# Systems of linear equations

a) $\begin{cases} x+y-z=0 \\ x+2y+3z=-5 \\ 2x-y-13z=17 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & 2 & 3 & -5 \\ 2 & -1 & -13 & 17 \end{array}\right]$ Augmented matrix

$R_2-R_1 \to R_2$ $R_3-2R_1 \to R_3$ $R_3+3R_2 \to R_3$

$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 4 & -5 \\ 2 & -1 & -13 & 17 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 4 & -5 \\ 0 & -3 & -11 & 17 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 1 & 2 \end{array}\right] \to$

$R_2-4R_3 \to R_2$ $R_1+R_3 \to R_1$ $R_1-R_2 \to R_1$

$\to \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & 2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & 2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$x=15$ $y=-13$ $z=2$

b) $\begin{cases} x+2y-z=3 \\ 2x+5y-2z=7 \\ -x+y+5z=-12 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 5 & -2 & 7 \\ -1 & 1 & 5 & -12 \end{array}\right]$

$R_2-2\cdot R_1 \to R_2$ $R_3+R_1 \to R_3$ $R_3-3\cdot R_2 \to R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ -1 & 1 & 5 & -12 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 3 & 4 & -9 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 4 & -12 \end{array}\right] \to$

$R_3/4 \to R_3$ $R_1+R_3 \to R_1$ $R_1-2R_2 \to R_1$

$\to \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right]$

$x=-2$ $y=1$ $z=-3$

c) $\begin{cases} x - 3y + 3z = 7 \\ x + 2y - z = -2 \\ 3x + 2y + 4z = 5 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & -3 & 3 & 7 \\ 1 & 2 & -1 & -2 \\ 3 & 2 & 4 & 5 \end{array}\right]$

$R_2 - R_1 \to R_2$ $R_3 - 3R_1 \to R_3$ $R_2/5 \to R_2$

$\left[\begin{array}{ccc|c} 1 & -3 & 3 & 7 \\ 0 & 5 & -4 & -9 \\ 3 & 2 & 4 & 5 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -3 & 3 & 7 \\ 0 & 5 & -4 & -9 \\ 0 & 11 & -5 & -16 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -3 & 3 & 7 \\ 0 & 1 & -0{,}8 & -1{,}8 \\ 0 & 11 & -5 & -16 \end{array}\right] \to$

$R_3 - 11 \cdot R_2 \to R_3$ $R_3/3{,}8 \to R_3$ $R_2 + 0{,}8R_3 \to R_2$

$\to \left[\begin{array}{ccc|c} 1 & -3 & 3 & 7 \\ 0 & 1 & -0{,}8 & -1{,}8 \\ 0 & 0 & 3{,}8 & 3{,}8 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -3 & 3 & 7 \\ 0 & 1 & -0{,}8 & -1{,}8 \\ 0 & 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -3 & 3 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] \to$

$\to$ $R_1 - 3R_3 \to R_1$ $R_1 + 3R_2 \to R_1$

$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$

$x = 1$

$y = -1$

$z = 1$

d) $\begin{cases} -3x + 6y - 9z = 3 \\ x - y - 2z = 0 \\ 5x + 5y - 7z = 63 \end{cases}$ $\left[\begin{array}{ccc|c} -3 & 6 & -9 & 3 \\ 1 & -1 & -2 & 0 \\ 5 & 5 & -7 & 63 \end{array}\right]$

$R_1/(-3) \to R_1$ $R_2 - R_1 \to R_2$ $R_3 - 5R_1 \to R_3$

$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 1 & -1 & -2 & 0 \\ 5 & 5 & -7 & 63 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & 1 & -5 & 1 \\ 5 & 5 & -7 & 63 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & 1 & -5 & 1 \\ 0 & 15 & -22 & 68 \end{array}\right] \to$

$R_3 - 15R_2 \to R_3$ $R_2 + 5R_3 \to R_2$ $R_1 - 3R_3 \to R_1$

$\to \left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & 1 & -5 & 1 \\ 0 & 0 & 53 & 53 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -2 & 0 & -4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 1 \end{array}\right] \to$

$53 \quad 53$

$1 \quad 1$

$R_1 + 2R_2 \to R_1$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 1 \end{array}\right] \qquad \begin{array}{l} x = 8 \\ y = 6 \\ z = 1 \end{array}$$

e) $\begin{cases} 3x + 2y - 4z = -12 \\ 3x - 3y + 2z = -15 \\ 4x + 6y + z = 0 \end{cases}$ $\qquad \left[\begin{array}{ccc|c} 3 & 2 & -4 & -12 \\ 3 & -3 & 2 & -15 \\ 4 & 6 & 1 & 0 \end{array}\right]$

$R_1/3 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & \frac{2}{3} & -\frac{4}{3} & -4 \\ 3 & -3 & 2 & -15 \\ 4 & 6 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & \frac{2}{3} & -\frac{4}{3} & -4 \\ 0 & -5 & 6 & -3 \\ 4 & 6 & 1 & 0 \end{array}\right] \to$$

$R_2 - 3R_1 \to R_2$

$R_3 - 4R_1 \to R_3$

$$\to \left[\begin{array}{ccc|c} 1 & \frac{2}{3} & -\frac{4}{3} & -4 \\ 0 & -5 & 6 & -3 \\ 0 & \frac{10}{3} & \frac{19}{3} & 16 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & \frac{2}{3} & -\frac{4}{3} & -4 \\ 0 & 1 & -\frac{6}{5} & \frac{3}{5} \\ 0 & \frac{10}{3} & \frac{19}{3} & 16 \end{array}\right] \to$$

$R_2/(-5) \to R_2$

$R_3 - \frac{10}{3} \cdot R_2 \to R_3$

$$\to \left[\begin{array}{ccc|c} 1 & \frac{2}{3} & -\frac{4}{3} & -4 \\ 0 & 1 & -\frac{6}{5} & \frac{3}{5} \\ 0 & 0 & \frac{31}{3} & 14 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & \frac{2}{3} & -\frac{4}{3} & -4 \\ 0 & 1 & -\frac{6}{5} & \frac{3}{5} \\ 0 & 0 & 1 & \frac{42}{31} \end{array}\right] \to$$

$R_3 / \left(\frac{31}{3}\right) \to R_3$

$R_2 + \frac{6}{5} R_3 \to R_2$

$$\begin{bmatrix} 1 & \frac{2}{3} & -\frac{4}{3} & -4 \\ 0 & 1 & 0 & \frac{69}{31} \\ 0 & 0 & 1 & \frac{42}{31} \end{bmatrix}$$

$R_1 + \frac{4}{3} \cdot R_3 \to R_1$

$$\begin{bmatrix} 1 & \frac{2}{3} & 0 & -\frac{68}{31} \\ 0 & 1 & 0 & \frac{69}{31} \\ 0 & 0 & 1 & \frac{42}{31} \end{bmatrix} \to$$

$R_1 - \frac{2}{3} \cdot R_2 \to R_1$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{114}{31} \\ 0 & 1 & 0 & \frac{69}{31} \\ 0 & 0 & 1 & \frac{42}{31} \end{bmatrix}$$

$x = -\frac{114}{31}$

$y = \frac{69}{31}$

$z = \frac{42}{31}$

f) $\begin{cases} 2x - 6y - 12z = 6 \\ 3x - 10y - 20z = 5 \\ 2x - 17z = -4 \end{cases}$

$$\begin{bmatrix} 2 & -6 & -12 & 6 \\ 3 & -10 & -20 & 5 \\ 2 & 0 & -17 & -4 \end{bmatrix}$$

$R_1 / 2 \to R_1$

$$\begin{bmatrix} 1 & -3 & -6 & 3 \\ 3 & -10 & -20 & 5 \\ 2 & 0 & -17 & -4 \end{bmatrix}$$

$R_2 - 3R_1 \to R_2$

$$\begin{bmatrix} 1 & -3 & -6 & 3 \\ 0 & -1 & -2 & -4 \\ 2 & 0 & -17 & -4 \end{bmatrix} \to$$

