$C_{21} = -\begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = -(1 \cdot (-2)) = 2$

$C_{22} = +\begin{vmatrix} 3 & 2 \\ 4 & -2 \end{vmatrix} = 3 \cdot (-2) - 2 \cdot 4 = -14$

$C_{23} = -\begin{vmatrix} 3 & 1 \\ 4 & 0 \end{vmatrix} = -(0 - 4) = -(-4) = 4$

$C_{31} = +\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = 3 - 2 \cdot 4 = -5$

$C_{32} = -\begin{vmatrix} 3 & 2 \\ -3 & 3 \end{vmatrix} = (9 - 2 \cdot (-3)) = -(9+6) = -15$

$C_{33} = +\begin{vmatrix} 3 & 1 \\ -3 & 4 \end{vmatrix} = 12 - (-3) = 15$

$C^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$ 3

$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

$A^{-1} = \frac{1}{-50} \cdot \begin{bmatrix} -8 & 2 & -5 \\ 6 & -14 & -15 \\ -16 & 4 & 15 \end{bmatrix} = -0{,}02 \begin{bmatrix} -8 & 2 & -5 \\ 6 & -14 & -15 \\ -16 & 4 & 15 \end{bmatrix} =$

$= \begin{bmatrix} 0{,}16 & -0{,}04 & 0{,}1 \\ -0{,}12 & 0{,}28 & 0{,}3 \\ 0{,}32 & -0{,}08 & -0{,}3 \end{bmatrix}$

Find inverse matrice

$B = \begin{bmatrix} -3 & -2 & 4 \\ -1 & 2 & 1 \\ -4 & 3 & -1 \end{bmatrix}$ $|B| = 45 \neq 0$ Invertible

$B^{-1} = \frac{1}{|B|} \cdot C^T$ $C^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$

$C_{11} = +\begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = -2 - 3 = -5$ $C_{12} = -\begin{vmatrix} -1 & 1 \\ -4 & -1 \end{vmatrix} = -(1 - (-4)) = -5$

$C_{13} = +\begin{vmatrix} -1 & 2 \\ -4 & 3 \end{vmatrix} = -3 - 2 \cdot (-4) = 5$ $C_{21} = -\begin{vmatrix} -2 & 4 \\ 3 & -1 \end{vmatrix} = -(2 - 12) = 10$

$C_{22} = +\begin{vmatrix} -3 & 4 \\ -4 & -1 \end{vmatrix} = 3 - (4 \cdot (-4)) = 19$ $C_{23} = -\begin{vmatrix} -3 & -2 \\ -4 & 3 \end{vmatrix} = -(-9 - 8) = 17.$

$$C_{31} = +\begin{vmatrix} -2 & 4 \\ 2 & 1 \end{vmatrix} = -2 - 8 = -10$$

$$C_{32} = -\begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -(-3-(-4)) = -1$$

$$C_{33} = +\begin{vmatrix} -3 & -2 \\ -1 & 2 \end{vmatrix} = -6 - ((-2)\cdot(-1)) = -6-2 = -8$$

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$B^{-1} = \frac{1}{45}\cdot\begin{bmatrix} -5 & 10 & -10 \\ -5 & 19 & -1 \\ 5 & 17 & -8 \end{bmatrix} = \begin{bmatrix} -\frac{1}{9} & \frac{2}{9} & -\frac{2}{9} \\ -\frac{1}{9} & \frac{19}{45} & -\frac{1}{45} \\ \frac{1}{9} & \frac{17}{45} & -\frac{8}{45} \end{bmatrix}$$

Find inverse matrix

$$C = \begin{bmatrix} 6 & -1 & 2 \\ -4 & -3 & 7 \\ -5 & 6 & 0 \end{bmatrix} \quad |C| = -295 \neq 0 \text{ Invertible}$$

$$C^{-1} = \frac{1}{|C|}\cdot K^T \qquad K^T = \begin{bmatrix} K_{11} & K_{21} & K_{31} \\ K_{12} & K_{22} & K_{32} \\ K_{13} & K_{23} & K_{33} \end{bmatrix}$$

