Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
A		2226873

Power Profile of a Cyclist

Summary

There are many types of bicycle road races. In an individual time trial, each athlete rides a set course. The winner is the athlete who completes the full course in the shortest time.

In the first model, we analyzed the biological mechanisms of the cyclist during one exercise. First, we considered the total energy output by the athlete. It is very important in our following models, for it limits the whole model. Second, we modeled the degree of muscle fatigue through biological theoretical knowledge to derive the energy output curve of athletes during competition. Then, by analyzing the Function Threshold Power data of world-class athletes and their previous race records, we derived the power curves for the athletes in two types, which are time trial cyclists and sprinters.

In the second model, we apply it to the strategic application of three bicycle road races. Before that, we established a physical model of factors such as wind resistance, air pressure, etc., to calculate the energy loss of these factors. Then, we analyzed the geographic data of these three courses to get the gradient of the gradient. And the Depth First Search algorithm is used to optimize the rider's riding strategy that is most suitable for a specific course.

We then expanded the first two models so they could be used in team time trials. We took into account the riding position of the cyclists and the amount of wind resistance borne by the cyclists in different positions in the team. After that, we could get the best use of energy for the team time trial. Therefore, we get the optimal way of team competition, which is much more meaningful.

To make the model completer and more realistic, we also considered the potential impact of weather conditions such as wind and atmospheric pressure. Then, we use Monte Carlo algorithm to analyze the effect on muscle fatigue when athletes failed to strictly enforce the power targets. Based on the above two conditions, we get the influence of some variables on our model and its sensitivity of it. In order to make our complex model more simple and more understandable, we use ring integral to optimize the model. More accurate models of wind direction, etc.

Key Words: Muscle Fatigue, Functional Threshold Power, Critical Power, Blood lactate accumulation, Depth First Search algorithm, Monte Carlo algorithm

Contents

1. Introduction

1.1 Background

Cycling has many types of competitions, such as cross-country competitions, mountain competitions, BMX, and so on. We're mainly looking at the bicycle road racing here, which includes big group races and time trials. We only consider the time trial case here. Time Trials are divided into Team Time Trials (TTT) and Individual Time Trials (ITT). Cyclists do not have the same probability of winning in these races. Therefore, it is necessary to guide the energy distribution of athletes in the game to achieve the goal of using the least time to complete the game and win.

The amount of energy a cyclist can generate and how long it lasts vary greatly from cyclist to cyclist. There is a limit to the total amount of energy an athlete can expend. FTP is related to the intensity of exercise at which lactate begins to accumulate in the blood, which is known as the lactate threshold (LT). It is a significant factor in human endurance performance. Therefore, we can use it to determine the energy output of the athlete and predict the fatigue of the athlete, to give a game strategy suitable for the athlete.

Cyclists are often divided into various types due to different factors such as body structure and muscle content. For example, time trial specialists, sprinters, rouleurs, etc. Their energy curves tend to be different. Cyclists always want to minimize their racing time. Therefore, when to apply force and when to rest becomes a very important thing. Different types of athletes have different abilities. So different athletes have different optimal energy use times.

1.2 Restatement of the Problem

To allow cyclists to finish in as little time as possible and to determine the relationship between cyclist position and energy, we are asked to develop a mathematical model which applies to any type of cyclist. Specific issues are as follows:

- Define energy curves related to the gender and type of an athlete.
- We ignore this consumption because the additional energy loss of the athlete during acceleration or deceleration is minimal and there is currently no data to support it.
- Determine the impact of weather conditions and athlete mistakes in executing the plan on the model.
- Extend the model and then in this way it can be used for a TTT (team time trial) with a team of six players.

2. Assumptions

We make some general assumptions to simplify our model:

- Only consider the effect of lactate buildup on athlete performance
- The points issued account for less market share and do not become the market leader.
- The cyclists in the team have the same strength, and the time for the cyclists to exchange orders can be ignored.
- Every driver in the team race has the same metrics.
- No other biological factors other than HR, LA, etc. are considered.
- The factor of consuming energy to increase heat is not included.
- A is calculated as a constant.

More detailed assumptions will be listed if needed.