$R_3 - 2R_1 \to R_3$

$(-1) R_2 \to R_2$

$R_3 - 6R_2 \to R_3$

$$\begin{bmatrix} 1 & -3 & -6 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 6 & -5 & -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & -6 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -17 & -34 \end{bmatrix} \to$$

1 2

$R_2 - 2R_3 \to R_2$ $\qquad$ $R_1 + 6R_3 \to R_1$

$$\to \left[\begin{array}{ccc|c} 1 & -3 & -6 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -3 & 0 & 15 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \to$$

$R_1 + 3R_2 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$x = 15$

$y = 0$

$z = 2$

9) $\begin{cases} 2x - y + z = 2 \\ 3x + y - 6z = -7 \\ x - y + 2z = 3 \end{cases}$ $\qquad$ $\left[\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 3 & 1 & -6 & -7 \\ 1 & -1 & 2 & 3 \end{array}\right]$

$R_1/2 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & -(1/2) & 1/2 & 1 \\ 3 & 1 & -6 & -7 \\ 1 & -1 & 2 & 3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -(1/2) & 1/2 & 1 \\ 0 & 5/2 & -(15/2) & -10 \\ 1 & -1 & 2 & 3 \end{array}\right] \to$$

$R_3 - R_1 \to R_3$ $\qquad$ $R_2/(5/2) \to R_2$

$$\to \left[\begin{array}{ccc|c} 1 & -(1/2) & 1/2 & 1 \\ 0 & 5/2 & -(15/2) & -10 \\ 0 & -(1/2) & 3/2 & 2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -(1/2) & 1/2 & 1 \\ 0 & 1 & -3 & -4 \\ 0 & -(1/2) & 3/2 & 2 \end{array}\right] \to$$

$R_3 + (1/2)R_2 \to R_3$ $\qquad$ $R_1 + (1/2)R_2 \to R_1$

$$\to \left[\begin{array}{ccc|c} 1 & -(1/2) & 1/2 & 1 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x - z = -1$ $\qquad$ $x = z - 1$

$y - 3z = -4$ $\qquad$ $y = 3z - 4$

$z = z \quad (-\infty; +\infty)$

h) $\begin{cases} 3x+2y+z=3 \\ x-y-z=2 \\ x+4y+3z=-1 \end{cases}$ $\left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 1 & -1 & -1 & 2 \\ 1 & 4 & 3 & -1 \end{array}\right]$

$R_1 - 3R_2 \to R_1$ $\quad$ $R_2 - R_1 \to R_2$

$\left[\begin{array}{ccc|c} 1 & 4 & 3 & -1 \\ 1 & -1 & -1 & 2 \\ 1 & 4 & 3 & -1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 4 & 3 & -1 \\ 0 & -5 & -4 & 3 \\ 1 & 4 & 3 & -1 \end{array}\right] \to$

$R_3 - R_1 \to R_3$ $\quad$ $R_2 / (-5) \to R_2$

$\to \left[\begin{array}{ccc|c} 1 & 4 & 3 & -1 \\ 0 & -5 & -4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 4 & 3 & -1 \\ 0 & 1 & 4/5 & -(3/5) \\ 0 & 0 & 0 & 0 \end{array}\right] \to$

$R_1 - 4R_2 \to R_1$

$\left[\begin{array}{ccc|c} 1 & 0 & -(1/5) & 7/5 \\ 0 & 1 & 4/5 & -(3/5) \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x - (1/5)z = 7/5 \quad x = 7/5 + (1/5)z$

$y + (4/5)z = -(3/5) \quad y = -(3/5 + (4/5)z)$

$z = z \quad (-\infty; +\infty)$

i) $\begin{cases} 2x+2y+z=2 \\ x-2y+2z=1 \\ -x+2y-2z=-1 \end{cases}$ $\left[\begin{array}{ccc|c} 2 & 2 & 1 & 2 \\ 1 & -2 & 2 & 1 \\ -1 & 2 & -2 & -1 \end{array}\right]$

$R_1 - R_2 \to R_1$ $\quad$ $R_2 - R_1 \to R_2$

$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 1 \\ 1 & -2 & 2 & 1 \\ -1 & 2 & -2 & -1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 4 & -1 & 1 \\ 0 & -6 & 3 & 0 \\ -1 & 2 & -2 & -1 \end{array}\right] \to$

$R_3 + R_1 \to R_3$ $\quad$ $R_3 + R_2 \to R_3$

$\to \left[\begin{array}{ccc|c} 1 & 4 & -1 & 1 \\ 0 & -6 & 3 & 0 \\ 0 & 6 & -3 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 4 & -1 & 1 \\ 0 & -6 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to$

$R_2 / (-6) \to R_2$ $\quad R_1 - 4R_2 \to R_1$

$$\to \left[\begin{array}{ccc|c} 1 & 4 & -1 & 1 \\ 0 & 1 & -(1/2) & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & (-1/2) & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + z = 1$ $\quad y - (1/2)z = 0$ $\quad z = z$

$x = 1 - z$ $\quad y = (1/2)z$ $\quad (-\infty; +\infty)$

j) $\begin{cases} 3x - 2y + 5z = 14 \\ 2x - 3y + 4z = 8 \\ x - 8y + 3z = 2 \end{cases}$ $\quad R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & -8 & 3 & 2 \\ 2 & -3 & 4 & 8 \\ 3 & -2 & 5 & 14 \end{array}\right] \to$$

$R_2 - 2R_1 \to R_2$ $\quad R_3 - 3R_1 \to R_3$

$$\to \left[\begin{array}{ccc|c} 1 & -8 & 3 & 2 \\ 0 & 13 & -2 & 4 \\ 3 & -2 & 5 & 14 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -8 & 3 & 2 \\ 0 & 13 & -2 & 4 \\ 0 & 22 & -4 & 8 \end{array}\right] \to$$

$R_2 / 13 \to R_2$ $\quad R_3 - 22 \cdot R_2 \to R_3$

$$\to \left[\begin{array}{ccc|c} 1 & -8 & 3 & 2 \\ 0 & 1 & -(2/13) & 4/13 \\ 0 & 22 & -4 & 8 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -8 & 3 & 2 \\ 0 & 1 & -(2/13) & 4/13 \\ 0 & 0 & -(8/13) & 16/13 \end{array}\right] \to$$

$R_3 / -(8/13) \to R_3$ $\quad R_2 + (2/13)R_3 \to R_2$

$$\to \left[\begin{array}{ccc|c} 1 & -8 & 3 & 2 \\ 0 & 1 & -(2/13) & 4/13 \\ 0 & 0 & 1 & -2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -8 & 3 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{array}\right] \to$$

$R_1 - 3R_3 \to R_1$ $\quad R_1 + 8R_2 \to R_1$

$$\to \left[\begin{array}{ccc|c} 1 & -8 & 0 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$x = 8$

$y = 0$

$z = -2$

**k)**

$$\begin{cases} 2x - 5y + z = -9 \\ x + 4y - 6z = 2 \\ 3x - 4y - 2z = -10 \end{cases}$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 4 & -6 & 2 \\ 2 & -5 & 1 & -9 \\ 3 & -4 & -2 & -10 \end{array}\right] \longrightarrow$$

$R_2 - 2 \cdot R_1 \to R_2$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 4 & -6 & 2 \\ 0 & -13 & 13 & -13 \\ 3 & -4 & -2 & -10 \end{array}\right]$$

$R_3 - 3 \cdot R_1 \to R_3$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 4 & -6 & 2 \\ 0 & -13 & 13 & -13 \\ 0 & -16 & 16 & -16 \end{array}\right] \longrightarrow$$

$-R_2 + R_3 \to R_3$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 4 & -6 & 2 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$-R_2 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 4 & -6 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow$$

$R_1 - 4R_2 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & -2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x - 2z = -2 \quad x = 2z - 2$

$y - z = 1 \quad y = z + 1$

$z = z \quad (-\infty; +\infty)$

**l)**

$$\begin{cases} 2x + y - z = 2 \\ x - y + 2z = 3 \\ x + y - z = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & 3 \\ 2 & 1 & -1 & 2 \end{array}\right] \longrightarrow$$

