# INTERNATIONAL MATHEMATICAL MODELING CHALLENGE


## CONTEST RESULTS AND PAPER

IM$^2$C promotes the teaching of mathematical modeling and applications at all educational levels for all students. It is based on the firm belief that students and teachers need to experience the power of mathematics to help better understand, analyze and solve real world problems outside of mathematics itself – and to do so in realistic contexts. The Challenge has been established in the spirit of promoting educational change.

## 2019 IM²C

The 5th Annual International Mathematical Challenge (IM²C) culminated with the five Meritorious Teams presenting their solution at the ICTMA Conference in China Hong Kong (SAR) . Congratulations to our Meritorious teams and all teams participating in the 2019 IM²C.

The IM²C continues to be a rewarding experience for students, advisors, schools, and judges. A total of 57 teams, with up to 4 students each, representing 33 countries and regions competed this year.

The purpose of the IM²C is to promote the teaching of mathematical modeling and applications at all educational levels for all students. It is based on the firm belief that students and teachers need to experience the underlying power of mathematics to help better understand, analyze, and solve real world problems outside of mathematics itself—and to do so in realistic contexts. The Challenge has been established in the spirit of promoting educational change.

For many years there has been an increased recognition of the importance of mathematical modeling from universities, government, and industry. Modeling courses have proliferated in undergraduate and graduate departments of mathematical sciences worldwide. Several university modeling competitions are flourishing. Yet at the school level, even amid signs of the growing recognition of modeling's centrality, there are only a few such competitions with many fewer students participating. One important way to influence secondary school culture, and teaching and learning practices, is to offer a high-level prestigious secondary-school contest that has both national and international recognition. With this in mind, we founded the International Mathematical Modeling Challenge (IM²C) in 2014 and launched the 1st annual Challenge in 2015."

The IM²C is a true team competition held over a number of days, with students able to use any inanimate resources. Real problems require a mix of different kinds of mathematics for their analysis and solution. And, real problems take time and teamwork. The IM²C provides students with a deeper experience of how mathematics can explain our world, and the satisfaction of applying mathematics to a real world problem to develop a model and solution.

### Plans for 2020

We invite countries to enter up to two teams, each with up to four students and one teacher/faculty advisor. The contest will begin in March and end in May. During that timeframe, teams will choose five (5) consecutive days to work together on the problem. The faculty advisor must then submit the paper and certify that students followed the contest rules.

The International Expert Panel will judge the papers in early June and will announce winners by late June. Papers will be designated as Outstanding, Meritorious, Honorable Mention, and Successful Participant with appropriate plaques and certificates given in the name of students, their advisor, and their schools.

In 2020 selected teams will present at ICME in Shanghai China. Complete information about IM²C is at **www.immchallenge.org**

### The IM²C International Organizing Committee

**Solomon Garfunkel**,
*COMAP, USA – Chair*

**Keng Cheng Ang**,
*National Institute of Education, Singapore*

**Fengshan Bai**,
*Tsinghua University, China*

**Alfred Cheung**,
*NeoUnion ESC Organization, China Hong Kong (SAR)*

**Frederick Leung**,
*University of Hong Kong, China Hong Kong (SAR)*

**Vladimir Dubrovsky**,
*Moscow State University, Russia*

**Henk van der Kooij**,
*Freudenthal Institute, The Netherlands*

**Mogens Allan Niss**,
*Roskilde University, Denmark*

**Ross Turner**,
*Australian Council for Educational Research, Australia*

**Jie "Jed" Wang**,
*University of Massachusetts, USA*

## The 2019 IM²C Problem

**What is the Earth's *carrying capacity* for human life?**

1. Identify and analyze the major factors that you consider crucial to limiting the Earth's carrying capacity for human life under current conditions.
2. Use mathematical modeling to determine the current carrying capacity of the Earth for human life under today's conditions and technology.
3. What can mankind realistically do to raise the carrying capacity of the Earth for human life in perceived or anticipated future conditions? What would those conditions be?

### IM²C Funding

Funding for planning and organizational activities is provided by IM²C co-founders and co-sponsors: *Consortium for Mathematics and its Applications* (COMAP), a not-for-profit company dedicated to the improvement of mathematics education, and *NeoUnion ESC Organization* in China Hong Kong (SAR).

Note that IM²C is aware of available resources and references that address and discuss this question. It is not sufficient to simply re-present any of these models or discussions, even if properly cited. Any successful paper must include development and analysis of your model.

Your submission should consist of:

- One-page Summary Sheet.
- Your solution of no more than 20 pages, for a maximum of 21 pages with your summary.
- A complete list of references with in-text citations.
- Note: Your reference list and any appendices do not count toward the 21-page limit and should appear after your completed solution.

**Glossary**

**Carrying Capacity:** The carrying capacity of a biological species in an environment is the maximum population size of the species that the environment can sustain indefinitely, given the food, habitat, water, and other necessities available in the environment.

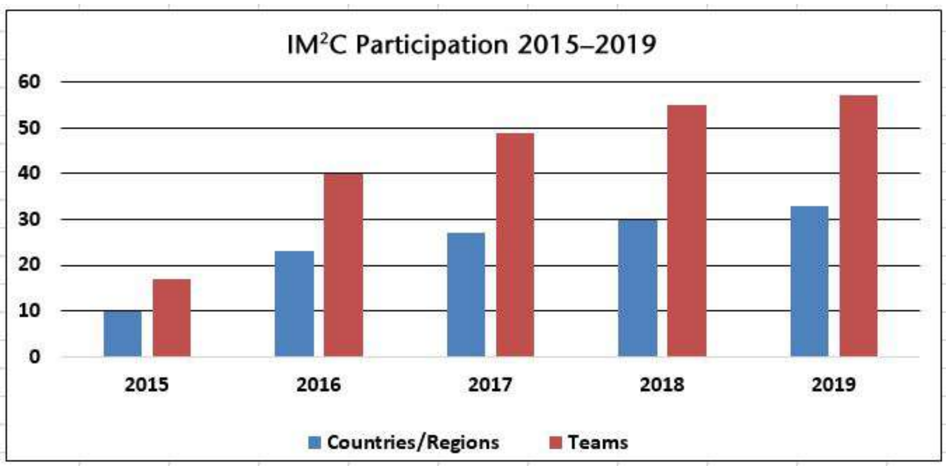
## The 2019 IM²C International Judges' Commentary

by Ruud Stolwijk

The IM²C judges wish to congratulate all students who took part in the 2019 IM²C. The judges were impressed by the efforts of all participating teams, the mathematics shown in the solutions, and the high quality of the final submissions. All of the 57 papers submitted (from 33 different countries/regions) showed great creativity in working on the Challenge. While the Expert Panel judges see only two papers from each participating country or region, we recognize that many more students participate in the Challenge. It is exciting to know that so many students are engaging in, and successfully completing, this mathematical modeling opportunity. We encourage students to continue to