3. Notation

SYMBOLS	DEFINITION
$\boldsymbol{E}$	The total energy a cyclist can expend
$\boldsymbol{E_\sigma}$	The athlete's energy consumption
$\boldsymbol{HR}$	Heart rate
$\boldsymbol{LA}$	Blood lactate accumulation
$\boldsymbol{FTP}$	Function Threshold Power
$\boldsymbol{CO}$	Cardiac output
$\boldsymbol{VO_{2_{max}}}$	Maximal oxygen consumption
$\boldsymbol{V}$	Velocity
$\boldsymbol{CP}$	Critical power
$\boldsymbol{W}$	Watts
$\boldsymbol{\eta}$	Mechanical efficiency
$\boldsymbol{P_{air}}$	Air resistance power
$\boldsymbol{P_{roll}}$	Rolling resistance power
$\boldsymbol{P_G}$	Gravity power

$\boldsymbol{A}$	Frontal area
$\boldsymbol{C_d}$	Air resistance coefficient
$\boldsymbol{\mu}$	Rolling resistance coefficient
$\boldsymbol{f_{air}}$	Air resistance to the competitor

Table 1: Notation

4. Cyclist energy model

4.1 Athlete total energy consumption limit

A recent study published in the journal *Science Advances*[12] points out that humans have limits to their endurance. The maximum sustained energy expenditure of the human body is approximately equal to 2.5 times the resting metabolic rate, with an average of 4000 kcal per day. According to the *Journal of Applied Physiology*[3], we can know that the mechanical efficiency of a professional cyclist in a race is about 22%. Therefore, we can calculate that the total energy E a cyclist can expend throughout the race is at least:

$$E = 4000kcal - (4000kcal/2.5) = 2400\ kcal = 10041600J$$

Currently, the world's top cyclist is Filippo Ganna. According to the data on Strava, we can know that his FTP (Function Threshold Power) is about 494W. FTP in cycling training refers to the maximum average power obtained during full-strength and stable cycling within 1 hour. Thus, we can get that the athlete's energy consumption E_σ in one hour is approximately equal to:

$$E_\sigma = 1 \times 60 \times 60 \times 494/22\% = 8083636.364J$$

The difference ΔE between the two is about:

$$\Delta E = E - E_\sigma = 10041600J - 8083636.364J = 1957963.636J$$

The total energy that can be expended is significantly greater than what the athlete expends in an hour. By studying the previous bicycle time trials, we can know that the competition time is about 1 hour, so there is no possibility that athletes consume more energy than the total energy that can be consumed in the bicycle time trial. Therefore, in the subsequent model, we no longer consider the limit of the total energy of the cyclist under the condition that the FTP is less than or equal to 494J.

4.2 Fatigue levels measurement model

When the rider outputs FTP, cyclists can maintain it for about 1 hour. As long as the power exceeds FTP for some time, lactic acid in the body will rapidly accumulate. Therefore, the time that can be maintained will be shortened. On the contrary, when the practical power is below FTP, it can be maintained longer.

Fatigue-induced changes in exercise performance span many disciplines. Many linear models are used to explain the unavoidable feeling of fatigue. Models of fatigue systems in prolonged exercise such as cycling time trials are closely related to cardiovascular, energy supply, neuromuscular fatigue, muscle trauma, biomechanics, thermoregulation, psychology, and central governors. Therefore, the fatigue system model is extremely complex.

The athlete's cardiac output will be the limiting factor for the athlete's exercise situation, where:

$$CO = HR \cdot V_{stroke}$$

However, the study by Warburton et al. showed that artificial plasma volume expansion in elite cyclists did not effect on ${VO_2}_{max}$. However, plasma volume expansion led to increases in cardiac output and hemoglobin. It suggested that the performance of ${VO_2}_{max}$ is not limited by cardiac output but may be limited by a combination of blood volume and hemoglobin content.