$R_2 - R_1 \to R_2$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & 2 \\ 2 & 1 & -1 & 2 \end{array}\right]$$

$R_3 - 2R_1 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & 2 \\ 0 & -1 & 1 & 0 \end{array}\right] \longrightarrow$$

$R_2 / (-2) \to R_2$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & (-3/2) & -1 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

$R_3 + R_2 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & -(3/2) & -1 \\ 0 & 0 & -(1/2) & -1 \end{array}\right] \longrightarrow$$

$-R_3 \cdot 2 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & -(3/2) & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right] \to$$

$R_2 + (3/2)R_3 \to R_2$

$R_1 + R_3 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_1 - R_2 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$x = 1$

$y = 2$

$z = 2$

**m)**

$$\begin{cases} W + 3x + 2y + 2z = 3 \\ W + x + 3y = 4 \\ 2W + 2y - 3z = 4 \\ W - 3x = 1 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 3 \\ 1 & 1 & 3 & 0 & 4 \\ 2 & 0 & 2 & -3 & 4 \\ 1 & -3 & 0 & 0 & 1 \end{array}\right]$$

1) $R_3 - 2R_1 \to R_3$
2) $R_2 - R_1 \to R_2$

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 3 \\ 0 & -2 & 1 & -2 & 1 \\ 0 & -6 & -2 & -7 & -2 \\ 1 & -3 & 0 & 0 & 1 \end{array}\right] \to$$

1) $R_2 / (-2) \to R_2$
2) $R_4 - R_1 \to R_4$

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 3 \\ 0 & 1 & -(1/2) & 1 & -(1/2) \\ 0 & -6 & -2 & -7 & -2 \\ 0 & -6 & -2 & -2 & -2 \end{array}\right] \to$$

1) $R_3 + 6R_2 \to R_3$
2) $R_4 + 6R_2 \to R_4$

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 3 \\ 0 & 1 & -(1/2) & 1 & -(1/2) \\ 0 & 0 & -5 & -1 & -5 \\ 0 & 0 & -5 & 4 & -5 \end{array}\right] \to$$

1) $R_3 / (-5) \to R_3$
2) $R_4 + 5R_3 \to R_4$

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 3 \\ 0 & 1 & -(1/2) & 1 & -(1/2) \\ 0 & 0 & 1 & 1/5 & 1 \\ 0 & 0 & 0 & 5 & 0 \end{array}\right] \to$$

1

$R_3 - (1/5)R_4 \to R_3$

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 3 \\ 0 & 1 & -(1/2) & 1 & -(1/2) \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 3 & 2 & 2 & 3 \\ 0 & 1 & -(1/2) & 0 & -(1/2) \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \to$$

$R_2 - R_4 \to R_2$

$R_1 - 2R_4 \to R_1$

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 0 & 3 \\ 0 & 1 & -(1/2) & 0 & -(1/2) \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 3 & 2 & 0 & 3 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$R_2 + (1/2)R_3 \to R_2$

$R_1 - 2R_3 \to R_1$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 0 & 1 \\ & & \equiv & & \\ & & \vdots & & \\ & & \equiv & & \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$R_1 - 3R_2 \to R_1$

$w = 1$
$x = 0$
$y = 1$
$z = 0$

h)

$$\begin{aligned} w + x + y + 2z &= 1 \\ w \quad\quad - 3y \quad\quad &= 2 \\ w - 3x \quad\quad + z &= -2 \\ x - 4y + z &= 0 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 1 \\ 1 & 0 & -3 & 0 & 2 \\ 1 & -3 & 0 & 1 & -2 \\ 0 & 1 & -4 & 1 & 0 \end{array}\right]$$

1) $R_2 - R_1 \to R_2$
2) $R_3 - R_1 \to R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 1 \\ 0 & -1 & -4 & -2 & 1 \\ 0 & -4 & -1 & -1 & -3 \\ 0 & 1 & -4 & 1 & 0 \end{array}\right] \to$$

1) $(-1) \cdot R_2 \to R_2$
2) $R_3 + 4R_2 \to R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 1 \\ 0 & 1 & 4 & 2 & 1 \\ 0 & 0 & 15 & 7 & -7 \\ 0 & 1 & -4 & 1 & 0 \end{array}\right] \to$$

$R_4 - R_2 \to R_4$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 1 \\ 0 & 1 & 4 & 2 & 1 \\ 0 & 0 & 15 & 7 & -7 \\ 0 & 0 & -8 & -1 & 1 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 1 \\ 0 & 1 & 4 & 2 & 1 \\ 0 & 0 & 1 & 7/15 & -(7/15) \\ 0 & 0 & -8 & -1 & 1 \end{array}\right] \to$$

$R_3/15 \to R_3$

$R_4 + 8R_3 \to R_4$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 1 \\ 0 & 1 & 4 & 2 & 1 \\ 0 & 0 & 1 & 7/15 & -(7/15) \\ 0 & 0 & 0 & 41/15 & -(41/15) \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 1 \\ 0 & 1 & 4 & 2 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \to$$

$1 \quad -1$

$R_3 - (7/15)R_4 \to R_3$

$R_2 - 2R_4 \to R_2$

$R_1 - 2R_4 \to R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 3 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \to$$

$R_2 - 4R_3 \to R_2$

$R_1 - R_3 \to R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$R_1 - R_2 \to R_1$

$w = 2$

$x = 1$

$y = 0$

$z = -1$

P)

$$\begin{cases} x + 4y - 6z - 3w = 3 \\ x - y + 2w = -5 \\ x + z + w = 1 \\ y + z + w = 0 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 1 & -1 & 0 & 2 & -5 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$R_2 - R_1 \to R_2$

$R_3 - R_1 \to R_3$

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 0 & -5 & 6 & 5 & -8 \\ 0 & -4 & 7 & 4 & -2 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] \longrightarrow$$

$R_2 / (-5) \to R_2$

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 0 & 1 & -(6/5) & -1 & 8/5 \\ 0 & -4 & 7 & 4 & -2 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] \longrightarrow$$

$R_3 + 4R_2 \to R_3$

$$\longrightarrow \left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 0 & 1 & -(6/5) & -1 & 8/5 \\ 0 & 0 & 11/5 & 0 & 22/5 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] \longrightarrow$$

$R_4 - R_2 \to R_4$

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 0 & 1 & -(6/5) & -1 & 8/5 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 11/5 & 2 & -(8/5) \end{array}\right] \longrightarrow$$

$R_4 - (11/5) R_3 \to R_4$

$$\longrightarrow \left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 0 & 1 & -(6/5) & -1 & 8/5 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 2 & -6 \end{array}\right] \longrightarrow$$

$R_2 + R_4 \to R_2$

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & -3 & 3 \\ 0 & 1 & -(6/5) & 0 & -(7/5) \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right] \longrightarrow$$

$R_1 + 3R_4 \to R_1$

$$\longrightarrow \left[\begin{array}{cccc|c} 1 & 4 & -6 & 0 & -6 \\ 0 & 1 & -(6/5) & 0 & -(7/5) \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right] \longrightarrow$$

$R_2 + (6/5) R_3 \to R_2$

$$\left[\begin{array}{cccc|c} 1 & 4 & -6 & 0 & -6 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right] \longrightarrow$$

$R_1 + 6R_3 \to R_1$

$$\longrightarrow \left[\begin{array}{cccc|c} 1 & 4 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$$

$R_1 - 4R_2 \to R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$$

$x = 2$

$y = 1$

$z = 2$

$w = -3$

8) $\begin{cases} 3x + 2y + z - w = 3 \\ x - y - 2z + 2w = 2 \\ 2x + 3y - z + w = 1 \\ -x + y + 2z - 2w = -2 \end{cases}$

$$\left[\begin{array}{cccc|c} 3 & 2 & 1 & -1 & 3 \\ 1 & -1 & -2 & 2 & 2 \\ 2 & 3 & -1 & 1 & 1 \\ -1 & 1 & 2 & -2 & -2 \end{array}\right] \longrightarrow$$