## The 2019 IM²C Meritorious Teams

| School, Location | Advisor | Team Members |
|---|---|---|
| St. Paul's Co-Educational College<br>China Hong Kong (SAR) | Chan Lung Chak | Au Yee Fong<br>Lam Justin<br>Yuen Shing Ho Kinsey<br>He Yixuan |
| Brisbane Boys College<br>Toowong, Australia | Chicri Maksoud | James English<br>Ethan Jonathan Waugh<br>Xavier Catford<br>Ometh Rajapakse |
| Manurewa High<br>Manurewa, Auckland,<br>New Zealand | Lawrence Naicker | Aimee Lew<br>Ella Guiao<br>Aaron Lew<br>John Chen |
| II Liceum Ogólnokształcące z Oddziałami Dwujęzycznymi<br>Warszawa, Poland | Zbigniew Luchcinski | Adam Harrison<br>Jakub Sliz<br>Maksymilian Wolski<br>Krzysztof Oldakowski |
| Utrechts Stedelijk Gymnasium<br>Utrecht, The Netherlands | Gert Welleweerd | Ties Bloemen<br>Lucas Baas<br>Daniel Kunenborg<br>Guido Siers |




### The 2019 IM²C Expert Panel

**Frank Giordano**,
*Naval Post Graduate School, USA – Chair*

**Konstantin K. Avilov**,
*Institute for Numerical Mathematics, Russia*

**Ruud Stolwijk**,
*Cito, The Netherlands*

**Liqiang Lu**,
*Fudan University, China*

**Jill Brown**,
*Australian Catholic University, Australia*

**Yang Wang**,
*The Hong Kong University of Science and Technology, China Hong Kong (SAR)*

**Henk van der Kooij**,
*Freudenthal Institute, The Netherlands*

**Kathleen Snook**
*COMAP, USA*

form teams, compete to represent their country or region, and aspire to be named among the top teams at the international level Expert Panel judging.

This year's challenge was to develop a mathematical model to determine the Earth's carrying capacity for human life. This problem posed three requirements. First, students identified and analyzed the major crucial factors that currently limit the Earth's carrying capacity for human life. Next, teams developed and used a mathematical model to determine the current carrying capacity under today's conditions and technology. Finally, students addressed how mankind could raise the carrying capacity in perceived or anticipated future conditions.

This year's problem was quite a different type of problem than in the former four IM²C in that it was fairly open ended. In many cases, teams found it difficult to find the appropriate mathematics to apply. Students performed analysis on data mined from available web sources, leading to a variety of descriptions of "the Earth's capacity for human life under today's conditions and technology," but not necessarily leading to a useful mathematical model to address the actual problem. Many teams concentrated their efforts on collecting, analyzing, and (sometimes) recalculating the data on annual supply rates and annual per capita demand rates of various resources. At that point, teams needed to analyze and look across these data, but many limited their mathematical model of carrying capacity to a very simple one that did not take into account that different critical resources are not independent. Thus, one of the main discriminators of "better papers" was inclusion of some interdependence of the resources into the mathematical model, as well as posing and solving some kind of optimization problem for such a model.

**Characteristics of the better papers**

From the 57 papers judged, five were considered to be Meritorious. The five countries/regions represented in this year's top category exemplify the growing international impact of IM²C. We congratulate these top teams.

In these Meritorious papers, teams made good choices for the factors considered crucial. Drinkable water, food, energy, and space (both for living and food production) turned out to be critical factors for teams to include in their discussions. The explanation of the factors, and their interdependence, was quite crucial with all Meritorious teams doing a good job addressing this aspect of the problem. Also, these teams appropriately justified their choices and uses of particular information, facts, figures, and graphs found on the Internet. This is quite an important issue for the IM²C judging: always justify the information you choose to use and what you do with it! Additionally, teams must document where they found their information by providing a reference to the source.

The beginning of a team's paper makes an important impression on the judges. A one-page summary at the start of the paper not only summarizes the team's work, but also serves as an invitation to the reader to read the rest of the paper. A good example of a nice summary can be found in paper 2019043 from Kamnoetvidya Science Academy (KVIS), Thailand.

The heart of the paper is in the modeling. An interesting modeling scheme was used in paper 2019032 from the Bayview Secondary School, Canada, where everything (including energy production and water supply via desalinization) was expressed in terms of land area used, with land size thus being the only limiting factor. In paper 2019038 from the Lauvyntyev Institute of Hydrodynamics, Russia, an optimization of human diet with respect to fats, carbohydrates, proteins, and energy consumption was used to determine the most efficient combination of crops to cultivate. As for choosing the limiting factors, there were several papers that also explored less obvious limiting factors like carbon emissions or non-recycled waste. Paper 2019018 from Brisbane Boy's College, Australia is a good example of a relatively short, well articulated, and straightforward presentation of the model.

Finally, the use of appendices is very suitable for both computer code and (big) data sets, since putting these items in appendices helps to increase the readability of the paper itself. Paper 2019017 from Escola Hou Kong, China Macau (SAR) provides a nice example of putting data used in the appendix. Please realize it is hardly likely the judges will read all computer code in the appendices, so the working of any computer program used must be succinctly, but fully, explained in the paper itself. Of course, a very important aspect in IM²C is lucid and coherent explanation of the mathematical model that includes presentation of underlying concepts and assumptions. Teams must make decisions about what to include in the main body of the paper and what to include as supporting material in the appendices.

High-level sophisticated mathematics is not required to make a good model, however in this year's problem, as stated before, many teams had difficulty putting together an integrated

**USA Participation**

In the USA, we invite all teams that successfully compete in the HiMCM contest and are awarded a designation of Meritorious or above (Meritorious, Finalist, or Outstanding) to compete in the IM²C. From these participants, U.S. Judges select the two top teams to move on and represent the USA in the IM²C international round. To participate in HiMCM in November 2019, visit **www.comap.com.**

model. In some cases, in attempts to use higher-level mathematics, teams showed quite a lot of mathematics that were not related to the problem itself. Teams should make sure they use appropriate mathematics and that they explain their modeling processes and procedures. In this year's Challenge all models developed were more or less linear. Paper 2019025 from Maimonides School, Chile provides an example of linear programming, that is quite simple and direct, but well explained.