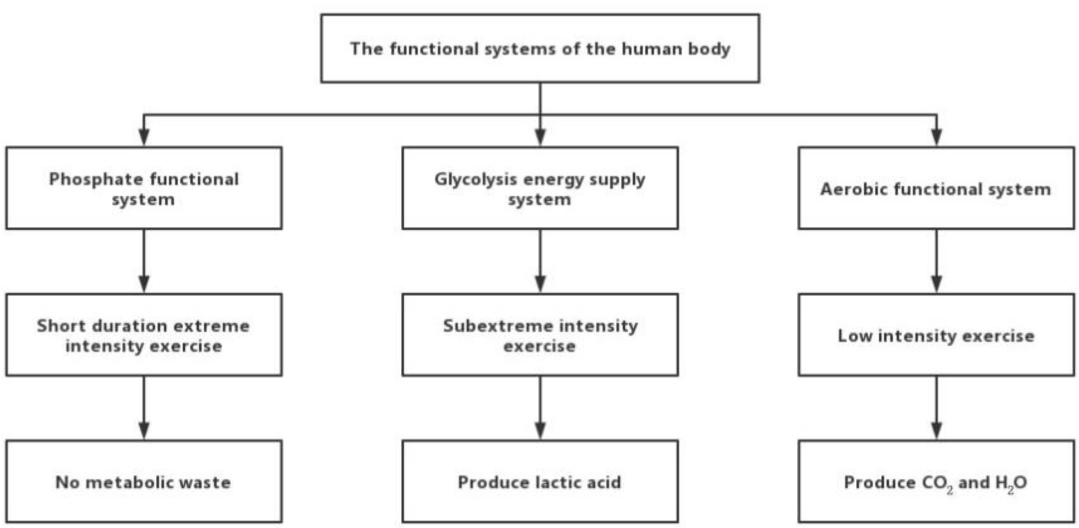

Figure 1 The functional systems of the human body

Blood doping and EPO (Erythropoietin) is alleged to be used by some famous cyclists such as Lance Armstrong. Therefore, we can get that hemoglobin content and red blood cell concentration will affect the performance to a certain extent. At the same time, on the physical level, some factors such as Muscle Blood Flow Occlusion, Oxygen Utilization Rate, and Neuromuscular Fatigue limit the performance of athletes. But due to their complex nature, we mainly consider the accumulation of metabolites which is the lactic acid in muscle or blood. We also need to consider the Fatigue mechanisms.

The human body produces lactic acid all the time. As long as the intensity is below the critical value, the body will break it down instantly. When we continue to increase the intensity, the production of lactic acid will continue to increase and the body's ability to break down will also increase. Therefore, the lactic acid concentration is in a state of dynamic equilibrium. However, when the intensity continues to increase, the lactic acid produced exceeds the standard, and lactic acid will accumulate in the blood in large quantities. Accumulation of lactic acid can cause rapid heartbeat and shortness of breath, which can affect muscle contraction and cause muscle fatigue, which in turn affects exercise performance and the power curve.

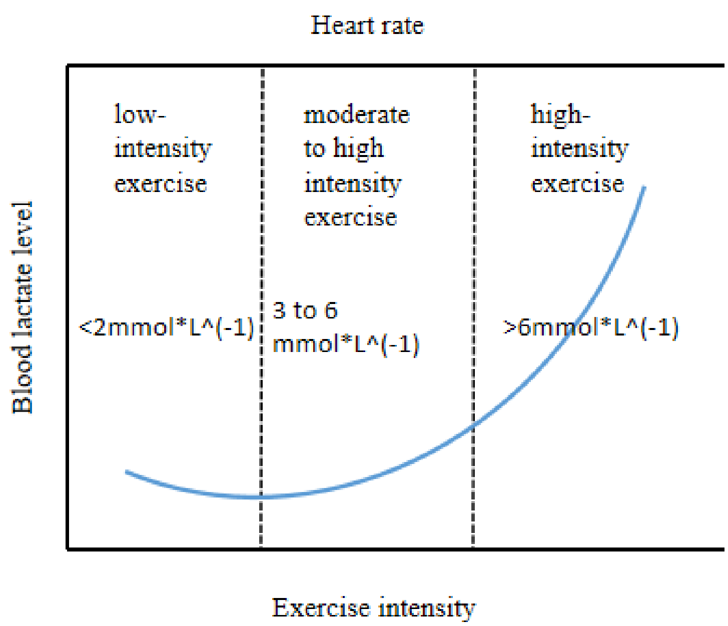

Figure 2 Blood lactate curve

Fatigue mechanisms are usually classified as being of two forms. The first one is peripheral fatigue, encompassing mechanisms of torque loss attributable to events distal to the neuromuscular junction. The second one is central fatigue, in which torque losses are attributable to events proximal to the neuromuscular junction.

4.3 The critical power of athletes

In order to study the mechanical efficiency and energy curve of athletes, we define critical power. It is traditionally defined as the external power output that could be sustained indefinitely.

In modern biology, *CP* is considered to represent the greatest metabolic rate that results in completely oxidative energy provision, where oxidative means energy supply through substrate-level phosphorylation reaches a steady state. The production rate of lactate inactive muscle matches its clearance rate in muscle and other tissues.