$R_1 \cdot (-1) \to R_1$

$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & 2 & 2 \\ 1 & -1 & -2 & 2 & 2 \\ 2 & 3 & -1 & 1 & 1 \\ 3 & 2 & 1 & -1 & 3 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & -1 & -2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 3 & -1 & 1 & 1 \\ 3 & 2 & 1 & -1 & 3 \end{array}\right] \longrightarrow$$

$R_2 - R_1 \to R_2$

$R_3 - 2R_1 \to R_3$

$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 3 & -3 & -3 \\ 3 & 2 & 1 & -1 & 3 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & -1 & -2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 3 & -3 & -3 \\ 0 & 5 & 7 & -7 & -3 \end{array}\right] \longrightarrow$$

$R_4 - 3R_1 \to R_4$

$R_3 \leftrightarrow R_2$

$R_2 / 5 \to R_2$

$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & 2 & 2 \\ 0 & 1 & 3/5 & -(3/5) & -(3/5) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 7 & -7 & -3 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & -1 & -2 & 2 & 2 \\ 0 & 1 & 3/5 & -(3/5) & -(3/5) \\ 0 & 0 & 4/1 & -4/1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \to$$

$R_4 \leftrightarrow R_3$

$R_3 - 5R_2 \to R_3$

$R_2 - (3/5) R_3 \to R_2$

$$\left[\begin{array}{cccc|c} 1 & -1 & -2 & 2 & 2 \\ 0 & 1 & 0 & 0 & -(3/5) \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -(3/5) \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \to$$

$R_1 + 2R_3 \to R_1$

$R_1 + R_2 \to R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 7/5 \\ 0 & 1 & 0 & 0 & -(3/5) \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x = 7/5$

$y = -(3/5)$

$z = w \quad (-\infty; +\infty)$

14

c) $\begin{cases} 2x+3y-z+3v+w=22 \\ 3x+y-4z+3v-w=0 \\ x+y+3z-4v+2w=6 \\ x+2y-3z+2v-2w=-6 \\ 2x+4v-5w=-7 \end{cases}$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 3 & -4 & 2 & 6 \\ 3 & 1 & -4 & 3 & -1 & 0 \\ 2 & 3 & -1 & 3 & 1 & 22 \\ 1 & 2 & -3 & 2 & -2 & -6 \\ 2 & 0 & 0 & 4 & -5 & -7 \end{array}\right]$$

1) $R_2 - 3R_1 \to R_2$
2) $R_3 - 2R_1 \to R_3$
3) $R_4 - R_1 \to R_4$
4) $R_5 - 2R_1 \to R_5$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 3 & -4 & 2 & 6 \\ 0 & -2 & -13 & 15 & -7 & -18 \\ 0 & 1 & -7 & 11 & -3 & 10 \\ 0 & 1 & -6 & 6 & -4 & -12 \\ 0 & -2 & -6 & 12 & -9 & -19 \end{array}\right]$$

1) $R_2/(-2) \to R_2$
2) $R_3 - R_2 \to R_3$
3) $R_4 - R_2 \to R_4$
4) $R_5 + 2R_2 \to R_5$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 3 & -4 & 2 & 6 \\ 0 & 1 & (13/2) & (-15/2) & (7/2) & 9 \\ 0 & 0 & -(27/2) & (37/2) & -(13/2) & 1 \\ 0 & 0 & -(25/2) & (27/2) & -(15/2) & -21 \\ 0 & 0 & 7 & -3 & -2 & -1 \end{array}\right]$$

1) $R_3/-(27/2) \to R_3$
2) $R_4 + (25/2)R_3 \to R_4$
3) $R_5 - 7R_3 \to R_5$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 3 & -4 & 2 & 6 \\ 0 & 1 & 13/2 & -(15/2) & 7/2 & 9 \\ 0 & 0 & 1 & -(37/27) & 13/27 & -(2/27) \\ 0 & 0 & 0 & -(98/27) & -(40/27) & -(592/27) \\ 0 & 0 & 0 & 178/27 & -(145/27) & -(13/27) \end{array}\right]$$

1) $R_4/-(98/27) \to R_4$
2) $R_5 - (178/27)R_4 \to R_5$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 3 & -4 & 2 & 6 \\ 0 & 1 & 13/2 & -(15/2) & 7/2 & 9 \\ 0 & 0 & 1 & -(37/27) & 13/27 & -(2/27) \\ 0 & 0 & 0 & 1 & 20/49 & 296/49 \\ 0 & 0 & 0 & 0 & -(395/49) & -(1975/49) \end{array}\right]$$

$-(395/49) \to 1$ $\quad -(1975/49) \to 5$

$R_1 - 2R_5 \to R_1$
$R_2 - (7/2)R_5 \to R_2$
$R_3 - (13/27)R_5 \to R_3$
$R_4 - (20/49)R_5 \to R_4$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 3 & -4 & 0 & -4 \\ 0 & 1 & 13/2 & -(15/2) & 0 & -(17/2) \\ 0 & 0 & 1 & -(37/27) & 0 & -(67/27) \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 5 \end{array}\right]$$

$R1 + 4R4 \to R1$
$R2 + (15/2)R4 \to R2$
$R3 + (37/27)R4 \to R3$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 3 & 0 & 0 & 12 \\ 0 & 1 & 13/2 & 0 & 0 & 43/2 \\ 0 & 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 5 \end{array}\right]$$

$R1 - 3R3 \to R1$
$R2 - (13/2)R3 \to R2$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 5 \end{array}\right]$$

$R1 - R2 \to R1$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 5 \end{array}\right] \quad \begin{array}{l} X = 1 \\ y = 2 \\ Z = 3 \\ V = 4 \\ W = 5 \end{array}$$

**2.** Find inverse matrix and determinant using Gauss-Jordan method.

$$A = \begin{bmatrix} 3 & 1 & 0 & 2 \\ 1 & 2 & -1 & 0 \\ -1 & 4 & 0 & 7 \\ 3 & 0 & -6 & 2 \end{bmatrix}$$

$R2 - (1/3)R1 \to R2$

$$\begin{bmatrix} 3 & 1 & 0 & 2 \\ 0 & 5/3 & -1 & -(2/3) \\ -1 & 4 & 0 & 7 \\ 3 & 0 & -6 & 2 \end{bmatrix} \longrightarrow$$

$R3 + (1/3)R1 \to R3$

$$\longrightarrow \begin{bmatrix} 3 & 1 & 0 & 2 \\ 0 & 5/3 & -1 & -(2/3) \\ 0 & 13/3 & 0 & 23/3 \\ 3 & 0 & -6 & 2 \end{bmatrix}$$

$R4 - R1 \to R4$

$$\longrightarrow \begin{bmatrix} 3 & 1 & 0 & 2 \\ 0 & 5/3 & -1 & -(2/3) \\ 0 & 13/3 & 0 & 23/3 \\ 0 & -1 & -6 & 0 \end{bmatrix} \longrightarrow$$

$R_3 - (13/5)R_2 \to R_3$

$$\begin{bmatrix} 3 & 1 & 0 & 2 \\ 0 & 5/3 & -1 & -(2/3) \\ 0 & 0 & 13/5 & 47/5 \\ 0 & -1 & -6 & 0 \end{bmatrix} \to \begin{bmatrix} 3 & 1 & 0 & 2 \\ 0 & 5/3 & -1 & -(2/3) \\ 0 & 0 & 13/5 & 47/5 \\ 0 & 0 & -(33/5) & -(2/5) \end{bmatrix} \to$$

$R_4 + (3/5)R_2 \to R_4$

$R_4 + (33/13)R_3 \to R_4$

$$\begin{bmatrix} 3 & 1 & 0 & 2 \\ 0 & 5/3 & -1 & -(2/3) \\ 0 & 0 & 13/5 & 47/5 \\ 0 & 0 & 0 & 305/13 \end{bmatrix}$$