The judges were quite surprised that a number of papers found the Earth's carrying capacity for human life to be less than the present population of about 7.7 billion people. And, in some cases, the carrying capacity was stated to be significantly less. These papers offered little or no explanation of how or why this result was possible or accurate, or that the result was in any way remarkable given our current population. Models and solutions must always be thought of within the real-life relevance of the problem as IM$^2$C problems are about real-life scenarios.

**Suggestions and advice for future participants**

- As stated above, the summary is the reader's introduction to the paper. The summary must not only describe the way the problem is solved, it must also invite the reader to continue to read the full paper. The summary is not the place for complex mathematical descriptions, but instead a general introduction to the problem, your solution process, and your final conclusion.
- As teams only have a short time to do the Challenge they must make assumptions to focus and simplify their work. Teams should only make assumptions that actually impact their model and they should justify their assumptions.
- The most important part of a team's submission is their model. All other parts of their paper support the development, use, and analysis of the model. The mathematics used must always be explained in a logical manner, since this is the heart of modeling!
- Make sure as you model you always stay in touch with reality. Since the problem is a real-life problem, it is quite essential that teams reflect and critically judge their mathematical solution as calculated.
- Since even a good model cannot be perfect, especially when developed in only five days of work, an analysis of the strengths and weaknesses is required.
- Ensure that your paper concludes with a short summary of the actual solution or findings to the requirements of the problem.
- With only 20 pages to introduce the problem, state assumptions and justifications, develop, solve, and apply the model, and do some analysis to include identifying strengths and weaknesses, there is not much room for tables of data or code. While you might include a short section of code or a small subsection of a data spreadsheet, computer code and (big) data sets should not be in the paper itself but in an appendix.
- Teams must properly document and reference all information taken from the Internet or other sources, including graphs, illustrations, and pictures. Teams can use in-line documentation, footnotes, or endnotes and should also include a Reference List.
- Teams should take notice of the page limitation of 20 pages, and the rules for font size and margins.


*The 2019 IM2C Meritorious Teams, their advisors, and contest officials.*

Finally, the IM$^2$C judges would like to compliment all teams on their effort, and thank them and their teachers/advisors for participating in the 2019 IM$^2$C. All teams did a great job in diving into a complex problem for five days. The result was quite a number of very creative papers. The judges (all mathematicians and teachers) had some stimulating discussions about the papers, as it was clear that the students gave some very thought provoking analyses. This shows that in many countries and schools, mathematical modeling is a growing field of interest that students enjoy and are very capable of doing. Well Done!

**General Advice to Teams Participating in Future IM²C**

The IM²C is definitely a challenge. Teams have to organize themselves, address all requirements of the problem, and write a report all in a short period of time. Budgeting time becomes critical so that you leave enough time to effectively communicate your work and results to the Challenge judges.

Our advice is to allow plenty of time to construct your report. In fact, consider outlining the report as soon as you begin working on the problem. This outline will guide your team in its work and provide a logical path to your solution for readers of your report. Remember, you are communicating with judges from many countries of the world. The judges are not necessarily familiar with the curricula of your school, so present the development of your model in a logical and easily understood fashion. Judges are not looking for the papers that use the most sophisticated mathematics. Do not force the mathematics upon a given scenario. Rather, begin with the simplest mathematics that solves the problem you have identified. Later, as appropriate, refine and enhance your model to increase its precision, or adjust your assumptions to find a more broadly appropriate solution.

Pictures, graphs, tables, and schedules can be quite effective and efficient in communicating your ideas. The use of relevant pictures and graphs can make a report clearer and more pleasant to read. Your report should include a combination of various representations: symbolic, graphical, and text that best present your model and solution. Realize, however, that large tables and extensive code or data might be better as supporting material located in an appendix.

The use of symbolic formulae and algorithms are quite essential in a mathematical modeling assignment. The use of unexplained formulae, however, will not make the report more convincing. The reader needs to believe that the writers themselves understand the formulae used. This is done through explanations and analyses of your modeling processes. The readers of your report, while experienced mathematicians, are not experts in all parts of the great world of math!

Appendices are very useful, but do not expect the judges to read them. While judges may refer to an appendix to check a reference or to get a general idea of your computer code, they will not fully read the appendices. Therefore, do not place anything critically important to the development of your model in an appendix.

Remember to list any sources you used during your work on the Challenge and to document in your paper where you used these sources (e.g. a graph or picture from a particular web site). Follow the rules of completing your solution report within the specified number of pages and in a font size of no smaller than 12 point type.

Overall, present the development and analysis of your model in a manner that a wide audience could understand. Consider who might be using your model and explain your model to that audience, as well as the judges. Ensure you close your report with a conclusion and a summary of your results.

**For more information about the IM²C, including the complete 2015–2019 results and sample papers, visit**

**www.immchallenge.org**

# Dynamic Decisions

**II Liceum Ogólnokształcące z Oddziałami Dwujęzycznymi im. Stefana Batorego**
**Warsaw, Poland**

**Advisor:**
Zbigniew Luchcinski

**Team Members:**
Adam Harrison,
Jakub Sliz,
Maksymilian Wolski,
Krzysztof Oldakowski

# Summary

With rapid growth of human population comes growth of human impact on Earth. It is predicted that in the next 30 years, our population may increase by 2 billion people. Although, human population seems to have no limits, Earth's resources are finite and may soon be depleted. Without said resources, people cannot satisfy their basic needs. This is why the notion of carrying capacity is of such importance and understanding it is key for future development. Our goal was to create models which can estimate current Earth's carrying capacity as well as propose a sensible solution to raising this number in the future.

We understand carrying capacity as maximum population that can both be environmentally sustainable and provide a decent life for all. The same idea was incorporated in Safe and Just Space Framework defined as doughnut-shaped "safe operating space" restrained by planetary and social boundaries. Based on SJS we selected 5 biophysical and 5 social factors that impact the carrying capacity the most.

Our base model was based on 2 methods used in the past. The first calculates carrying capacity by estimating how many people can live on Earth based on space they need. The result that we got using this method was 12 billion. The other method focuses on choosing one natural resource and estimating how many people it can satisfy indefinitely. In this model we used the most commonly analysed resources - water and food and arrived at 31 billion and 13 billion respectively.

