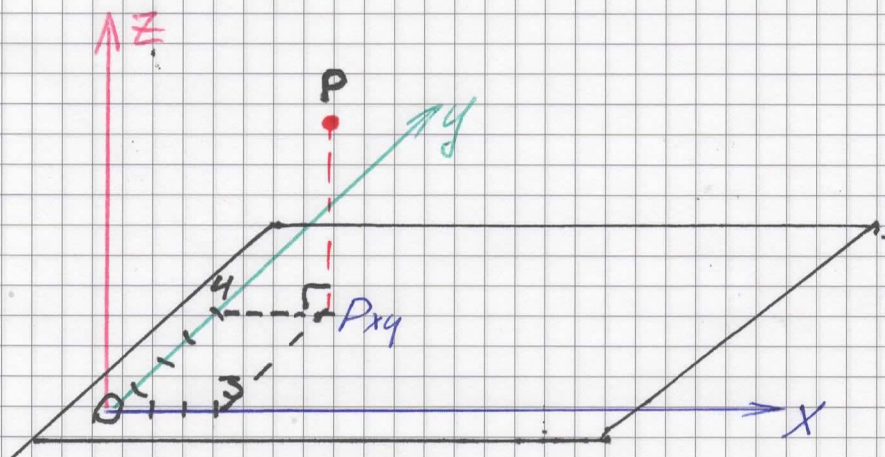


# Introduction to vectors

1 Draw a right-handed coordinate system and locate the points:

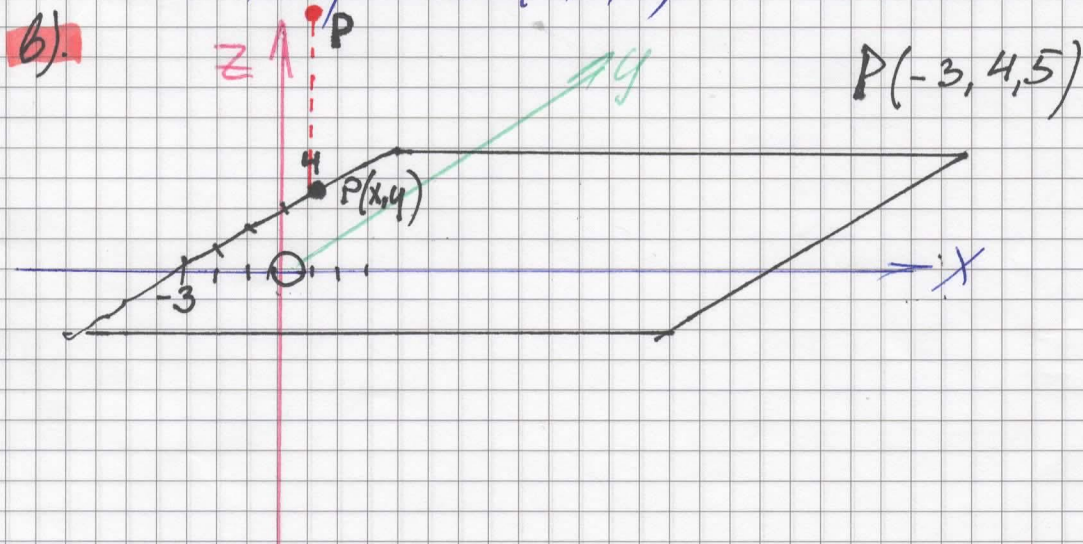
a)  $P(3, 4, 5)$

Draw a plane defined by  $xy$ -axis



Locate  $P(x, y) = P(3, 4)$  which is projection of  $P$  on plane  $xy$ .