$|A| = 3 \cdot (5/3) \cdot (13/5) \cdot \frac{305}{13} =$

$= 305 \neq 0$ Invertible

$$\left[\begin{array}{cccc|cccc} 3 & 1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 4 & 0 & 7 & 0 & 0 & 1 & 0 \\ 3 & 0 & -6 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1/3 \to R_1$, $R_2 - R_1 \to R_2$, $R_3 + R_1 \to R_3$, $R_4 - 3R_1 \to R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 1/3 & 0 & 2/3 & 1/3 & 0 & 0 & 0 \\ 0 & 5/3 & -1 & -(2/3) & -(1/3) & 1 & 0 & 0 \\ 0 & 13/3 & 0 & 23/3 & 1/3 & 0 & 1 & 0 \\ 0 & -1 & -6 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_2/(5/3) \to R_2$, $R_3 - (13/3)R_2 \to R_3$, $R_4 + R_2 \to R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 1/3 & 0 & 2/3 & 1/3 & 0 & 0 & 0 \\ 0 & 1 & -(3/5) & -(2/5) & -(1/5) & 3/5 & 0 & 0 \\ 0 & 0 & 13/5 & 47/5 & 6/5 & -(13/5) & 1 & 0 \\ 0 & 0 & -(33/5) & -(2/5) & -(6/5) & 3/5 & 0 & 1 \end{array}\right]$$

$R_3/(13/5) \to R_3$, $R_4 + (33/5)R_3 \to R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 1/3 & 0 & 2/3 & 1/3 & 0 & 0 & 0 \\ 0 & 1 & -(3/5) & -(2/5) & -(1/5) & 3/5 & 0 & 0 \\ 0 & 0 & 1 & 47/13 & 6/13 & -1 & 5/13 & 0 \\ 0 & 0 & 0 & 305/13 & 24/13 & -6 \cdot 33/13 & 1 & \end{array}\right]$$

$$R4 / (305/13) \rightarrow \left[\begin{array}{cccc|cccc} 1 & 1/3 & 0 & 2/3 & 1/3 & 0 & 0 & 0 \\ 0 & 1 & -(3/5) & -(2/5) & -(1/5) & 3/5 & 0 & 0 \\ 0 & 0 & 1 & 47/13 & 6/13 & -1 & 5/13 & 0 \\ 0 & 0 & 0 & 1 & 24/305 & \frac{-78}{305} & \frac{33}{305} & \frac{13}{305} \end{array}\right]$$

$$R3 - (47/13)R4 \Rightarrow R3 \rightarrow \left[\begin{array}{cccc|cccc} 0 & 0 & 1 & 0 & 54/305 & -(23/305) & \frac{-2}{305} & \frac{-47}{305} \end{array}\right]$$

$$R2 + (2/5)R4 \Rightarrow \left[\begin{array}{cccc|cccc} 0 & 1 & -(3/5) & 0 & -(257/1525) & 759/1525 & 66/1525 & 26/1525 \end{array}\right]$$

$$R1 - (2/3)R4 \Rightarrow \left[\begin{array}{cccc|cccc} 1 & 1/3 & 0 & 0 & 257/915 & 52/305 & -(22/305) & -(26/915) \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 1/3 & 0 & 0 & 257/915 & 52/305 & -(22/305) & -(26/915) \\ 0 & 1 & -(3/5) & 0 & -(257/1525) & 759/1525 & 66/1525 & 26/1525 \\ 0 & 0 & 1 & 0 & 54/305 & -(23/305) & -(2/305) & -(47/305) \\ 0 & 0 & 0 & 1 & 24/305 & -(78/305) & 33/305 & 13/305 \end{array}\right]$$

$$R2 + (3/5)R3 \rightarrow R2 \left[\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & -(19/305) & 138/305 & 12/305 & -(23/305) \end{array}\right]$$

$$R1 - (1/3)R2 \rightarrow R1 \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 92/305 & 6/305 & -(26/305) & -(1/305) \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 92/305 & 6/305 & -(26/305) & -(1/305) \\ -(19/305) & 138/305 & 12/305 & -(23/305) \\ 54/305 & -(23/305) & -(2/305) & -(47/305) \\ 24/305 & -(78/305) & 33/305 & 13/305 \end{bmatrix}$$

Find determinant and inverse matrix

$$B = \begin{bmatrix} -2 & 0 & -1 & 4 \\ 0 & 6 & -3 & 9 \\ 4 & 0 & -5 & 6 \\ -1 & 3 & 0 & -5 \end{bmatrix}$$

$R_3 + 2R_1 \to R_3$

$R_4 - (1/2)R_1 \to R_4$

$$\begin{bmatrix} -2 & 0 & -1 & 4 \\ 0 & 6 & -3 & 9 \\ 0 & 0 & -7 & 14 \\ 0 & 3 & 1/2 & -7 \end{bmatrix}$$

$R_4 - (1/2)R_2 \to R_4$

$$\begin{bmatrix} -2 & 0 & -1 & 4 \\ 0 & 6 & -3 & 9 \\ 0 & 0 & -7 & 14 \\ 0 & 0 & 2 & -(23/2) \end{bmatrix}$$

$R_4 + (2/7)R_3 \to R_4$

$$\begin{bmatrix} -2 & 0 & -1 & 4 \\ 0 & 6 & -3 & 9 \\ 0 & 0 & -7 & 14 \\ 0 & 0 & 0 & -7,5 \end{bmatrix}$$

$|B| = (-2) \cdot 6 \cdot (-7) \cdot (-7,5) = -630$

$$\left[\begin{array}{cccc|cccc} -2 & 0 & -1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 6 & -3 & 9 & 0 & 1 & 0 & 0 \\ 4 & 0 & -5 & 6 & 0 & 0 & 1 & 0 \\ -1 & 3 & 0 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1/(-2)$, $R_2/6$, $R_3 - R_1 \cdot 4$, $R_4 + R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1/2 & -2 & -(1/2) & 0 & 0 & 0 \\ 0 & 1 & -(1/2) & 3/2 & 0 & 1/6 & 0 & 0 \\ 0 & 0 & -7 & 14 & 2 & 0 & 1 & 0 \\ 0 & 3 & 1/2 & -7 & -(1/2) & 0 & 0 & 1 \end{array}\right]$$

$R_3/(-7)$, $R_4 - 3R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1/2 & -2 & -(1/2) & 0 & 0 & 0 \\ 0 & 1 & -(1/2) & 3/2 & 0 & 1/6 & 0 & 0 \\ 0 & 0 & 1 & -2 & -(2/7) & 0 & -(1/7) & 0 \\ 0 & 0 & 2 & -(23/2) & -(1/2) & -(1/2) & 0 & 1 \end{array}\right]$$

$R_4 - 2R_3 \to$

$$\left[\begin{array}{cccc|cccc} 0 & 0 & 0 & -(15/2) & 1/14 & -(1/2) & 2/7 & 1 \end{array}\right]$$

$R_4/-(15/2)$

$$\left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 1 & -(1/105) & 1/15 & -(4/105) & -(2/15) \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1/2 & -2 & -(1/2) & 0 & 0 & 0 \\ 0 & 1 & -(1/2) & 3/2 & 0 & 1/6 & 0 & 0 \\ 0 & 0 & 1 & -2 & -(2/7) & 0 & -(1/7) & 0 \\ 0 & 0 & 0 & 1 & -(1/105) & 1/15 & -(4/105) & -(2/15) \end{array}\right]$$

$R_3 + 2R_4 \to R_3 \left[\begin{array}{cccc|cccc} 0 & 0 & 1 & 0 & -(32/105) & 2/15 & -(23/105) & -(4/15) \end{array}\right]$

$R_2 - (3/2)R_4 \to R_2 \left[\begin{array}{cccc|cccc} 0 & 1 & -(1/2) & 0 & 1/70 & 1/15 & 2/35 & 1/5 \end{array}\right]$