Our main, Dynamic model was based on Safe and Just Space framework and used System Dynamics approach to calculate carrying capacity. The first one provided us with a conceptualization of the problem while the latter enabled us to calculate interdependent factors in respect to population. We set thresholds that our key factors cannot transgress with so as to they fit in the SJS. Consequently, using Python, for every country, we calculated for what population the factors do not overpass the thresholds. We then summed the optimal population for each country and got 7.6 billion as a result. This amount is incredibly close to the one that scientist have most oftenly predicted - 8 billion.

By expanding the Dynamic Model by Q-learning algorithm we created President Model that enabled us to predict future conditions and consequently calculate future Earth's carrying capacity. President model is a simulation in which a World President controlling the Planet makes decisions with aim to increase the carrying capacity of Earth. Using Q-learning the President achieved optimistic results of increasing Earth's carrying capacity up to 13 billion people.

# Introduction

Human population has lately surpassed 7.6 billion people and is rapidly growing. This growth is especially concerning and over the course of past years environmentalists tried to bring attention to the problem at hand. It is projected that there may be as many as 9.8 billion people in 2050, and 11.2 billion in 2100[1].

Every person consumes Earth's resources whether it is food that we eat, water which we drink or oil that we need to run our cars etc. As human population rises, the total consumption increases as well. An increase in the standard of life, together with the spread of a consumptionist culture over the last 50 years has caused a further stress on the environment. The Earth doesn't have endless resources however. At some point there will be just too many people and the Earth will no longer be able to provide for them.

Determining the number of people the Earth can hold and what it is impacted by is critical to the establishing of well-thought policies and sustainable development. We have to find a balance between the well-being of our species and the state of our environment here on Earth.

# Problem restatement

Many scientist mistakenly took carrying capacity as population equilibrium - a point at which population stabilizes and does not change over the years[2]. Said stabilisation may be caused not by reaching Earth's carrying capacity but by halting fertility with the use of contraceptives etc.

In our understanding carrying capacity is the population of humans which can live indefinitely on Earth. When the human population exceeds the carrying capacity, then the overconsumption leads to depletion of Earth's resources; and/or the waste and pollution produced by people destroys the environment and poisons humans[3]. At carrying capacity, Earth can support people persistently.

Using this definition of carrying capacity we divide our problem into 3 main parts:

- Finding key factors that limit the Earth's carrying capacity.
- Calculating Earth's current carrying capacity.
- Predicting how Earth's carrying capacity is going to change in the future and finding ways to increase it.

# General Assumptions

1. **Trade of resources**
   We do not take into account the trade of resources as their production in the world remains the same regardless of their location of use and consequently does not influence the Earth's carrying capacity.
2. **Time spread of our data is negligible**
   In our model we used data from various sources and since some were lacking information from recent years (2008-2017), we were forced to make do with what we found. However, this time inconsistency is low enough to be considered irrelevant.

# Key factors

To fully understand the carrying capacity, we must first identify what influences it and how does it happen. Choosing the most impactful factors is especially important since they will be a pivot point of our models.

## Safe and Just Space

It is a framework proposed by Kate Raworth that contradicts the current idea of constant growth and further economic developments and instead concentrates on a fair division of the world's supplies. The SJS[4] focuses on sustainable environmental development while maintaining decent and just life for people. Thus, a doughnut-shaped space bounded by planetary and social boundaries is introduced (Fig. 1). It is a "safe operating space" for people to thrive in, while not damaging the environment and providing basic need for humans. Our model strongly focuses on calculating the carrying capacity by calculating the maximum population that can live within the Safe and Just Space.

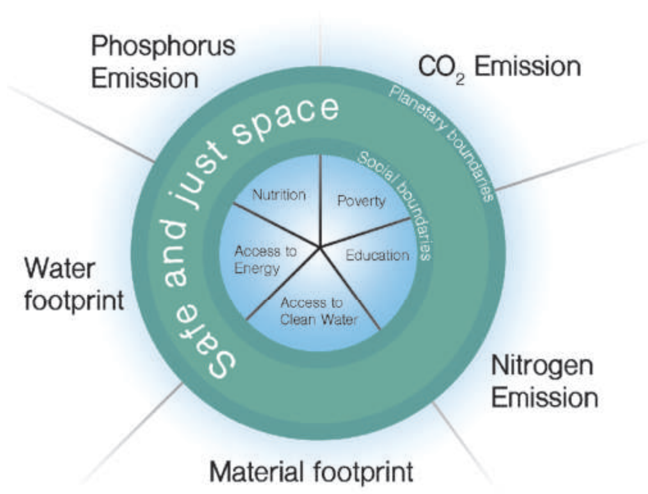



*Fig. 1 - Visualization of Safe and Just Space*

## Factor selection

In accordance with the SJS framework we selected 10 factors that will help us determine the carrying capacity of our planet. We divided them into two groups: biophysical factors and social factors. The first group consists of biophysical factors that impact the environmental well-being, whereas the other group contains social factors, which influence the quality of life of Earth's inhabitants.[5]

### Biophysical factors

- $CO_2$ emission

  The excessive amount of carbon dioxide in Earth's atmosphere is one of the main reasons why global warming is happening. The raise of global temperatures is dangerous, it causes a rise in  sea levels as well as a decrease in crop yields, which may cause forced population displacement and food shortage. Though some may not see the consequences of high density of CO2 now, its future impact will be irreversible.[6]

- Nitrogen emission

  Nitrogen is released when petrol combusts in car engines in the presence of air, and because of the production of artificial fertilisers. Nowadays such fertilisers play a significant role in increasing crop yield. This comes with a cost however. Extensive nitrogen use devastates the soil, may cause acid rains and pollutes water. It can also cause breathing diseases such as asthma[7].

- Phosphorus emissions

  It is mostly a by-product of burning fossil-fuels and biomass, but also a result of the production of fertilisers. It may influence the carbon cycle[8] in unpredictable ways and cause the eutrophication of lakes. Their overuse can cause a decrease in water resources.

- Material footprint

  Material footprint shows the amount of raw materials (minerals, fossil fuels, and biomass) associated with the final demand for goods and services, no matter where it is extracted  It is important as we are slowly running out of mined resources. According to some predictions we have already reached peak oil - the time half of the oil in the world has been extracted. Oil is used to generate electricity, but is also used in many products such as plastics. It is therefore crucial that we find alternatives, as oil is a very useful resource and we might not have it soon. Lately the reserves of rare metals as chromium have also been running low. They are used in many everyday objects, which are essential to our lives. The low availability may cause a rise in prices and thus prevent poorer people from accessing new technologies[9].