$$\begin{bmatrix} +\begin{vmatrix} -3 & 7 \\ 6 & 0 \end{vmatrix} & -\begin{vmatrix} -1 & 2 \\ 6 & 0 \end{vmatrix} & +\begin{vmatrix} -1 & 2 \\ -3 & 7 \end{vmatrix} \\ -\begin{vmatrix} -4 & 7 \\ -5 & 0 \end{vmatrix} & +\begin{vmatrix} 6 & 2 \\ -5 & 0 \end{vmatrix} & -\begin{vmatrix} 6 & 2 \\ -4 & 7 \end{vmatrix} \\ +\begin{vmatrix} -4 & -3 \\ -5 & 6 \end{vmatrix} & -\begin{vmatrix} 6 & -1 \\ -5 & 6 \end{vmatrix} & +\begin{vmatrix} 6 & -1 \\ -4 & -3 \end{vmatrix} \end{bmatrix}$$

$$-7\cdot 6 \;;\; -(0-6\cdot 2) \;;\; -7-2\cdot(-3)$$

$$-(0-7\cdot(-5)) \;;\; -2\cdot(-5) \;;\; -(6\cdot 7-2\cdot(-4))$$

$$-4\cdot 6-(-3)(-5) \;;\; -(36-5) \;;\; 6\cdot(-3)-4$$

$$K^T = \begin{bmatrix} -42 & 12 & -1 \\ -35 & 10 & -50 \\ -39 & -31 & -22 \end{bmatrix}$$

$$C^{-1} = \frac{1}{-295}\cdot K^T = \begin{bmatrix} \frac{42}{295} & -\frac{12}{295} & \frac{1}{295} \\ \frac{7}{59} & -\frac{2}{59} & \frac{10}{59} \\ \frac{39}{295} & \frac{31}{295} & \frac{22}{295} \end{bmatrix}$$

Find inverse matrix

$$D=\begin{bmatrix}-3 & -1 & 2\\ 4 & -5 & 1\\ 0 & \pi & 0\end{bmatrix}$$

$|D| = 11\pi \neq 0$ Invertible

$$D^{-1}=\frac{1}{|D|}\cdot C^T=\frac{1}{|D|}\cdot\begin{bmatrix}C_{11} & C_{21} & C_{31}\\ C_{12} & C_{22} & C_{32}\\ C_{13} & C_{23} & C_{33}\end{bmatrix}$$

$$\begin{bmatrix}+\begin{vmatrix}-5 & 1\\ \pi & 0\end{vmatrix} & -\begin{vmatrix}-1 & 2\\ \pi & 0\end{vmatrix} & +\begin{vmatrix}-1 & 2\\ -5 & 1\end{vmatrix}\\ -\begin{vmatrix}4 & 1\\ 0 & 0\end{vmatrix} & +\begin{vmatrix}-3 & 2\\ 0 & 0\end{vmatrix} & -\begin{vmatrix}-3 & 2\\ 4 & 1\end{vmatrix}\\ +\begin{vmatrix}4 & -5\\ 0 & \pi\end{vmatrix} & -\begin{vmatrix}-3 & -1\\ 0 & \pi\end{vmatrix} & +\begin{vmatrix}-3 & -1\\ 4 & -5\end{vmatrix}\end{bmatrix}\quad\begin{matrix}0-\pi\,;\,-(0-2\pi)\,;\,-1-(-10)\\ 0\;;\;0\;;\;-(-3-8)\\ 4\pi-0\,;\,-(-3\pi)\,;\,|-3||-5|\neq 4\end{matrix}$$

$$C^T=\begin{bmatrix}-\pi & 2\pi & 9\\ 0 & 0 & 11\\ 4\pi & 3\pi & 19\end{bmatrix}\qquad D^{-1}=\begin{bmatrix}-\frac{1}{11} & \frac{2}{11} & \frac{9}{11\pi}\\ 0 & 0 & \frac{1}{\pi}\\ \frac{4}{11} & \frac{3}{11} & \frac{19}{11\pi}\end{bmatrix}$$