$R_1 + 2R_4 \to R_1 \left[\begin{array}{cccc|cccc} 1 & 0 & 1/2 & 0 & -(109/210) & 2/15 & -(8/105) & -(4/15) \end{array}\right]$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1/2 & 0 & -(109/210) & 2/15 & -(8/105) & -(4/15) \\ 0 & 1 & -(1/2) & 0 & 1/70 & 1/15 & 2/35 & 1/5 \\ 0 & 0 & 1 & 0 & -(32/105) & 2/15 & -(23/105) & -(4/15) \\ 0 & 0 & 0 & 1 & -(1/105) & 1/15 & -(4/105) & -(2/15) \end{array}\right]$$

$R_2 + (1/2)R_3 \to R_2$

$R_1 - (1/2)R_3 \to R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -(11/30) & 1/15 & 1/30 & -(2/15) \\ 0 & 1 & 0 & 0 & -(29/210) & 2/15 & -(11/210) & 1/15 \\ 0 & 0 & 1 & 0 & -(32/105) & 2/15 & -(23/105) & -(4/15) \\ 0 & 0 & 0 & 1 & -(1/105) & 1/15 & -(4/105) & -(2/15) \end{array}\right]$$

$$B^{-1} = \left[\begin{array}{cccc} -(11/30) & 1/15 & 1/30 & -(2/15) \\ -(29/210) & 2/15 & -(11/210) & 1/15 \\ -(32/105) & 2/15 & -(23/105) & -(4/15) \\ -(1/105) & 1/15 & -(4/105) & -(2/15) \end{array}\right]$$

Find determinant and inverse matrix

$$C=\begin{bmatrix}4&5&1&0\\-1&2&0&4\\2&0&-1&7\\0&1&0&4\end{bmatrix}$$

$R_2+(1/4)R_1 \rightarrow$
$R_3-(1/2)R_1 \rightarrow$

$$\begin{bmatrix}4&5&1&0\\0&13/4&1/4&4\\0&-(5/2)&-(3/2)&7\\0&1&0&4\end{bmatrix}$$

$R_3+(10/13)R_2 \rightarrow$
$R_4-(4/13)R_2 \rightarrow$

$$\begin{bmatrix}4&5&1&0\\0&13/4&1/4&4\\0&0&-(17/13)&131/13\\0&0&-(1/13)&36/13\end{bmatrix}$$

$R_4-(1/17)R_3 \rightarrow$

$$\begin{bmatrix}4&5&1&0\\0&13/4&1/4&4\\0&0&-(17/13)&131/13\\0&0&0&37/17\end{bmatrix}$$

$|C| = 4\cdot(13/4)\cdot(-(17/13))\cdot(37/17) = -37$

$R_1/4 \rightarrow R_1$
$R_2+R_1 \rightarrow R_2$
$R_3-2R_1 \rightarrow R_3$

$$\left[\begin{array}{cccc|cccc}1&5/4&1/4&0&1/4&0&0&0\\0&13/4&1/4&4&1/4&1&0&0\\0&-(5/2)&-(3/2)&7&-(1/2)&0&1&0\\0&1&0&4&0&0&0&1\end{array}\right]$$

$R_2/(13/4)$
$R_3+(5/2)R_2$
$R_4-R_2$

$$\left[\begin{array}{cccc|cccc}1&5/4&1/4&0&1/4&0&0&0\\0&1&1/13&16/13&1/13&4/13&0&0\\0&0&-(17/13)&131/13&-(4/13)&10/13&1&0\\0&0&-(1/13)&36/13&-(1/13)&-(4/13)&0&1\end{array}\right]$$

$R_3/(-17/13)$
$R_4+(1/13)R_3$

$$\left[\begin{array}{cccc|cccc}1&5/4&1/4&0&1/4&0&0&0\\0&1&1/13&16/13&1/13&4/13&0&0\\0&0&1&-(131/17)&4/17&-(10/17)&-(13/17)&0\\0&0&0&37/17&-(1/17)&-(6/17)&-(1/17)&1\end{array}\right]$$

$$
\begin{array}{l}
\uparrow R_2-(16/13)R_4 \\
R_3+(131/17)R_4 \\
R_4/(37/17)
\end{array}
\left[\begin{array}{cccc|cccc}
1 & 5/4 & 1/4 & 0 & 1/4 & 0 & 0 & 0 \\
0 & 1 & 1/13 & 0 & 53/481 & 244/481 & 16/481 & -(272/481) \\
0 & 0 & 1 & 0 & 1/37 & -(68/37) & -(36/37) & 131/37 \\
0 & 0 & 0 & 1 & -(1/37) & -(6/37) & -(1/37) & 17/37
\end{array}\right]
$$

$$
\begin{array}{l}
R_1-(1/4)R_3 \\
\uparrow R_2-(1/13)R_3
\end{array}
\left[\begin{array}{cccc|cccc}
1 & 5/4 & 0 & 0 & 9/37 & 17/37 & 9/37 & -(131/148) \\
0 & 1 & 0 & 0 & 4/37 & 24/37 & 4/37 & -(31/37) \\
0 & 0 & 1 & 0 & 1/37 & -(68/37) & -(36/37) & 131/37 \\
0 & 0 & 0 & 1 & -(1/37) & -(6/37) & -(1/37) & 17/37
\end{array}\right]
$$

$R_1-(5/4)R_2 \Rightarrow R_1$

$$
\left[\begin{array}{cccc|cccc}
1 & 0 & 0 & 0 & 4/37 & -(13/37) & 4/37 & 6/37 \\
0 & 1 & 0 & 0 & 4/37 & 24/37 & 4/37 & -(31/37) \\
0 & 0 & 1 & 0 & 1/37 & -(68/37) & -(36/37) & 131/37 \\
0 & 0 & 0 & 1 & -(1/37) & -(6/37) & -(1/37) & 17/37
\end{array}\right]
$$

$$
C^{-1} = \left[\begin{array}{cccc}
4/37 & -(13/37) & 4/37 & 6/37 \\
4/37 & 24/37 & 4/37 & -(31/37) \\
1/37 & -(68/37) & -(36/37) & 131/37 \\
-(1/37) & -(6/37) & -(1/37) & 17/37
\end{array}\right]
$$

Find determinant and inverse matrix

$$
D = \left[\begin{array}{cccc}
2 & 4 & 0 & 1 \\
0 & -2 & 5 & 0 \\
-1 & 4 & 3 & 7
\end{array}\right]
$$

Matrix is not square.
Determinant is not defined.
Inverse matrix is not defined.

Find the inverse matrix and the determinant.

$$E = \begin{bmatrix} 4 & 5 & 0 & 0 & 2 \\ +1 & 0 & -3 & 0 & +1 \\ 9 & -4 & -3 & 0 & 0 \\ 4 & 3 & 2 & 4 & -2 \\ 2 & 0 & -1 & 0 & -1 \end{bmatrix} \leftarrow R_2 = R_2 \cdot (-1)$$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & -3 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 5 & 12 & 0 & -2 & 1 & 4 & 0 & 0 & 0 \\ 0 & -4 & 24 & 0 & -9 & 0 & 9 & 1 & 0 & 0 \\ 0 & 3 & 14 & 4 & -6 & 0 & 4 & 0 & 1 & 0 \\ 0 & 0 & 5 & 0 & -3 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \leftarrow R_2 - 4R_1 \\ \leftarrow R_3 - 9R_1 \\ \leftarrow R_4 - 4 \cdot R_1 \\ \leftarrow R_5 - 2 \cdot R_1 \end{array} \downarrow$$