- Water footprint

  Water footprint shows the amount of water used by a country in a year. Water is called the staple of life as humans consist of up to 60% water. It is necessary in the production of food, in many industrial processes and is also used domestically, for drinking and sanitation[11]. It is therefore essential that our usage of water does not exceed the renewable reserves as people in many places in the world are already experiencing water scarcity or are subjected to water stress. The situation is not predicted to get better as the proportion of people affected by water stress is likely to increase, partly due to climate change[10].

## Social factors

- Nutrition

  Receiving enough calories is a basic human need and a crucial aspect of living. Without it our organism cannot maintain it's immunity. In developing countries even one in every 3 children is malnourished. This results in stunting (insufficient height for a given age) and wasting (insufficient weight for a given height.).

- Percent of population living in poverty

  According to World Bank definitions extreme poverty is an income below $1.90/day. Without enough money, people cannot buy most needed things and thus fulfil their basic needs. This why it is such an important factor[12].

- Education

  Teaching young people about the environment is a first step to make a change. People need to be educated, especially in poor-countries, about family planning and ecology. It also allows for the creation of highly skilled workers who are crucial to running a modern economy.

- Access to electricity

  It's one of the basic human needs in order to live a decent life. These days almost everything depends on electricity - our smartphones, TVs or electric kettles. It frees us from the dependence on sunlight and allows us to work after sunset. Though such technology is not crucial for survival, it significantly impacts overall happiness of people and gives them more time to spend on education, work, and recreation[13].

- Access to clean water

  No person can live without water. In many places due to the lack of safe drinking water sources people suffer from diarrhea, which causes many death especially among the youngest. Only in 2015, 1.8 million people died due to polluted water.[14]

# Our Models

## Historical results

Many attempts to calculate Earth's carrying capacity were carried out in the past. The first known attempt has been carried out by Antonie van Leeuwenhoek, where he estimated the population density of the Netherlands, and then extrapolated it over the habitable area of the Earth. He arrived at a result of 13 billion. Other models were based on finding a resource which could limit the human population such as food or water consumption and then find the maximum number of humans that could be sustained by that resource indefinitely. At the start we tried both these methods[15]

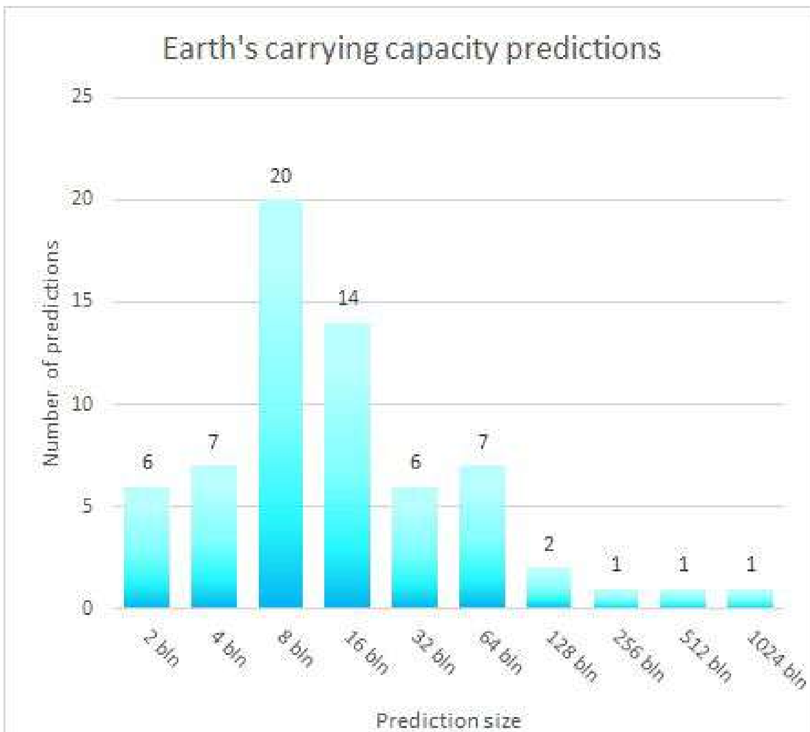



*Fig. 2 - Chart of most commonly predicted Earth's carrying capacities.*

## Basic Model

### Description

In our basic model we calculate the Earth's carrying capacity by measuring how many people can be indefinitely sustained by a one given resource. Similarly to past researches the resources we chose are: water, nutrition, and area as those are one of the most basic human needs.

### Model assumptions

1. People have to eat at least 2000 kcal daily and we can obtain 8,278,200 kcal/year from a hectare of field[16].
2. We also assumed that the density of population in the countries of European Union (117.3 people/$km^2$) is an optimal one.

### Calculations

The model calculates carrying capacity using the following formula.

$$carrying\ capacity = \frac{total\ amount\ of\ the\ resource}{need\ per\ capita}$$

These are our results:

| | Need per capita per year | Available earth resources per year | Result |
|---|---|---|---|
| Water[17] | 1385 $m^3$ | 42 809 * $10^9$ $m^3$ | **~ 31 billions** |
| Nutrition[18] | 730 000 kcal | 9.46 * $10^{15}$ kcal | **~ 13 billions** |
| Area[19] | 8 525 $m^2$ | 104 * $10^6$ $km^2$ | **~ 12 billions** |

The carrying capacity of this model is the lowest of the calculated amounts, which is **12 billions**.

## Dynamic Model

### Use of Safe and Just Space

In this model we make use of the Safe and Just Space Framework[20]. The Earth's carrying capacity is then the maximum population that fits in the Safe and Just Space. We set the planetary and social boundaries as thresholds our Key Factors must not transgress. We make sure that for each country the population at carrying capacity is both environmentally sustainable and socially just, as suggested by Kate Raworth.

