

CRAMER'S RULE

Solve systems of equations

$$a) \begin{cases} x + y - z = 0 \\ x + 2y + 3z = -5 \\ 2x - y - 13z = 17 \end{cases}$$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & 3 \\ 2 & -1 & -13 \end{vmatrix} = 1$$

$$= 1 \cdot 2 \cdot (-13) + 1 \cdot 3 \cdot 2 + 1 \cdot (-1) \cdot (-1) - \\ - 2 \cdot 2 \cdot (-1) - 3 \cdot (-1) \cdot 1 - 1 \cdot 1 \cdot (-13) = \\ = 1$$

$$\Delta_x = \begin{vmatrix} 0 & 1 & -1 \\ -5 & 2 & 3 \\ 17 & -1 & -13 \end{vmatrix} = 0 \cdot 2 \cdot (-13) + 1 \cdot 3 \cdot 17 + (-5) \cdot (-1) \cdot (-1) - \\ - 17 \cdot 2 \cdot (-1) - 3 \cdot (-1) \cdot 0 - 1 \cdot (-5) \cdot (-13) = 15$$

$$\Delta_y = \begin{vmatrix} 1 & 0 & -1 \\ 1 & -5 & 3 \\ 2 & 17 & -13 \end{vmatrix} = -13$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & -5 \\ 2 & -1 & 17 \end{vmatrix} = 2$$

$$x = \frac{\Delta_x}{\Delta} = \frac{15}{1} = 15$$

$$15 - 13 - 2 = 0 \quad \checkmark$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-13}{1} = -13$$

$$15 + 2(-13) + 3 \cdot 2 = -5$$

$$15 - 26 + 6 = -5$$

$$-5 = -5 \quad \checkmark$$

$$z = \frac{\Delta_z}{\Delta} = \frac{2}{1} = 2$$

$$2 \cdot 15 - (-13) - 13 \cdot 2 = 17$$

$$\text{Answer: } \begin{cases} x = 15 \\ y = -13 \\ z = 2 \end{cases}$$

$$30 + 13 - 26 = 17$$

$$17 = 17 \quad \checkmark$$

$$b) \begin{cases} x + 2y - z = 3 \\ 2x + 5y - 2z = 7 \\ -x + y + 5z = -12 \end{cases}$$

$$\Delta = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 5 & -2 \\ -1 & 1 & 5 \end{vmatrix} = 4$$

$$\Delta x = \begin{vmatrix} 3 & 2 & -1 \\ 7 & 5 & -2 \\ -12 & 1 & 5 \end{vmatrix} = -8$$

$$x = \frac{\Delta x}{\Delta} = \frac{-8}{4} = -2$$

$$\Delta y = \begin{vmatrix} 1 & 3 & -1 \\ 2 & 7 & -2 \\ -1 & -12 & 5 \end{vmatrix} = 4$$

$$y = \frac{\Delta y}{\Delta} = \frac{4}{4} = 1$$

$$\Delta z = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -1 & 1 & -12 \end{vmatrix} = -12$$

$$z = \frac{\Delta z}{\Delta} = \frac{-12}{4} = -3$$

$$c) \begin{cases} x - 3y + 3z = 7 \\ x + 2y - z = -2 \\ 3x + 2y + 4z = 5 \end{cases}$$

$$\Delta = \begin{vmatrix} 1 & -3 & 3 \\ 1 & 2 & -1 \\ 3 & 2 & 4 \end{vmatrix} = 19$$

$$\Delta x = \begin{vmatrix} 7 & -3 & 3 \\ -2 & 2 & -1 \\ 5 & 2 & 4 \end{vmatrix} = 19$$

$$x = \frac{\Delta x}{\Delta} = \frac{19}{19} = 1$$

$$\Delta y = \begin{vmatrix} 1 & 7 & 3 \\ 1 & -2 & -1 \\ 3 & 5 & 4 \end{vmatrix} = -19$$