$R2 \leftrightarrows R4$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & -3 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 3 & 14 & 4 & -6 & 0 & 4 & 0 & 1 & 0 \\ 0 & -4 & 24 & 0 & -9 & 0 & 9 & 1 & 0 & 0 \\ 0 & 5 & 12 & 0 & -2 & 1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & -3 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = R_2/3$, $R_3 = R_3 + 4R_2$, $R_4 = R_4 - 5R_2$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & -3 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 14/3 & 4/3 & -2 & 0 & 4/3 & 0 & 1/3 & 0 \\ 0 & 0 & 128/3 & 16/3 & -17 & 0 & 43/3 & 1 & 4/3 & 0 \\ 0 & 0 & -(34/3) & -(20/3) & 8 & 1 & -(8/3) & 0 & -(5/3) & 0 \\ 0 & 0 & 5 & 0 & -3 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_3 = R_3 / (128/3)$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & -3 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 14/3 & 4/3 & -2 & 0 & 4/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & 1/8 & -(51/128) & 0 & 43/128 & 3/128 & 1/32 & 0 \\ 0 & 0 & -(34/3) & -(20/3) & 8 & 1 & -(8/3) & 0 & -5/3 & 0 \\ 0 & 0 & 5 & 0 & -3 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_1 = R_1 + 3R_3$
$R_2 = R_2 - (14/3)R_3$
$R_4 = R_4 + (34/3)R_3$
$R_5 = R_5 - 5R_3$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 3/8 & -(25/128) & 0 & 1/128 & 9/128 & 3/32 & 0 \\ 0 & 1 & 0 & 3/4 & -(9/64) & 0 & -(15/64) & -(7/64) & 3/16 & 0 \\ 0 & 0 & 1 & 1/8 & -(51/128) & 0 & 43/128 & 3/128 & 1/32 & 0 \\ 0 & 0 & 0 & -(21/4) & 223/64 & 1 & 73/64 & 17/64 & -(21/16) & 0 \\ 0 & 0 & 0 & -(5/8) & -(129/128) & 0 & 41/128 & -(15/128) & -(5/32) & 1 \end{array}\right]$$

$R_4 = R_4 / (-(21/4))$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 3/8 & -(25/128) & 0 & 1/128 & 9/128 & 3/32 & 0 \\ 0 & 1 & 0 & 3/4 & -(9/64) & 0 & -(15/64) & -(7/64) & 3/16 & 0 \\ 0 & 0 & 1 & 1/8 & -(51/128) & 0 & 43/128 & 3/128 & 1/32 & 0 \\ 0 & 0 & 0 & 1 & -(223/336) & -(4/21) & -(73/336) & -(17/336) & 1/4 & 0 \\ 0 & 0 & 0 & -(5/8) & -(129/128) & 0 & 41/128 & -(15/128) & -(5/32) & 1 \end{array}\right]$$

$R_1 - (3/8)R_4$
$R_2 - (3/4)R_4$
$R_3 - (1/8)R_4$
$R_5 + (5/8)R_4$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 3/56 & 1/14 & 5/56 & 5/56 & 0 & 0 \\ 0 & 1 & 0 & 0 & 5/14 & 1/7 & -(1/14) & -(1/14) & 0 & 0 \\ 0 & 0 & 1 & 0 & -(53/168) & 1/42 & 61/168 & 5/168 & 0 & 0 \\ 0 & 0 & 0 & 1 & -(223/336) & -(4/21) & -(73/336) & -(17/336) & 1/4 & 0 \\ 0 & 0 & 0 & 0 & -(239/168) & -(5/42) & 31/168 & -(25/168) & 0 & 1 \end{array}\right]$$

$R_5 = R_5 / -(239/168)$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 3/56 & 1/14 & 5/56 & 5/56 & 0 & 0 \\ 0 & 1 & 0 & 0 & 5/14 & 1/7 & -(1/14) & -(1/14) & 0 & 0 \\ 0 & 0 & 1 & 0 & -(53/168) & 1/42 & 61/168 & 5/168 & 0 & 0 \\ 0 & 0 & 0 & 1 & -(223/336) & -(4/21) & -(73/336) & -(17/336) & 1/4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 20/239 & -(31/239) & 25/239 & 0 & -(168/239) \end{array}\right]$$

$R_1 - (3/56)R_5$
$R_2 - (5/14)R_5$
$R_3 + (53/168)R_5$
$R_4 + (223/336)R_5$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 0 & 16/239 & 23/239 & 20/239 & 0 & 9/239 \\ 0 & 1 & 0 & 0 & 0 & 27/239 & -(6/239) & -(26/239) & 0 & 60/239 \\ 0 & 0 & 1 & 0 & 0 & 12/239 & 77/239 & 15/239 & 0 & -(53/239) \\ 0 & 0 & 0 & 1 & 0 & -(129/956) & -(145/478) & 9/478 & 1/4 & -(223/478) \\ 0 & 0 & 0 & 0 & 1 & 20/239 & -(31/239) & 25/239 & 0 & -(168/239) \end{array}\right]$$

$$E^{-1} = \begin{bmatrix} 16/239 & 23/239 & 20/239 & 0 & 9/239 \\ 27/239 & -(6/239) & -(26/239) & 0 & 60/239 \\ 12/239 & 77/239 & 15/239 & 0 & -(53/239) \\ -(129/956) & -(145/478) & 9/478 & 1/4 & -(223/478) \\ 20/239 & -(31/239) & 25/239 & 0 & -(168/239) \end{bmatrix}$$

Determinant calculation using Gaussian elimination.

$$\begin{bmatrix} 4 & 5 & 0 & 0 & 2 \\ -1 & 0 & 3 & 0 & -1 \\ 9 & -4 & -3 & 0 & 0 \\ 4 & 3 & 2 & 4 & -2 \\ 2 & 0 & -1 & 0 & -1 \end{bmatrix}$$

$R1 \leftrightarrow R2$ (Det. sign flip)

$$\begin{bmatrix} -1 & 0 & 3 & 0 & -1 \\ 0 & 5 & 12 & 0 & -2 \\ 0 & -4 & 24 & 0 & -9 \\ 0 & 3 & 14 & 4 & -6 \\ 0 & 0 & 5 & 0 & -3 \end{bmatrix}$$

$R2 + 4R1$
$R3 + 9R1$
$R4 + 4R1$
$R5 + 2R1$

$R2 \leftrightarrow R4$ (Det. sign same (second flip))

$$\begin{bmatrix} -1 & 0 & 3 & 0 & -1 \\ 0 & 3 & 14 & 4 & -6 \\ 0 & 0 & 128/3 & 16/3 & -17 \\ 0 & 0 & -(34/3) & -(20/3) & 8 \\ 0 & 0 & 5 & 0 & -3 \end{bmatrix}$$

$R3 + (4/3)R2$
$R4 - (5/3)R2$

$$\begin{bmatrix} -1 & 0 & 3 & 0 & -1 \\ 0 & 3 & 14 & 4 & -6 \\ 0 & 0 & 128/3 & 16/3 & -17 \\ 0 & 0 & 0 & -(21/4) & 223/64 \\ 0 & 0 & 0 & -(5/8) & -(129/128) \end{bmatrix}$$

$R4 + (17/64)R3$
$R5 - (15/128)R3$

$R_5 = R_5 - (5/42) \cdot R_4$

$$\begin{bmatrix} -1 & 0 & 3 & 0 & -1 \\ 0 & 3 & 14 & 4 & -6 \\ 0 & 0 & 128/3 & 16/3 & -17 \\ 0 & 0 & 0 & -(21/4) & 223/64 \\ 0 & 0 & 0 & 0 & -(239/168) \end{bmatrix}$$

$$|E| = (+1) \cdot 3 \cdot (128/3) \cdot (+21/4) \cdot (-239/168) = \overset{32}{}$$

$$= 32 \cdot 21 \cdot (-(239/168)) = -160608/168 =$$

$$= -956$$

Find inverse matrix and determinant

$$F = \begin{bmatrix} 2 & -1 & 5 & 4 & 0 \\ 0 & 0 & 11 & 0 & 0 \\ 0 & -1 & 5 & 2 & 1 \\ 2 & 3 & 7 & -6 & 2 \\ 3 & 0 & 7 & 0 & 3 \end{bmatrix}$$

$R_4 - R_1$

$R_5 - (3/2) R_1$

$$\begin{bmatrix} 2 & -1 & 5 & 4 & 0 \\ 0 & 0 & 11 & 0 & 0 \\ 0 & -1 & 5 & 2 & 1 \\ 0 & 4 & 2 & -10 & 2 \\ 0 & 3/2 & -(1/2) & -6 & 3 \end{bmatrix}$$