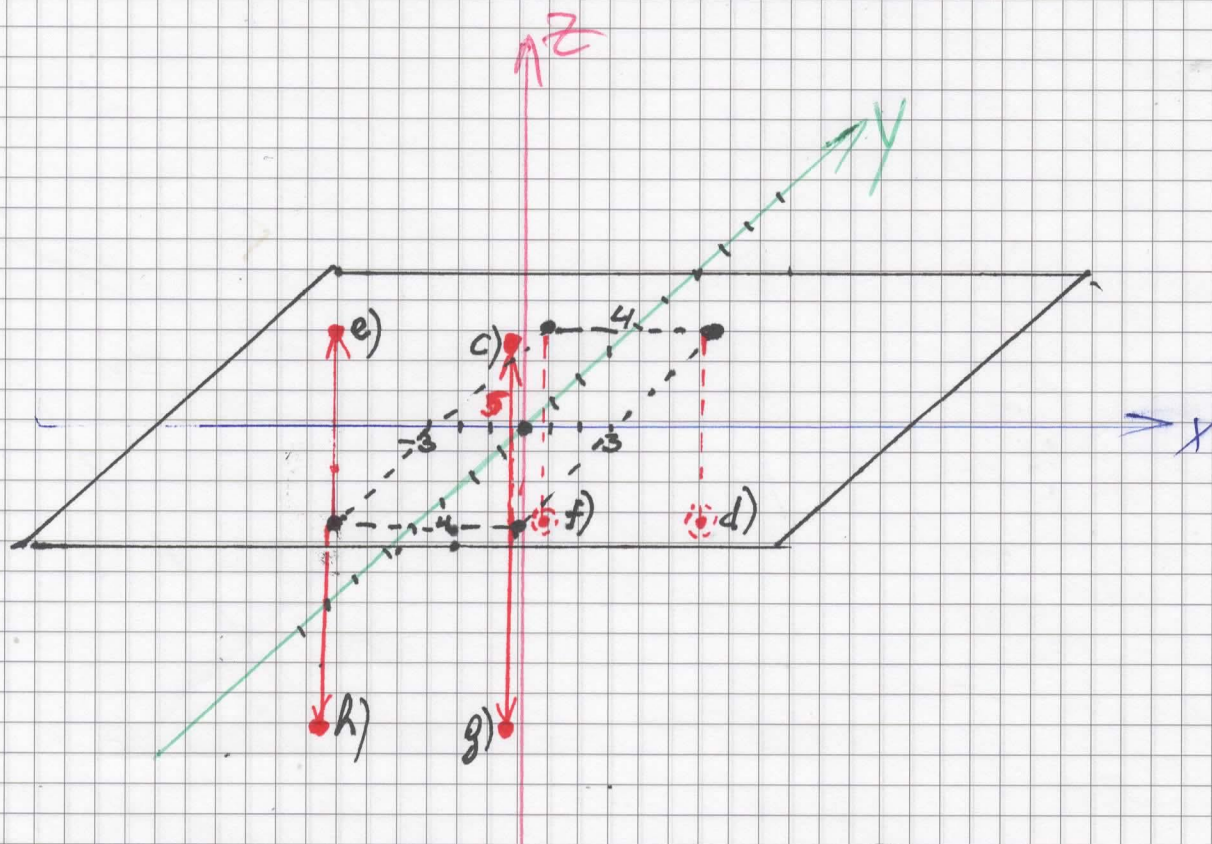
Lift  $P(x, y)$  for 5 measurement units over the  $xy$ -plane under right angle ( $90^\circ$ ) to get  $P'(3, 4, 5)$





- c)  $(3, -4, 5)$  d)  $(3, 4, -5)$  e)  $(-3, -4, 5)$   
 f)  $(-3, 4, -5)$  g)  $(3, -4, -5)$  h)  $(-3, -4, -5)$

2.