$$y = \frac{\Delta y}{\Delta} = \frac{-19}{19} = -1$$

$$\Delta z = \begin{vmatrix} 1 & -3 & 7 \\ 1 & 2 & -2 \\ 3 & 2 & 5 \end{vmatrix} = 19$$

$$z = \frac{\Delta z}{\Delta} = \frac{19}{19} = 1$$

$$d) \begin{cases} -3x + 6y - 9z = 3 \\ x - y - 2z = 0 \\ 5x + 5y - 7z = 63 \end{cases}$$

$$\Delta = \begin{vmatrix} -3 & 6 & -9 \\ 1 & -1 & -2 \\ 5 & 5 & -7 \end{vmatrix} = -159$$

$$\Delta x = \begin{vmatrix} 3 & 6 & -9 \\ 0 & -1 & -2 \\ 63 & 5 & -7 \end{vmatrix} = -1272$$

$$x = \frac{\Delta x}{\Delta} = \frac{-1272}{-159} = 8$$

$$\Delta y = \begin{vmatrix} -3 & 3 & -9 \\ 1 & 0 & -2 \\ 5 & 63 & -7 \end{vmatrix} = -954$$

$$y = \frac{\Delta y}{\Delta} = \frac{-954}{-159} = 6$$

$$\Delta z = \begin{vmatrix} -3 & 6 & 3 \\ 1 & -1 & 0 \\ 5 & 5 & 63 \end{vmatrix} = -159$$

$$z = \frac{\Delta z}{\Delta} = \frac{-159}{-159} = 1$$

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e)
$$\begin{cases} 3x + 2y - 4z = -12 \\ 3x - 3y + 2z = -15 \\ 4x + 6y + z = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} 3 & 2 & -4 \\ 3 & -3 & 2 \\ 4 & 6 & 1 \end{vmatrix} = -155$$

$$\Delta x = \begin{vmatrix} -12 & 2 & -4 \\ -15 & -3 & 2 \\ 0 & 6 & 1 \end{vmatrix} = 570$$

$$x = \frac{\Delta x}{\Delta} = \frac{570}{-155} = -\frac{114}{31}$$

$$\Delta y = \begin{vmatrix} 3 & -12 & -4 \\ 3 & -15 & 2 \\ 4 & 0 & 1 \end{vmatrix} = -345$$

$$y = \frac{\Delta y}{\Delta} = \frac{-345}{-155} = \frac{69}{31}$$

$$\Delta z = \begin{vmatrix} 3 & 2 & -12 \\ 3 & -3 & -15 \\ 4 & 6 & 0 \end{vmatrix} = -210$$

$$z = \frac{\Delta z}{\Delta} = \frac{-210}{-155} = \frac{42}{31}$$

f)
$$\begin{cases} 2x - 6y - 12z = 6 \\ 3x - 10y - 20z = 5 \\ 2x \quad \quad - 17z = -4 \end{cases}$$

$$\Delta = \begin{vmatrix} 2 & -6 & -12 \\ 3 & -10 & -20 \\ 2 & 0 & -17 \end{vmatrix} = 34$$

$$\Delta x = \begin{vmatrix} 6 & -6 & -12 \\ 5 & -10 & -20 \\ -4 & 0 & -17 \end{vmatrix} = 510$$

$$x = \frac{\Delta x}{\Delta} = \frac{510}{34} = 15$$

$$\Delta y = \begin{vmatrix} 2 & 6 & -12 \\ 3 & 5 & -20 \\ 2 & -4 & -17 \end{vmatrix} = 0$$

$$y = \frac{\Delta y}{\Delta} = \frac{0}{34} = 0$$

$$\Delta z = \begin{vmatrix} 2 & -6 & 6 \\ 3 & -10 & 5 \\ 2 & 0 & -4 \end{vmatrix} = 68$$

$$z = \frac{\Delta z}{\Delta} = \frac{68}{34} = 2$$