### Thresholds

The thresholds are values the factors cannot overpass. They are based on guidelines for sustainable environmental and social development from various sources.

| | Threshold | Data source |
|---|---|---|
| Nutrition | 2000 kcal / day per capita | World Bank |
| Population living in poverty | 5% | World Bank |
| Access to education | 75% | World Bank |
| Access to electricity | 87% | World Bank |
| Access to clean water | 95% | The Guardian datablog |

| | Threshold | Data Source |
|---|---|---|
| $CO_2$ emission | 4 Tons / year per capita | World Bank |
| Water footprint | 713 $m^3$/year per capita | Hoekstra, A.Y. & Mekonnen, M.M. (2012) 'The water footprint of humanity', *Proceedings of the National Academy of Sciences* |
| Material footprint | 7.2 t / year per capita | Eora MRIO Database |
| Phosphorus emission | 0.89 kg / year per capita | Eora MRIO Database |
| Nitrogen emission | 8.9 kg / year per capita | Eora MRIO Database |

## System Dynamics

Our goal is to calculate for what maximal population do our Key Factors do not transgress the thresholds. Yet, with a change in population comes a change in every Key Factor and Key Factors are also dependent on other Key Factors. Therefore we have a system of interdependent Key Factors. Calculation of such in respect to population is extremely complex. Fortunately System Dynamics approach makes it simpler.

The notion of using System Dynamics in order to model population was first introduced in 1971 by J.W.Forrester in his book *World Dynamics*[21]. The World2 model developed by Forrester is a "simple" 5th-order differential equation that uses self-feedback loops in order to model population in respect to pollution, natural resources and other. A year later Dennis Meadows et. al. published *Limits to Growth*[22] in which he presented a more sophisticated and complex model than World2 called World3. We based our model on World3 with use of Safe and Just Space.

## Calculations

Our model strongly relies on Python code that we used to apply System Dynamic to our model. BPTK_PY library equipped us with proper tool to do so. Yet, data needs to be prepared beforehand.

Firstly, after collecting data concerning Key Factors for every country, we calculate the correlations between population and all Key Factors (using linear and logarithmic fit) as well as between Biophysical factors and Social factors. This will enable us to see which factors correlate and where the change will be induced on others.

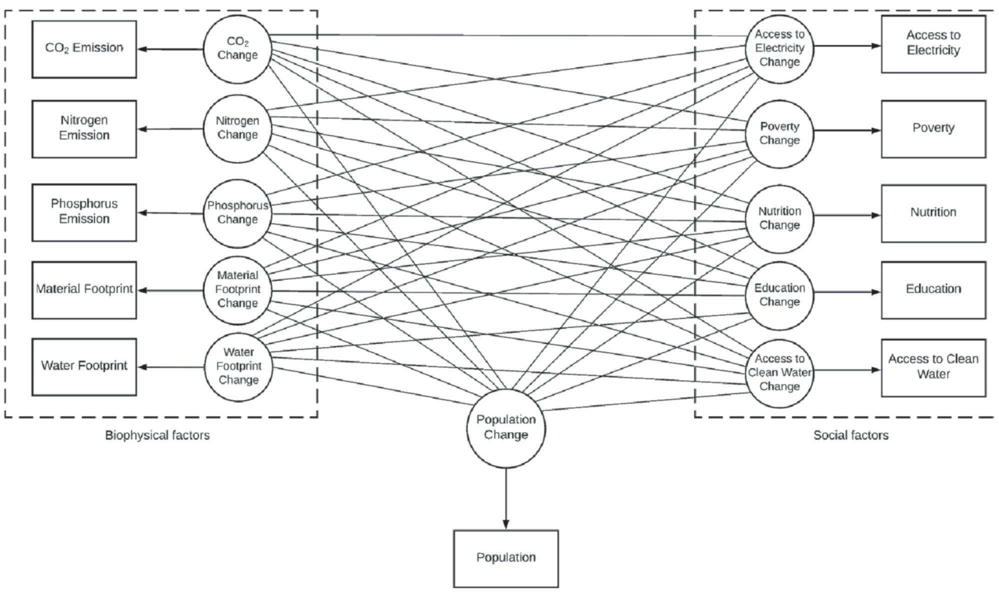



*Fig 3. A simplified scheme of system dynamics approach.*

Next, correlation coefficient must be calculated. We will achieve this by using the best fit ($R^2$) from linear of logarithmic correlation. From those calculations we will be able to see how much change does each factor induces on other factors.

Fig. 3 shows a simplified scheme of our use of System Dynamics. Boxes represent total value of a Key Factor which will change according to the change of a Key Factor shown as a circle. The Key Factor changes are interdependent. Population change induces change on the Key Factors' change while remaining unchanged by them.

Now, for each country, using the Python code, we can calculate how change in population of a country changes the state of its Key Factors.

Finally, our code calculates what is the country's maximum population for which none of the given country's Key Factors transgresses set thresholds. This maximum population is the Carrying capacity of said country. Summing results for each and every country will give us the Earth's carrying capacity for human population.

## Models results and comparison

According to Dynamic Model we are currently at Earth's carrying capacity - 7.6 Billion. It is a significantly lower number than the one Base Model estimates - 12 Billion. However, the result is a close match with the most often predicted number, which is 8 Billion[23].

## Strengths and weaknesses

### Strengths

- System Dynamics takes interdependence of Key Factors into consideration
- Social Factors are included in the model
- Ecc is calculated for every country separately
- The model allows for analysis of how each Factor contributes to Carrying Capacity and thus it enables us to find ways to increase it

### Weaknesses

- Lack of insight control of model calculations because of the fact that we work on Python library
- Economic processes are not taken into account

# Raising Earth's Carrying Capacity

## Introduction

Our environment is constantly changing. This is an undeniable fact. Moreover with the change of our environment a resulting increase in awareness about the problem must follow. With a massive influx of natural disasters, warming and cooling periods, different types of weather patterns, drastical emission of CO2 and much more, people need to be aware of what types of environmental problems our planet is facing.

Our planet is poised at the brink of a severe environmental crisis. Current environmental problems make us vulnerable to disasters and tragedies, now and in the future. We are in a state of planetary emergency, with environmental problems piling up high around us. Unless we address the various issues prudently and seriously we are surely doomed for disaster. Current environmental problems require urgent attention.

## Possible solutions

Saving the environment is no longer just a problem for environmentalists and policymakers. Our very own existence may soon be put into question. Individuals, NGO's, corporations, and governments must come together and join hands to protect what is left of our planet so that the future is not wiped out before it's time for a curtain call.

In order to raise Earth's carrying capacity we researched many solutions in different branches that have a great impact on reducing for example CO2 emission. This possible solutions will be used in President Model in which every solution will have its own *impact table*.