Find inverse matrix

$$E=\begin{bmatrix}5 & -7 & 2\\ 8 & -1 & 3\\ 4 & 0 & -5\end{bmatrix}$$

$|E| = -331 \neq 0$ Invertible

$$\begin{bmatrix}+\begin{vmatrix}-1 & 3\\ 0 & -5\end{vmatrix} & -\begin{vmatrix}-7 & 2\\ 0 & -5\end{vmatrix} & +\begin{vmatrix}-7 & 2\\ -1 & 3\end{vmatrix}\\ -\begin{vmatrix}8 & 3\\ 4 & -5\end{vmatrix} & +\begin{vmatrix}5 & 2\\ 4 & -5\end{vmatrix} & -\begin{vmatrix}5 & 2\\ 8 & 3\end{vmatrix}\\ +\begin{vmatrix}8 & -1\\ 4 & 0\end{vmatrix} & -\begin{vmatrix}5 & -7\\ 4 & 0\end{vmatrix} & +\begin{vmatrix}5 & -7\\ 8 & -1\end{vmatrix}\end{bmatrix}\quad\begin{matrix}5\;;\;-(35)\;;\;-21-(-2)\\ -(-40-12)\;;\;-25-8\;;\;-(15-16)\\ -(-4)\;;\;-(-(-28)\;;\\ -5-(-7\cdot 8)\end{matrix}$$

$$C^T = \begin{bmatrix} 5 & -35 & -19 \\ 52 & -33 & 1 \\ 4 & -28 & 51 \end{bmatrix}$$

$$E^{-1} = \frac{1}{-331} \cdot C^T = \begin{bmatrix} -\frac{5}{331} & \frac{35}{331} & \frac{19}{331} \\ -\frac{52}{331} & \frac{33}{331} & -\frac{1}{331} \\ -\frac{4}{331} & \frac{28}{331} & -\frac{51}{331} \end{bmatrix}$$

Find inverse matrix $|F| = -57 \neq 0$ invertible

$$F = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -3 \\ 0 & -3 & -6 \end{bmatrix} \qquad F^{-1} = \frac{1}{|F|} \cdot C^T$$

$$\begin{bmatrix} +\begin{vmatrix} 2 & -3 \\ -3 & -6 \end{vmatrix} & -\begin{vmatrix} -1 & 0 \\ -3 & -6 \end{vmatrix} & +\begin{vmatrix} -1 & 0 \\ 2 & -3 \end{vmatrix} \\ -\begin{vmatrix} -1 & -3 \\ 0 & -6 \end{vmatrix} & +\begin{vmatrix} 3 & 0 \\ 0 & -6 \end{vmatrix} & -\begin{vmatrix} 3 & 0 \\ -1 & -3 \end{vmatrix} \\ +\begin{vmatrix} -1 & 2 \\ 0 & -3 \end{vmatrix} & -\begin{vmatrix} 3 & -1 \\ 0 & -3 \end{vmatrix} & +\begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} \end{bmatrix} \begin{matrix} -12 & -(-3)\cdot(-3); & -(6) \; ; 3 \\ -(6-0); & -18; & -(-9) \\ 3; & -(-9); & 6-1 \end{matrix}$$

$$C^T = \begin{bmatrix} -21 & -6 & 3 \\ -6 & -18 & 9 \\ 3 & 9 & 5 \end{bmatrix}$$

$$F^{-1} = \frac{1}{-57} \cdot C^T = \begin{bmatrix} \frac{7}{19} & \frac{2}{19} & -\frac{1}{19} \\ \frac{2}{19} & \frac{6}{19} & -\frac{3}{19} \\ -\frac{1}{19} & -\frac{3}{19} & -\frac{5}{57} \end{bmatrix}$$

Find inverse matrix $|G| = \sqrt{15} - \sqrt{12} \neq 0$ Invertible

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3} & \sqrt{6} \\ 0 & \sqrt{2} & \sqrt{5} \end{bmatrix} \qquad G^{-1} = \frac{1}{|G|} \cdot C^T$$

$$\begin{bmatrix} +\begin{vmatrix} \sqrt{3} & \sqrt{6} \\ \sqrt{2} & \sqrt{5} \end{vmatrix} & -\begin{vmatrix} 0 & 0 \\ \sqrt{2} & \sqrt{5} \end{vmatrix} & +\begin{vmatrix} 0 & 0 \\ \sqrt{3} & \sqrt{6} \end{vmatrix} \\ -\begin{vmatrix} 0 & \sqrt{6} \\ 0 & \sqrt{5} \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 0 & \sqrt{5} \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & \sqrt{6} \end{vmatrix} \\ +\begin{vmatrix} 0 & \sqrt{3} \\ 0 & \sqrt{2} \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & \sqrt{2} \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 0 & \sqrt{3} \end{vmatrix} \end{bmatrix} = \begin{bmatrix} \sqrt{15}-\sqrt{12} & 0 & 0 \\ 0 & \sqrt{5} & -\sqrt{6} \\ 0 & -\sqrt{2} & \sqrt{3} \end{bmatrix}$$