2. Find components of vector  $\overrightarrow{P_1 P_2}$

a)  $P_1 (4, 8) \quad P_2 (3, 7)$

$$\overrightarrow{P_1 P_2} = (3 - 4, 7 - 8) = (-1, -1)$$

$$x = -1; y = -1$$

b)  $P_1 (3, -5) \quad P_2 (-4, -7)$

$$\overrightarrow{P_1 P_2} = (-4 - 3, -7 - (-5)) = (-7, -2)$$

$$x = -7; y = -2$$

c)  $P_1 (-5, 0) \quad P_2 (-3, 1) \quad \overrightarrow{P_1 P_2} = (-3 - (-5), 1 - 0) = (2, 1)$

$$x = 2 \quad y = 1$$

d)  $P_1 (3, -7, 2) \quad P_2 (-2, 5, -4) \quad \overrightarrow{P_1 P_2} = (-2 - 3, 5 - (-7), -4 - 2)$   

$$= (-5, 12, -6)$$

$$x = -5$$

$$y = 12$$

$$z = -6$$



e)  $P_1(-1, 0, 2)$   $P_2(0, -1, 0)$

$$P_1P_2 = (0 - (-1), -1 - 0, 0 - 2) = (1, -1, -2)$$

$$x = 1 \quad y = -1 \quad z = -2$$

3) Let  $u = (-3, 1, 2)$   $v = (4, 0, -8)$   $w = (6, -1, -4)$   
Find components

a)  $v - w = (4, 0, -8) - (6, -1, -4)$

$$x = 4 - 6 = -2 \quad y = 0 - (-1) = 1 \quad z = -8 - (-4) = -4$$

b)  $6u + 2v = 6 \cdot (-3, 1, 2) + 2 \cdot (4, 0, -8) = (-18, 6, 12) + (8, 0, -16) = (-10, 6, -4)$

$$x = -10 \quad y = 6 \quad z = -4$$

c)  $-v + u = -(4, 0, -8) + (-3, 1, 2) = (-4, 0, 8) + (-3, 1, 2) = (-7, 1, 10)$

$$x = -7 \quad y = 1 \quad z = 10$$

d)  $5 \cdot (v - 4u) = 5 \cdot ((4, 0, -8) - 4 \cdot (-3, 1, 2)) = 5 \cdot ((4, 0, -8) - (-12, 4, 8)) = 5 \cdot (16, -4, -16) = (80, -20, -80)$

$$x = 80 \quad y = -20 \quad z = -80$$

e)  $-3(v - 8w) = -3((4, 0, -8) - 8 \cdot (6, -1, -4)) = -3 \cdot (-44, 8, 24) = (132, -24, -72)$

$$x = 132 \quad y = -24 \quad z = -72$$

4) Find the norm of  $v$

a)  $v(4, -3) \quad |v| = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

b)  $v(2, 3) \quad |v| = \sqrt{4 + 9} = \sqrt{13} \approx 3,6$

c)  $v(2, 2, 2) \quad |v| = \sqrt{4 + 4 + 4} = \sqrt{12} \approx 3,46$



d)  $v(-7, 2, -1)$

$$|v| = \sqrt{49+4+1} = \sqrt{54} \approx 7,35$$

e)  $v(2, 6, 1)$

$$|v| = \sqrt{4+36+1} = \sqrt{41} \approx 6,4$$

5. Find the distance between  $P_1$  and  $P_2$

a)  $P_1(-3, 6)$   $P_2(-1, -4)$

$$\begin{aligned} \text{Distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \\ &= \sqrt{2^2 + (-10)^2} = \sqrt{104} \approx 10,2 \end{aligned}$$

b)  $P_1(7, -5, 1)$   $P_2(-7, -2, -1)$

$$\begin{aligned} \text{Distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \\ &= \sqrt{(-14)^2 + 3^2 + (-2)^2} = \\ &= \sqrt{196 + 9 + 4} = \sqrt{209} \approx 14,46 \end{aligned}$$

c)  $P_1(3, 3, 3)$   $P_2(6, 0, 3)$

$$\text{Dist.} = \sqrt{3^2 + (-3)^2 + 0^2} = \sqrt{18} \approx 4,24$$