### New resources of energy:

- Nuclear fusion[24] and cold fusion[25]
- Nuclear power[24]

### Global environmental improvements:

- Reforestation[26]

### Environment focused politics:

- Electrical cars policy

Change in population diet and food production:

- Grow only corps for human[16]

Resource efficiency:

- Liquid coal or ultra supercritical effectiveness of burning [27]

Reduction of greenhouse gases:

- NO with carbon monoxide on copper-cobalt oxide[28]
- Capturing carbon dioxide[29]

Renewable energy:

- Space solar[30]
- Tidal [31]
- Geothermal[32]

Trash management:

- Plastic decomposing bacteria[33]

## President Model

### Model overview and justification

President Model is based on combination of creating probability decision tree and agent based algorithm Q-learning. In this model we want to check what is the best combinations of decisions to raise carrying capacity. Our model could be a great tool in creating import decisions starting from CEO's to country presidents.

In President Model creating an environment for agent is vital, therefore we tried many different possible scenarios. Q-learning used in the model perfectly fits to this problem because of its simplicity in application, fast learning time and ability to adapt to environment with given set of rules.

### Model Assumptions

1. Possible solutions proposed by us can have measurable impact on factors limiting Earth's carrying capacity.
2. We assume that with time chance of more advanced decision showing in decision tree will rise (eg. current technology is not able to create cold fusion but with time this technology might be more accessible).

## Model

In our model we will use Q-learning reinforcement algorithm because of its adequacy and simplicity. The goal of Q-learning is to learn a policy, which tells the president (agent) what action to take under what circumstances. It does not require a model of the environment, and it can handle problems with stochastic transitions and rewards, without requiring adaptations.

For any finite decision process, Q-learning finds a policy that is optimal in the sense that it maximizes the expected value of the total reward in our case Earth's carrying capacity over any and all successive steps, starting from the current state (7.6 billion). Q-learning can identify an optimal action-selection policy given infinite exploration time and a partly-random policy. "Q" names the function that returns the reward used to provide the reinforcement and can be said to stand for the "quality" of an action taken in a given state.

$$Q(s,a) \leftarrow Q(s,a) + \alpha(R + \gamma Q(s',a') - Q(s,a))$$

This update rule to estimate the value of Q is applied at every time step of the president's interaction with the environment. The terms used are explained below:

$s -$ *current Earth's carrying capacity*

$a -$ *current decision chosen by the president* ( *eg. Nuclear Power*)

$s' -$ *future Earth's carrying capacity*

$a' -$ *best future decision that will maximize Earth's carrying capacity*

$R$ $-$ *current change in population* (*reward*)

$\gamma -$ *reward discounting factor* (*the higher the president will prefer long* $-$ *term solutions* )

$\alpha -$ *years taken to update estimation of* $Q(s,a)$

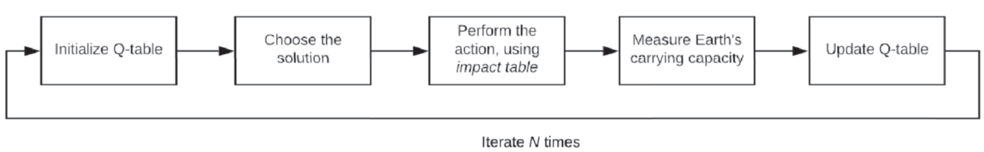



*Fig. 4 - Q-learning algorithm flow chart*

For our model we created an environment which is representation of decision making processes where each decision impacts Earth's carrying capacity. Each decision has its own *impact table*. *Impact table* is a prognosed impact on key factors that we proposed before. Eg.

$$Cold\,fusion \ = \ \{"CO_2 \ emission" : \ -0.34\% \ ... \ etc\}$$

Then after each decision we calculate Earth's carrying capacity using System Dynamics model. This leaves us with graph (environment) in which Q-learning model will try to find the best combination of decisions that raise Earth's carrying capacity.

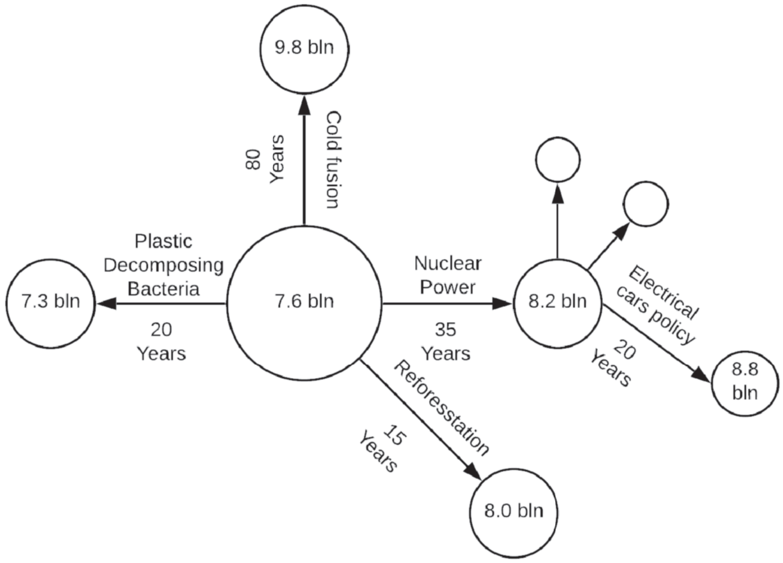
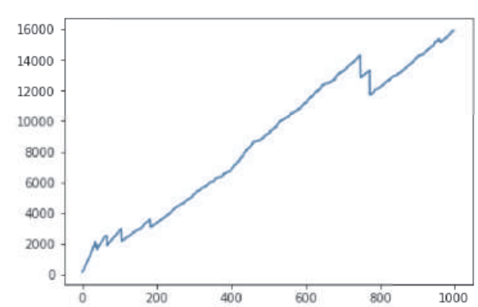



*Fig. 5 - Example of environment where edges represent decisions over time and nodes the resulting increase or decrease in Earth's carrying capacity*

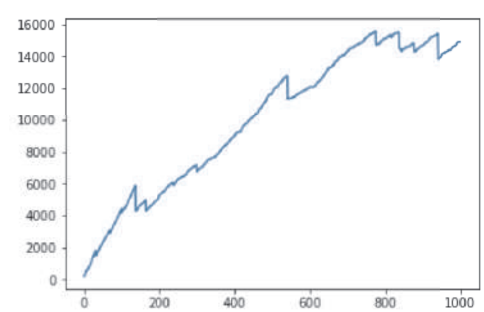
Having created the environment, we can run our model on 4 gamma parameters.