$$C^T = \begin{bmatrix} \sqrt{15}-\sqrt{12} & 0 & 0 \\ 0 & \sqrt{5} & -\sqrt{6} \\ 0 & -\sqrt{2} & \sqrt{3} \end{bmatrix}$$

$$G^{-1} = \frac{1}{\sqrt{15}-\sqrt{12}} \cdot C^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{5}}{\sqrt{15}-\sqrt{12}} & -2\sqrt{2}-\sqrt{10} \\ 0 & -\frac{\sqrt{2}}{\sqrt{15}-\sqrt{12}} & \sqrt{5}+2 \end{bmatrix}$$

Find inverse matrix $|H| = \pi^2 - \pi\sqrt{6} \neq 0$ Invertible

$$H = \begin{bmatrix} -1 & 0 & \sqrt{2} \\ -\sqrt{3} & 0 & \pi \\ 0 & \pi & 0 \end{bmatrix} \qquad H^{-1} = \frac{1}{|H|} \cdot C^T$$

$$\begin{bmatrix} +\begin{vmatrix} 0 & \pi \\ \pi & 0 \end{vmatrix} & -\begin{vmatrix} 0 & \sqrt{2} \\ \pi & 0 \end{vmatrix} & +\begin{vmatrix} 0 & \sqrt{2} \\ 0 & \pi \end{vmatrix} \\ -\begin{vmatrix} -\sqrt{3} & \pi \\ 0 & 0 \end{vmatrix} & +\begin{vmatrix} -1 & \sqrt{2} \\ 0 & 0 \end{vmatrix} & -\begin{vmatrix} -1 & \sqrt{2} \\ -\sqrt{3} & \pi \end{vmatrix} \\ +\begin{vmatrix} -\sqrt{3} & 0 \\ 0 & \pi \end{vmatrix} & -\begin{vmatrix} -1 & 0 \\ 0 & \pi \end{vmatrix} & +\begin{vmatrix} -1 & 0 \\ -\sqrt{3} & 0 \end{vmatrix} \end{bmatrix} \begin{matrix} -\pi^2; \sqrt{2}\pi; 0 \\ 0; 0; -(-\pi - (\sqrt{2} \cdot (-\sqrt{3}))) \\ -\sqrt{3}\pi; \pi; 0 \end{matrix}$$

$$C^T = \begin{bmatrix} -\pi^2 & \sqrt{2}\pi & 0 \\ 0 & 0 & \pi - \sqrt{6} \\ -\sqrt{3}\pi & \pi & 0 \end{bmatrix}$$

$$H^{-1} = \frac{1}{\pi^2 - \sqrt{6}\pi} \cdot C^T =$$

$$= \begin{bmatrix} -\frac{\pi^2 + \pi\sqrt{6}}{\pi^2 - 6} & \frac{\pi\sqrt{2} + 2\sqrt{3}}{\pi^2 - 6} & 0 \\ 0 & 0 & \frac{1}{\pi} \\ -\frac{\pi\sqrt{3} + 3\sqrt{2}}{\pi^2 - 6} & \frac{\pi + \sqrt{6}}{\pi^2 - 6} & 0 \end{bmatrix}$$