$R_3 \leftrightarrow -R_2$

$$\begin{bmatrix} 2 & -1 & 5 & 4 & 0 \\ 0 & -1 & 5 & 2 & 1 \\ 0 & 0 & -11 & 0 & 0 \\ 0 & 4 & 2 & -10 & 2 \\ 0 & 3/2 & -(1/2) & -6 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 5 & 4 & 0 \\ 0 & -1 & 5 & 2 & 1 \\ 0 & 0 & -\pi & 0 & 0 \\ 0 & 0 & 22 & -2 & 6 \\ 0 & 0 & 7 & -3 & 9/2 \end{bmatrix}$$

$R_4 + 4R_2$

$R_5 + (3/2)R_2$

$$\begin{bmatrix} 2 & -1 & 5 & 4 & 0 \\ 0 & -1 & 5 & 2 & 1 \\ 0 & 0 & -\pi & 0 & 0 \\ 0 & 0 & 0 & -2 & 6 \\ 0 & 0 & 0 & -3 & 9/2 \end{bmatrix}$$

$R_4 + (22/\pi)R_3$

$R_5 + (7/\pi)R_3$

$$\begin{bmatrix} 2 & -1 & 5 & 4 & 0 \\ 0 & -1 & 5 & 2 & 1 \\ 0 & 0 & -\pi & 0 & 0 \\ 0 & 0 & 0 & -2 & 6 \\ 0 & 0 & 0 & 0 & -(9/2) \end{bmatrix}$$

$R_5 - (3/2)R_4$

$$|F| = 2 \cdot (+1) \cdot (+\pi) \cdot (+2) \cdot (+9/2) = 2 \cdot \pi \cdot 9 = 18\pi$$

$$\left[\begin{array}{ccccc|ccccc} 2 & -1 & 5 & 4 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \pi & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 5 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 3 & 7 & -6 & 2 & 0 & 0 & 0 & 1 & 0 \\ 3 & 0 & 7 & 0 & 3 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1/2$

$R_4 - 2R_1$

$R_5 - 3R_1$

$$\left[\begin{array}{ccccc|ccccc} 1 & -(1/2) & 5/2 & 2 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \pi & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 5 & 2 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 2 & -10 & 2 & -1 & 0 & 0 & 1 & 0 \\ 0 & 3/2 & -(1/2) & -6 & 3 & -(3/2) & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$R_2 \cdot (-1)$

$$\left[\begin{array}{ccccc|ccccc} 1 & -(1/2) & 5/2 & 2 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1 & -5 & -2 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & \widetilde{\pi} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 4 & 2 & -10 & 2 & -1 & 0 & 0 & 1 & 0 \\ 0 & 3/2 & -(1/2) & -6 & 3 & -(3/2) & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 + (1/2) R_2$

$R_4 - 4R_2$

$R_5 - (3/2) R_2$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 1 & -(1/2) & 1/2 & 0 & -(1/2) & 0 & 0 \\ 0 & 1 & -5 & -2 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & \widetilde{\pi} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 22 & -2 & 6 & -1 & 0 & 4 & 1 & 0 \\ 0 & 0 & 7 & -3 & 9/2 & -(3/2) & 0 & 3/2 & 0 & 1 \end{array}\right]$$

$R_3 \leftrightarrow R_5$

$R_3 / 7$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 1 & -(1/2) & 1/2 & 0 & -(1/2) & 0 & 0 \\ 0 & 1 & -5 & -2 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -(3/7) & 9/14 & -(3/14) & 0 & 3/14 & 0 & 1/7 \\ 0 & 0 & 22 & -2 & 6 & -1 & 0 & 4 & 1 & 0 \\ 0 & 0 & \widetilde{\pi} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right]$$

$R_2 + 5R_3$

$R_4 - 22R_3$

$R_5 - \widetilde{\pi} \cdot R_3$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 1 & -(1/2) & 1/2 & 0 & -(1/2) & 0 & 0 \\ 0 & 1 & 0 & -(29/7) & 31/14 & -(15/14) & 0 & 1/14 & 0 & 5/7 \\ 0 & 0 & 1 & -(3/7) & 9/14 & -(3/14) & 0 & 3/14 & 0 & 1/7 \\ 0 & 0 & 0 & 52/7 & -(57/7) & 26/7 & 0 & -(5/7) & 1 & -(22/7) \\ 0 & 0 & 0 & (3/7\widetilde{\pi}) & -(9/14\widetilde{\pi}) & 3/14\widetilde{\pi} & 1 & -(3/14\widetilde{\pi}) & 0 & -(1/7\widetilde{\pi}) \end{array}\right]$$

$R_4 / (52/7)$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 1 & -(1/2) & 1/2 & 0 & -(1/2) & 0 & 0 \\ 0 & 1 & 0 & -(29/7) & 31/14 & -(15/14) & 0 & 1/14 & 0 & 5/7 \\ 0 & 0 & 1 & -(3/7) & 9/14 & -(3/14) & 0 & 3/14 & 0 & 1/7 \\ 0 & 0 & 0 & 1 & -(57/52) & 1/2 & 0 & -(5/52) & 7/52 & -(11/26) \\ 0 & 0 & 0 & 3/7\widetilde{\pi} & -(9/14\widetilde{\pi}) & (3/14\widetilde{\pi} & 1 & -(3/14\widetilde{\pi}) & 0 & -(1/7\widetilde{\pi}) \end{array}\right]$$

$$
\begin{array}{l}
R_1 - R_4 \\
R_2 + (29/7)R_4 \\
R_3 + (3/7)R_4 \\
\\
R_5 - \left(\frac{3}{4\pi}\right)\cdot R_4
\end{array}
\left[\begin{array}{ccccc|ccccc}
1 & 0 & 0 & 0 & 31/52 & 0 & 0 & -(21/52) & -(7/52) & 11/26 \\
0 & 1 & 0 & 0 & -(121/52) & 1 & 0 & -(17/52) & 29/52 & -(27/26) \\
0 & 0 & 1 & 0 & 9/52 & 0 & 0 & 9/52 & 3/52 & -(1/26) \\
0 & 0 & 0 & 1 & -(57/52) & 1/2 & 0 & -(5/52) & 7/52 & -(11/26) \\
0 & 0 & 0 & 0 & -(9/52\pi) & 0 & 1 & -(9/52\pi) & -(3/52\pi) & 1/26\pi
\end{array}\right]
$$

$$R_5 / -(9/52\pi)$$

$$
\left[\begin{array}{ccccc|ccccc}
0 & 0 & 0 & 0 & 1 & 0 & -(52/9\pi) & 1 & 1/3 & -(2/9)
\end{array}\right]
$$

$$
\begin{array}{l}
R_1 - (31/52)\,R_5 \\
R_2 + (121/52)\,R_5 \\
R_3 - (9/52)\,R_5 \\
R_4 + (57/52)\,R_5 \\
\\
\end{array}
\left[\begin{array}{ccccc|ccccc}
1 & 0 & 0 & 0 & 0 & 0 & 31/9\pi & -1 & -(1/3) & 5/9 \\
0 & 1 & 0 & 0 & 0 & 1 & -(121/9\pi) & 2 & 4/3 & -(14/9) \\
0 & 0 & 1 & 0 & 0 & 0 & 1/\pi & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1/2 & -(19/3\pi) & 1 & 1/2 & -(2/3) \\
0 & 0 & 0 & 0 & 1 & 0 & -(52/9\pi) & 1 & 1/3 & -(2/9)
\end{array}\right]
$$

$$
F^{-1} = \begin{bmatrix}
0 & \frac{31}{9\pi} & -1 & -\frac{1}{3} & \frac{5}{9} \\
1 & -\frac{121}{9\pi} & 2 & \frac{4}{3} & -\frac{14}{9} \\
0 & \frac{1}{\pi} & 0 & 0 & 0 \\
\frac{1}{2} & -\frac{19}{3\pi} & 1 & \frac{1}{2} & -\frac{2}{3} \\
0 & -\frac{52}{9\pi} & 1 & \frac{1}{3} & -\frac{2}{9}
\end{bmatrix}
$$