$\gamma = 0.6$

Results: +0.63

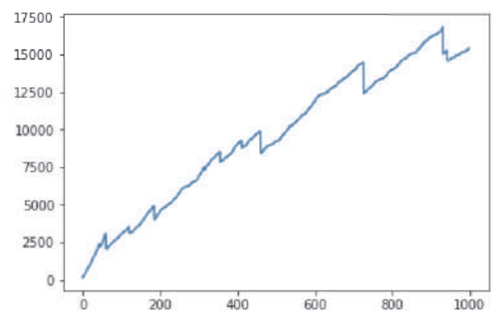
Earth's carrying capacity: 12.3 bln




$\gamma = 0.9$

Results: +0.73

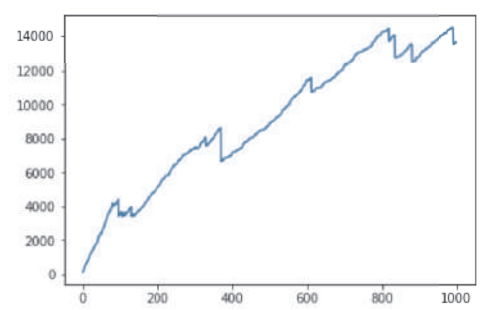
Earth's carrying capacity: 13.14 bln




γ = 0.85

Results: +0.68%

Earth's carrying capacity: 12.76 bln




γ = 0.95

Results: +0.78

Earth's carrying capacity: 13.54 bln

*Fig. 6 - Q-learning reaching best results over number of simulations*

This results will leave us with average **70%** raise in future Earth's carrying capacity which gives us **13 billion** people that our Earth's can hold.

## Strengths and Weaknesses

### Strengths

- Our agent (president) based model can be used in various environments with ability to adapt to it.

### Weaknesses

- We base impact of each solution based on our research but in reality the impact might be bigger or smaller.
- Complex world decision can't be shown in a simple graph form because they don't include internal correlation between each node.

# Future work

The environmental crisis is complex, requiring many solutions. However, it is our responsibility to protect and improve the environment. In future we would like to improve our results by providing the model with bigger amount of data. We would also considered more factors to precisely define how many happy and eco-friendly habitants can the Earth support. With more data acquired we could make variants of basic model by changing the values of thresholds.

# Appendix

## References


1. The World Population Prospects: The 2017 Revision
2. Letter to the Editor / Ecological Modelling 192 (2006) 317–320 Cang Hui
3. Letter to the Editor / Ecological Modelling 192 (2006) 317–320 Cang Hui
4. Raworth Kate. (2013). Defining a Safe and Just Space for Humanity.
5. W. O'Neill, Daniel & L. Fanning, Andrew & Lamb, William & Steinberger, Julia. (2018). A good life for all within planetary boundaries. Nature Sustainability.
6. Edenhofer, O & Pichs-Madruga, R & Sokona, Youba & Farahani, E & Kadner, Susanne & Seyboth, K. (2014). Climate change 2014: mitigation of climate change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change.
7. Review of the Primary National Ambient Air Quality Standards for Oxides of Nitrogen
8. Significant contribution of combustion-related emissions to the atmospheric phosphorus budget Rong Wang, Yves Balkanski, Olivier Boucher, Philippe Ciais, Josep Peñuelas Shu Tao
9. The material footprint of nations Thomas O. Wiedmann, Heinz Schandl, Manfred Lenzen, Daniel Moran, Sangwon Suh, James West, Keiichiro Kanemoto
10. The Water Footprint Assessment Manual Setting the Global Standard The Water Footprint Assessment Manual Arjen Y. Hoekstra, Ashok K. Chapagain, Maite M. Aldaya and Mesfin M. Mekonnen
11. The Water Footprint Assessment Manual Setting the Global Standard The Water Footprint Assessment Manual Arjen Y. Hoekstra, Ashok K. Chapagain, Maite M. Aldaya and Mesfin M. Mekonnen
12. Datt, G. and Ravallion, M. (1992) "Growth and Redistribution Components of Changes in Poverty Measures: A decomposition with applications to Brazil and India in the 1980s" Journal of Development economics
13. Anne-Sophie Garrigou. The impact of bringing electricity to rural and vulnerable populations
14. Pollution, health, and the planet: time for decisive action Pamela Das, Richard Horton
15. Cohen, Joel. (1995). Population Growth and Earth's Human Carrying Capacity. Science (New York, N.Y.).
16. Cassidy, Emily & West, Paul & Gerber, James & A Foley, Jonathan. (2013). Redefining Agricultural Yields: from Tonnes to People Nourished per Hectare.
17. Ruddell, Benjamin. (2018). Threshold Based Footprints (for Water).
18. Nayak, Aparna. (2015). Food Security. Journal of Agricultural Studies.

19. The EU in the world - 2018 edition
20. W. O'Neill, Daniel & L. Fanning, Andrew & Lamb, William & Steinberger, Julia. (2018). A good life for all within planetary boundaries. Nature Sustainability.
21. 1971. *World Dynamics* J.W.Forrester
22. H. Meadows, Donella & L. Meadows, Dennis & Jorgen Randers, J & W. Behrens, William & of Rome, Club. (1972). The Limits of Growth.
23. One Planet, How Many People? A Review of Earth's Carrying Capacity A discussion paper for the year of RIO+20
24. Buongiorno, Jacopo & Corradini, Michael & Parsons, John & Petti, David. (2019). Nuclear Energy in a Carbon-Constrained World: Big Challenges and Big Opportunities.
25. The Curious Story of the Muon-Catalyzed Fusion Reaction Joshua Yoon
26. Hansen, James & Sato, Makiko & Kharecha, Pushker & Beerling, David & Berner, Robert & Masson-Delmotte, Valerie & Pagani, Mark & Raymo, Maureen & L. Royer, Dana & Zachos, J.C.. (2008). Target Atmospheric CO2: Where should Humanity Aim?
27. Coal – Energy for Sustainable Development. (2012) World Coal Association.
28. Chen, Xia & Zhang, Junfeng & Huang, Yan & Tong, Zhiquan & Huang, Ming. (2009). Catalytic reduction of nitric oxide with carbon monoxide on copper-cobalt oxides supported on nano-titanium dioxide.
29. David W. Keith et al. (2018). A Process for Capturing CO2 from the Atmosphere.