④ Find the determinant

$$A=\begin{bmatrix} 3 & 1 & 0 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 0 & 7 \\ 3 & 0 & -6 & 2 \end{bmatrix}$$

$$\begin{matrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{matrix}$$

$-6R_2+R_4 \to R_4$

$$\begin{bmatrix} 3 & 1 & 0 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 0 & 7 \\ -3 & -12 & 0 & 2 \end{bmatrix}$$

$$|A| = (-1)(-1)\cdot \begin{vmatrix} 3 & 1 & 2 \\ -1 & 4 & 7 \\ -3 & -12 & 2 \end{vmatrix}$$

$$|A| = 3\cdot 4\cdot 2 + (1\cdot(-1)(-12)\cdot 2) + 1\cdot 7(-3) - (2\cdot 4(-3) + 1(-1)\cdot 2 + 7(-12)\cdot 3) =$$

$$= 24+24-21-(-24-2-252) = 27+278 = 305$$

Find the determinant

$$B=\begin{bmatrix} -2 & 0 & -1 & 4 \\ 0 & 6 & -3 & 9 \\ 4 & 0 & -5 & 6 \\ -1 & 3 & 0 & -5 \end{bmatrix}$$

$2R_1+R_3 \to R_3$

$$\begin{bmatrix} -2 & 0 & -1 & 4 \\ 0 & 6 & -3 & 9 \\ 0 & 0 & -7 & 14 \\ -1 & 3 & 0 & -5 \end{bmatrix}$$

$-2R_4+R_1 \to R_1$

$$\begin{bmatrix} 0 & -6 & -1 & 14 \\ 0 & 6 & -3 & 9 \\ 0 & 0 & -7 & 14 \\ -1 & 3 & 0 & -5 \end{bmatrix}$$

$$|B| = (-1)\cdot(-1)\cdot \begin{vmatrix} -6 & -1 & 14 \\ 6 & -3 & 9 \\ 0 & -7 & 14 \end{vmatrix}$$

$$|B| = 18\cdot 14 - 6\cdot 7\cdot 14 - (9(-7)(-6) + 6(-1)\cdot 14) =$$

$$= 252 - 588 - (378-84) = -336 - 294 = -630$$

Find the determinant

$$C=\begin{bmatrix} 4 & 5 & 1 & 0 \\ -1 & 2 & 0 & 4 \\ 2 & 0 & -1 & 7 \\ 0 & 1 & 0 & 4 \end{bmatrix}$$

$$\begin{matrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{matrix}$$

$R1+R3 \to R3$

$$\begin{bmatrix} 4 & 5 & 1 & 0 \\ -1 & 2 & 0 & 4 \\ 6 & 5 & 0 & 7 \\ 0 & 1 & 0 & 4 \end{bmatrix}$$

$$|C| = 1 \cdot 1 \cdot \begin{vmatrix} -1 & 2 & 4 \\ 6 & 5 & 7 \\ 0 & 1 & 4 \end{vmatrix}$$

$$|C| = -20+24-(-7+48) = 4-41 = -37$$

Find the determinant

$$D=\begin{bmatrix} 2 & 4 & 0 & 1 \\ 0 & -2 & 5 & 0 \\ -1 & 4 & 3 & 11 \end{bmatrix}$$

Matrix is not square.

Determinant not defined.

Find the determinant

$$E=\begin{bmatrix} 4 & 5 & 0 & 0 & 2 \\ -1 & 0 & 3 & 0 & -1 \\ 9 & -4 & -3 & 0 & 0 \\ 4 & 3 & 2 & 4 & -2 \\ 2 & 0 & -1 & 0 & -1 \end{bmatrix}$$

$$\begin{matrix} + & - & + & - & + \\ - & + & - & + & - \\ + & - & + & - & + \\ - & + & - & + & - \\ + & - & + & - & + \end{matrix}$$

$$|E| = +4 \begin{vmatrix} 4 & 5 & 0 & 2 \\ -1 & 0 & 3 & -1 \\ 9 & -4 & -3 & 0 \\ 2 & 0 & -1 & -1 \end{vmatrix}$$

$2R4+R1 \to R1$

$$\begin{bmatrix} 8 & 5 & -2 & 0 \\ -1 & 0 & 3 & -1 \\ 9 & -4 & -3 & 0 \\ 2 & 0 & -1 & -1 \end{bmatrix}$$

