Vector products

(1) Find $u \cdot v$

a) $u = (1, -5, 4)$ $v = (3, 3, 3)$

$u \cdot v = 1 \cdot 3 + (-5) \cdot 3 + 4 \cdot 3 = 3 + (-15) + 12 = 0$

b) $u = (-2, 2, 3)$ $v = (1, 7, -4)$

$u \cdot v = (-2) \cdot 1 + 2 \cdot 7 + 3 \cdot (-4) = -2 + 14 - 12 = 0$

(2) Find the angle between u and v

a) $u = (6, 1, 4)$ $v = (2, 0, -3)$

$\alpha = arccos \left(u \cdot v / (|u| \cdot |v|) \right)$

$u \cdot v = 6 \cdot 2 + 1 \cdot 0 + 4(-3) = 12 + 0 - 12 = 0$

$|u| = \sqrt{6^2 + 1^2 + 4^2} = \sqrt{53} \approx 7,28$

$|v| = \sqrt{2^2 + 0^2 + (-3)^2} = \sqrt{13} \approx 3,6$

$cos \alpha = 0 / (7,28 \cdot 3,6) = 0$

$\alpha = arccos (0) = 90°$

b) $u = (0, 0, -1)$ $v = (1, 1, 1)$

$u \cdot v = -1$ $|u| = 1$ $|v| \approx 1,7321$

$cos \alpha = -1 / (1 \cdot 1,7321) \approx -0,5774$

$\alpha = arccos (-0,5774)$

$\alpha = 125,26°$

c) $u = (-6, 0, 4)$ $v = (3, 1, 6)$

$u \cdot v = 6$ $|u| \approx 7,2111$ $|v| \approx 6,7823$

$cos \alpha = 6 / 7,2111 \cdot 6,7823 \approx 0,1227$

$\alpha = arccos (0,1227) \approx 82,95°$

d) $u = (2, 4, -8)$ $v = (5, 3, 7)$

$u \cdot v = -34$ $|u| \approx 9,1652$ $|v| \approx 9,1104$

$\cos \alpha = -34 / 9,1652 \cdot 9,1104 \approx -0,4072$

$\alpha = \arccos(-0,4072) \approx 114,03°$

③ Find the orthogonal projection of u on a.

a) $u = (6, 2)$ $a = (3; -9)$

$\text{proj}_a(u) = (u \cdot a / a \cdot a) \times a$

Dot products:

$u \cdot a = 6 \cdot 3 - 9 \cdot 2 = 0$

$a \cdot a = 90$

Scalar multiplication

$\text{proj}_a(u) = 0 \times (3, -9) = (0, 0)$

b) $u = (-1, -2)$ $a = (-2, 3)$

Dot products

$u \cdot a = (-1) \cdot (-2) + (-2) \cdot 3 = 2 + (-6) = -4$

$a \cdot a = (-2) \cdot (-2) + 3 \cdot 3 = 4 + 9 = 13$

$\text{proj}_a(u) = -0,3077 \cdot (-2, 3) = (0,6154, -0,9231)$

c) $u = (3, 1, -7)$ $a = (1, 0, 5)$

$u \cdot a = -32$ $a \cdot a = 26$

$\text{proj}_a(u) = -1,2308 \times (1, 0, 5) = (-1,2308; 0; -6,1538)$

d) $u = (1, 0, 0)$ $a = (4, 3, 8)$

$u \cdot a = 4$ $a \cdot a = 89$ $u \cdot a / a \cdot a \approx 0,0449$

$\text{proj}_a(u) = 0,0449 \times (4, 3, 8) = (0,1798; 0,1348; 0,3596)$

4. Let $u = (3, 2, -1)$ $v = (0, 2, -3)$ $W = (2, 6, 7)$

a) $V \times W$

$(0, 2, -3) \times (2, 6, 7)$

$x = 2 \cdot 7 - 6(-3) = 14 + 18 = 32$

$y = -(0 \cdot 7 - (-3) \cdot 2) = -6$

$z = 0 \cdot 6 - 2 \cdot 2 = -4$

$$\begin{bmatrix} i & j & k \\ 0 & 2 & -3 \\ 2 & 6 & 7 \end{bmatrix}$$

$(V \times W) = (32, -6, -4)$

B) $u \times (V - 2W)$

$V - 2W = (0, 2, -3) - (4, 12, 14) = (-4, -10, -17)$

$(3, 2, -1) \times (-4, -10, -17)$

$$\begin{bmatrix} i & j & k \\ 3 & 2 & -1 \\ -4 & -10 & -17 \end{bmatrix}$$

$x = 2 \cdot (-17) - ((-1) \cdot (-10)) =$
$= -34 - 10 = -44$

$y = -(3 \cdot (-17) - ((-1) \cdot (-4))) = -(-51 - 4) = -(-55) = 55$

$z = 3 \cdot (-10) - 2 \cdot (-4) = -30 + 8 = -22$

$u \times (V - 2W) = (-44, 55, -22)$

c) $(u \times V) - 2W$

$(3, 2, -1) \times (0, 2, -3)$

$x = 2 \cdot (-3) - (-1) \cdot 2 = -6 + 2 = -4$

$y = -(3 \cdot (-3) - 0) = 9$

$z = 3 \cdot 2 - 0 = 6$

$$\begin{bmatrix} i & j & k \\ 3 & 2 & -1 \\ 0 & 2 & -3 \end{bmatrix}$$

$(-4, 9, 6) - 2 \cdot (2, 6, 7) = (-4, 9, 6) - (4, 12, 14) =$
$= ((-4) - 4, 9 - 12, 6 - 14) = (-8, -3, -8)$

$(u \times V) - 2W = (-8, -3, -8)$

d) $u \times (v \times w)$

$v \times w = (32, -6, -4)$

$(3, 2, -1) \times (32, -6, -4)$

$$\begin{bmatrix} i & j & k \\ 3 & 2 & -1 \\ 32 & -6 & -4 \end{bmatrix}$$

$X = 2 \cdot (-4) - (-1) \cdot (-6) = -8 - 6 = -14$

$y = -(3 \cdot (-4) - (-1) \cdot (32)) = -(-12 + 32) = -20$

$Z = 3 \cdot (-6) - 2 \cdot 32 = -18 - 64 = -82$

$u \times (v \times w) = (-14, -20, -82)$

e) $(u \times v) \times w$

$$\begin{bmatrix} i & j & k \\ -4 & 9 & 6 \\ 2 & 6 & 7 \end{bmatrix}$$

$u \times v = (-4, 9, 6)$

$(u \times v) \times w = (-4, 9, 6) \times (2, 6, 7)$

$X = 9 \cdot 7 - 6 \cdot 6 = 27$

$y = -((-4) \cdot 7 - 6 \cdot 2) = -(-28 - 12) = 40$

$Z = (-4) \cdot 6 - 9 \cdot 2 = -24 - 18 = -42$

$(u \times v) \times w = (27, 40, -42)$

f) $(u \times v) \times (v \times w)$

$(-4, 9, 6) \times (32, -6, -4)$

$$\begin{bmatrix} i & j & k \\ -4 & 9 & 6 \\ 32 & -6 & -4 \end{bmatrix}$$

$X = 9 \cdot (-4) - 6 \cdot (-6) = 0$

$y = -((-4) \cdot (-4) - 6 \cdot 32) = -(16 - 192) = 176$

$Z = (-4) \cdot (-6) - 9 \cdot 32 = 24 - 288 = -264$

$(u \times v) \times (v \times w) = (0, 176, -264)$

5. Find the scalar triple product $u \cdot (v \times w)$

a) $u = (-1, 2, 4)$ $v = (3, 4, -2)$ $W = (-1, 2, 5)$

$$\begin{bmatrix} i & j & k \\ 3 & 4 & -2 \\ -1 & 2 & 5 \end{bmatrix}$$

$v \times w = (3, 4, -2) \times (-1, 2, 5)$

$X = 4 \cdot 5 - (-2) \cdot 2 = 20 + 4 = 24$

$y = -(3 \cdot 5 - (-2) \cdot (-1)) = -(15 - 2) = -13$

$Z = 3 \cdot 2 - 4(-1) = 6 + 4 = 10$

$u \cdot (v \times w) = (-1, 2, 4) \cdot (24, -13, 10) =$

$= (-1) \cdot 24 + 2 \cdot (-13) + 4 \cdot 10 = 40 - 24 - 26 = -10$

6) $u = (3, -1, 6)$ $v = (2, 4, 3)$ $w = (5, -1, 2)$

$$\begin{bmatrix} i & k & j \\ 2 & 4 & 3 \\ 5 & -1 & 2 \end{bmatrix}$$

$v \times w = (2, 4, 3) \times (5, -1, 2)$

$x = 4 \cdot 2 - 3(-1) = 8 + 3 = 11$

$y = -(2 \cdot 2 - 3 \cdot 5) = -(4 - 15) = 11$

$z = 2 \cdot (-1) - 4 \cdot 5 = -2 - 20 = -22$

$u \cdot (v \times w) = (3, -1, 6) \cdot (11, 11, -22) = 3 \cdot 11 + (-1) \cdot 11 + 6 \cdot (-22) =$

$= 33 - 11 - 132 = -110$