$-R4+R2 \to R2$

$$\begin{bmatrix} 8 & 5 & -2 & 0 \\ -3 & 0 & 4 & 0 \\ 9 & -4 & -3 & 0 \\ 2 & 0 & -1 & -1 \end{bmatrix}$$

$$|E| = +4 \cdot (-1)(+1) \cdot \begin{vmatrix} 8 & 5 & -2 \\ -3 & 0 & 4 \\ 9 & -4 & -3 \end{vmatrix} = -4 \cdot (-24+180-(45-128)) =$$

$$= -4 \cdot 239 = -956$$

Find the determinant

$$F=\begin{bmatrix} 2 & -1 & 5 & 4 & 0 \\ 0 & 0 & \pi & 0 & 0 \\ 0 & -1 & 5 & 2 & 1 \\ 2 & 3 & 7 & -6 & 2 \\ 3 & 0 & 7 & 0 & 3 \end{bmatrix} \qquad |F| = -\pi \cdot \begin{vmatrix} 2 & -1 & 4 & 0 \\ 0 & -1 & 2 & 1 \\ 2 & 3 & -6 & 2 \\ 3 & 0 & 0 & 3 \end{vmatrix}$$

Col 1 − Col 4 → Col 1

$$\begin{bmatrix} 2 & -1 & 4 & 0 \\ -1 & -1 & 2 & 1 \\ 0 & 3 & -6 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$|F| = -\pi \cdot 3 \cdot \begin{vmatrix} 2 & -1 & 4 \\ -1 & -1 & 2 \\ 0 & 3 & -6 \end{vmatrix} = -\pi \cdot 3 \cdot (-6) = 18\pi$$

(5.) Find inverse matrix $|A| = 305 \neq 0$ invertible

$$A=\begin{bmatrix} 3 & 1 & 0 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 0 & 7 \\ 3 & 0 & -6 & 2 \end{bmatrix} \qquad A^{-1} = \frac{1}{|A|} \cdot \begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} \\ C_{12} & C_{22} & C_{32} & C_{42} \\ C_{13} & C_{23} & C_{33} & C_{43} \\ C_{14} & C_{24} & C_{34} & C_{44} \end{bmatrix}$$

$$\begin{bmatrix} +\begin{vmatrix} 2 & -1 & 0 \\ 4 & 0 & 7 \\ 0 & -6 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 0 & 2 \\ 4 & 0 & 7 \\ 0 & -6 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 2 \\ 2 & -1 & 0 \\ 0 & -6 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 0 & 2 \\ 2 & -1 & 0 \\ 4 & 0 & 7 \end{vmatrix} \\ -\begin{vmatrix} 1 & -1 & 0 \\ -1 & 0 & 7 \\ 3 & -6 & 2 \end{vmatrix} + \begin{vmatrix} 3 & 0 & 2 \\ -1 & 0 & 7 \\ 3 & -6 & 2 \end{vmatrix} - \begin{vmatrix} 3 & 0 & 2 \\ 1 & 1 & 0 \\ 3 & -6 & 2 \end{vmatrix} + \begin{vmatrix} 3 & 0 & 2 \\ 1 & -1 & 0 \\ -1 & 0 & 7 \end{vmatrix} \\ +\begin{vmatrix} 1 & 2 & 0 \\ -1 & 4 & 7 \\ 3 & 0 & 2 \end{vmatrix} - \begin{vmatrix} 3 & 1 & 2 \\ -1 & 4 & 7 \\ 3 & 0 & 2 \end{vmatrix} + \begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & 0 \\ 3 & 0 & 2 \end{vmatrix} - \begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & 0 \\ -1 & 4 & 7 \end{vmatrix} \\ -\begin{vmatrix} 1 & 2 & -1 \\ -1 & 4 & 0 \\ 3 & 0 & -6 \end{vmatrix} + \begin{vmatrix} 3 & 1 & 0 \\ -1 & 4 & 0 \\ 3 & 0 & -6 \end{vmatrix} - \begin{vmatrix} 3 & 1 & 0 \\ 1 & 2 & -1 \\ 3 & 0 & -6 \end{vmatrix} + \begin{vmatrix} 3 & 1 & 0 \\ 1 & 2 & -1 \\ -1 & 4 & 0 \end{vmatrix} \end{bmatrix}$$

$$\begin{bmatrix} 92 & -(-6) & -26 & -1 \\ -19 & 138 & 12 & -23 \\ 54 & -23 & -2 & -47 \\ -(-24) & -78 & 33 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{305} \cdot \begin{bmatrix} 92 & 6 & -26 & -1 \\ -19 & 138 & 12 & -23 \\ 54 & -23 & -2 & -47 \\ 24 & -78 & 33 & 13 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{92}{305} & \frac{6}{305} & \frac{-26}{305} & \frac{-1}{305} \\ \frac{-19}{305} & \frac{138}{305} & \frac{12}{305} & \frac{-23}{305} \\ \frac{54}{305} & \frac{-23}{305} & \frac{-2}{305} & \frac{-47}{305} \\ \frac{24}{305} & \frac{-78}{305} & \frac{33}{305} & \frac{13}{305} \end{bmatrix}$$

Find inverse matrix $|B| = -630 \neq 0$ Invertible

$$B = \begin{bmatrix} -2 & 0 & -1 & 4 \\ 0 & 6 & -3 & 9 \\ 4 & 0 & -5 & 6 \\ -1 & 3 & 0 & -5 \end{bmatrix} \qquad B^{-1} = \frac{1}{|B|} \cdot C^T$$

$$\begin{bmatrix}
+\begin{vmatrix} 6 & -3 & 9 \\ 0 & -5 & 6 \\ 3 & 0 & -5 \end{vmatrix} - \begin{vmatrix} 0 & -1 & 4 \\ 0 & -5 & 6 \\ 3 & 0 & -5 \end{vmatrix} + \begin{vmatrix} 0 & -1 & 4 \\ 6 & -3 & 9 \\ 3 & 0 & -5 \end{vmatrix} - \begin{vmatrix} 0 & -1 & 4 \\ 6 & -3 & 9 \\ 0 & -5 & 6 \end{vmatrix} \\
-\begin{vmatrix} 0 & -3 & 9 \\ 4 & -5 & 6 \\ -1 & 0 & -5 \end{vmatrix} + \begin{vmatrix} -2 & -1 & 4 \\ 4 & -5 & 6 \\ -1 & 0 & -5 \end{vmatrix} - \begin{vmatrix} -2 & -1 & 4 \\ 0 & -3 & 9 \\ -1 & 0 & -5 \end{vmatrix} + \begin{vmatrix} -2 & -1 & 4 \\ 0 & -3 & 9 \\ 4 & -5 & 6 \end{vmatrix} \\
+\begin{vmatrix} 0 & 6 & 9 \\ 4 & 0 & 6 \\ -1 & 3 & -5 \end{vmatrix} - \begin{vmatrix} -2 & 0 & 4 \\ 4 & 0 & 6 \\ -1 & 3 & -5 \end{vmatrix} + \begin{vmatrix} -2 & 0 & 4 \\ 0 & 6 & 9 \\ -1 & 3 & -5 \end{vmatrix} - \begin{vmatrix} -2 & 0 & 4 \\ 0 & 6 & 9 \\ 4 & 0 & 6 \end{vmatrix} \\
-\begin{vmatrix} 0 & 6 & -3 \\ 4 & 0 & -5 \\ -1 & 3 & 0 \end{vmatrix} + \begin{vmatrix} -2 & 0 & -1 \\ 4 & 0 & -5 \\ -1 & 3 & 0 \end{vmatrix} - \begin{vmatrix} -2 & 0 & -1 \\ 0 & 6 & -3 \\ -1 & 3 & 0 \end{vmatrix} + \begin{vmatrix} -2 & 0 & -1 \\ 0 & 6 & -3 \\ 4 & 0 & -5 \end{vmatrix}
\end{bmatrix}$$

$$B^{-1} = \frac{1}{-630} \cdot \begin{bmatrix} 231 & -42 & -21 & 84 \\ 87 & -84 & 33 & -42 \\ 192 & -84 & 138 & 168 \\ 6 & -42 & 24 & 84 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{231}{-630} & \frac{42}{630} & \frac{21}{630} & \frac{84}{-630} \\ \frac{87}{-630} & \frac{84}{630} & \frac{33}{-630} & \frac{42}{630} \\ \frac{192}{-630} & \frac{84}{630} & \frac{138}{-630} & \frac{168}{-630} \\ \frac{6}{-630} & \frac{42}{630} & \frac{24}{-630} & \frac{84}{-630} \end{bmatrix} =$$

$$= \left[\begin{array}{c|c|c|c} \frac{-11}{30} & \frac{1}{15} & \frac{1}{30} & \frac{-2}{15} \\ \hline \frac{-29}{210} & \frac{2}{15} & \frac{-11}{210} & \frac{1}{15} \\ \hline \frac{-32}{105} & \frac{2}{15} & \frac{-23}{105} & \frac{-4}{15} \\ \hline \frac{-1}{105} & \frac{1}{15} & \frac{-4}{105} & \frac{-2}{15} \end{array}\right]$$

Find the inverse matrix $|C| = -37 \neq 0$ invertible

$$C = \begin{bmatrix} 4 & 5 & 1 & 0 \\ -1 & 2 & 0 & 4 \\ 2 & 0 & -1 & 7 \\ 0 & 1 & 0 & 4 \end{bmatrix}$$

$$C^{-1} = \frac{1}{|C|} \cdot K^T$$

K-cofactor adjugate matrix.

$$\begin{bmatrix}
+\begin{vmatrix} 2 & 0 & 4 \\ 0 & -1 & 7 \\ 1 & 0 & 4 \end{vmatrix} & -\begin{vmatrix} 5 & 1 & 0 \\ 0 & -1 & 7 \\ 1 & 0 & 4 \end{vmatrix} & +\begin{vmatrix} 5 & 1 & 0 \\ 2 & 0 & 4 \\ 1 & 0 & 4 \end{vmatrix} & -\begin{vmatrix} 5 & 1 & 0 \\ 2 & 0 & 4 \\ 0 & -1 & 7 \end{vmatrix} \\
-\begin{vmatrix} -1 & 0 & 4 \\ 2 & -1 & 7 \\ 0 & 0 & 4 \end{vmatrix} & +\begin{vmatrix} 4 & 1 & 0 \\ 2 & -1 & 7 \\ 0 & 0 & 4 \end{vmatrix} & -\begin{vmatrix} 4 & 1 & 0 \\ -1 & 0 & 4 \\ 0 & 0 & 4 \end{vmatrix} & +\begin{vmatrix} 4 & 1 & 0 \\ -1 & 0 & 4 \\ 2 & -1 & 7 \end{vmatrix} \\
+\begin{vmatrix} -1 & 2 & 4 \\ 2 & 0 & 7 \\ 0 & 1 & 4 \end{vmatrix} & -\begin{vmatrix} 4 & 5 & 0 \\ 2 & 0 & 7 \\ 0 & 1 & 4 \end{vmatrix} & +\begin{vmatrix} 4 & 5 & 0 \\ -1 & 2 & 4 \\ 0 & 1 & 4 \end{vmatrix} & -\begin{vmatrix} 4 & 5 & 0 \\ -1 & 2 & 4 \\ 2 & 0 & 7 \end{vmatrix} \\
-\begin{vmatrix} -1 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 0 \end{vmatrix} & +\begin{vmatrix} 4 & 5 & 1 \\ 2 & 0 & -1 \\ 0 & 1 & 0 \end{vmatrix} & -\begin{vmatrix} 4 & 5 & 1 \\ -1 & 2 & 0 \\ 0 & 1 & 0 \end{vmatrix} & +\begin{vmatrix} 4 & 5 & 1 \\ -1 & 2 & 0 \\ 2 & 0 & -1 \end{vmatrix}
\end{bmatrix}$$

$$C^{-1} = \frac{1}{-37} \cdot \begin{bmatrix} -4 & 13 & -4 & -6 \\ -4 & -24 & -4 & 31 \\ -1 & 68 & 36 & -131 \\ 1 & 6 & 1 & -17 \end{bmatrix} = \begin{bmatrix} \frac{4}{37} & \frac{-13}{37} & \frac{4}{37} & \frac{6}{37} \\ \frac{4}{37} & \frac{24}{37} & \frac{4}{37} & \frac{-31}{37} \\ \frac{1}{37} & \frac{-68}{37} & \frac{-36}{37} & \frac{131}{37} \\ \frac{-1}{37} & \frac{-6}{37} & \frac{-1}{37} & \frac{17}{37} \end{bmatrix}$$

Find inverse matrix.

$$D = \begin{bmatrix} 2 & 4 & 0 & 1 \\ 0 & -2 & 5 & 0 \\ -1 & 4 & 3 & \pi \end{bmatrix}$$

Matrix is not square.
Determinant is not defined.
Inverse matrix is not defined.