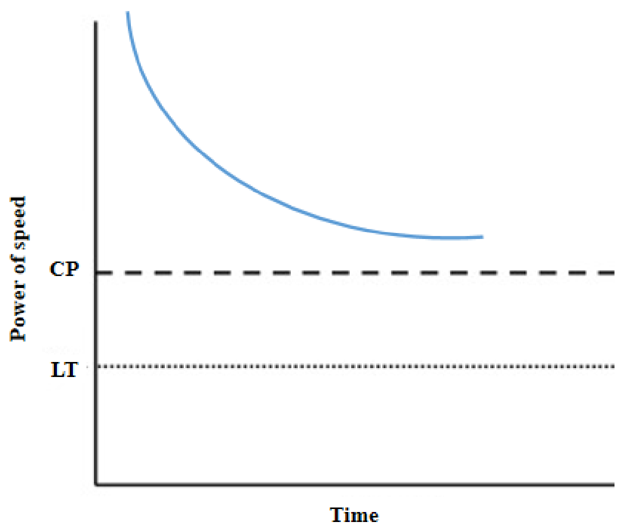

Figure 3 Critical power curve

The *CP* of a professional athlete is between 80%-90% $VO_{2_{max}}$. We calculated that the *CP* of the cyclist Filippo Ganna studied above is about 480, which is in line with the data support.

Notably, although the *CP* parameter is well defined as the maximum oxidative metabolic rate that can be maintained without a discontinuous reduction in the additional work done *F*, its physiological determinants are more difficult to address. While seemingly mathematically accurate, the physiological interpretation of this model is extremely complex. There is a close qualitative relationship between $VO_{2_{max}}$ development and the dynamic changes in *F* during high-intensity exercise. Therefore, *CP* should be considered an integral part of an integrated bioenergy system. However, there will always be some error in the estimation of *CP* and it varies slightly from day to day in the same subject.

4.4 Fatigue mechanisms

4.4.1 High-intensity exercise model

In the case of high-intensity exercise, we have to consider central fatigue. We compared the data of each node in *Exercise intensity during competition time trials in professional road cycling*[7] and then found that the relationship between *HR* and *W* in a steady state is shown in the figure below:

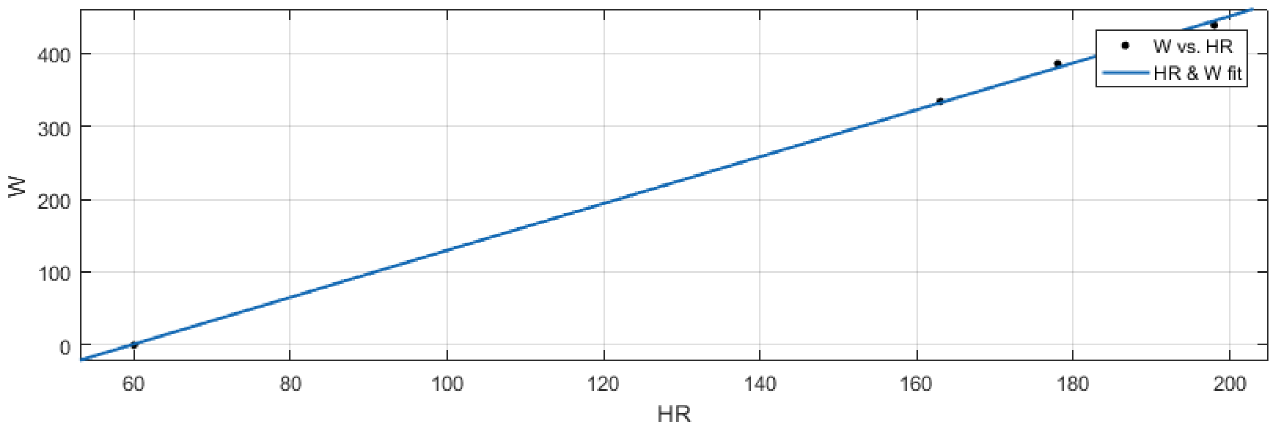

Figure 4 The relationship between *HR* and *W*

We can know that R-square: 0.9994, the function fitting result is more real and reliable. Therefore, we can get that *W* and *HR* have a linear relationship.

By analyzing the data of *HR* and *LA*, we can get:

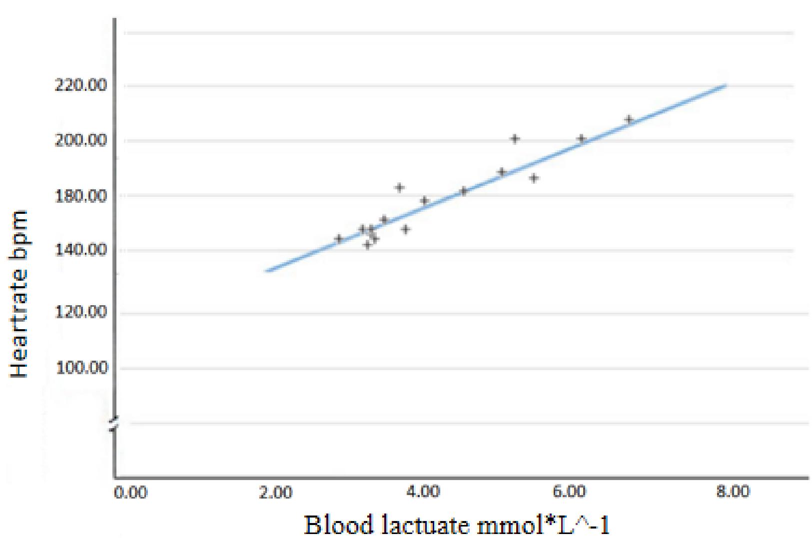

Figure 5 The relationship between *HR* and *LA*

Obviously, HR also has a linear relationship with LA.

However, due to the limitations of humans, the upper limit of LA is $9.9mmol \cdot L^{-1}$, which limits the time that athletes can exercise above the aerobic interval. At the same time, the closer LA is to the upper limit, the more fatigued the muscles, and the lower the mechanical efficiency of the athlete.

4.4.2 Low-intensity exercise model

In the case of low-intensity exercise, we have to consider peripheral fatigue, which is a slow process. The accumulation of metabolites has less effect. Below *CT*, the progressive development of peripheral fatigue is slow. However, it occurs in the face of measurably invariant metabolic and cardiorespiratory function. Thus, it is obvious that metabolic factors are unlikely to be responsible for such a decline. We know that the exercise intensity that can be tolerated within 3 hours will drop below *CP*:

$$W = CP + f(t)$$

The image obtained by curve fitting shows that:

$$\begin{cases} f(t) > 0, & t < 2.5h \\ f(t) < 0, & t > 2.5h \end{cases}$$

4.5 Power curves of the cyclists

Filippo Ganna is the top-time trial cyclist in the world. Mark Cavendish is the world's top sprinter cyclist. They represent the characteristics of world-class cyclists. Based on the above fatigue model, we analyzed the power data of Filippo Ganna and Mark Cavendish in important stages of the Grand Tour in the past five years. From the power data, we get the maximum power output at a specific time. We then corrected the image using *FTP* from the cyclists. The following two images are obtained:

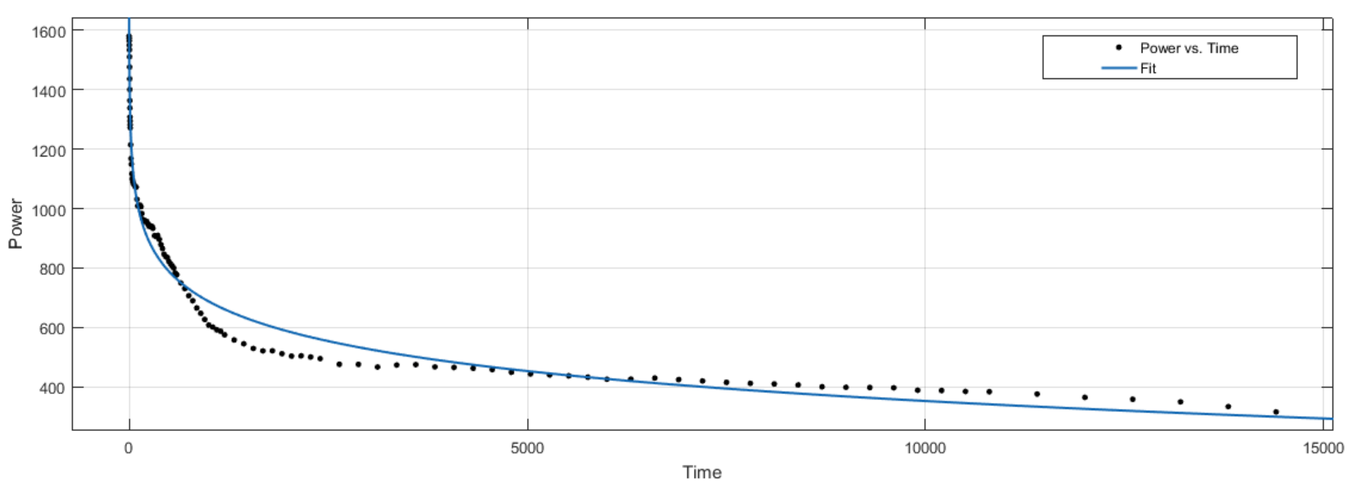

Figure 6 Power curve of sprinters

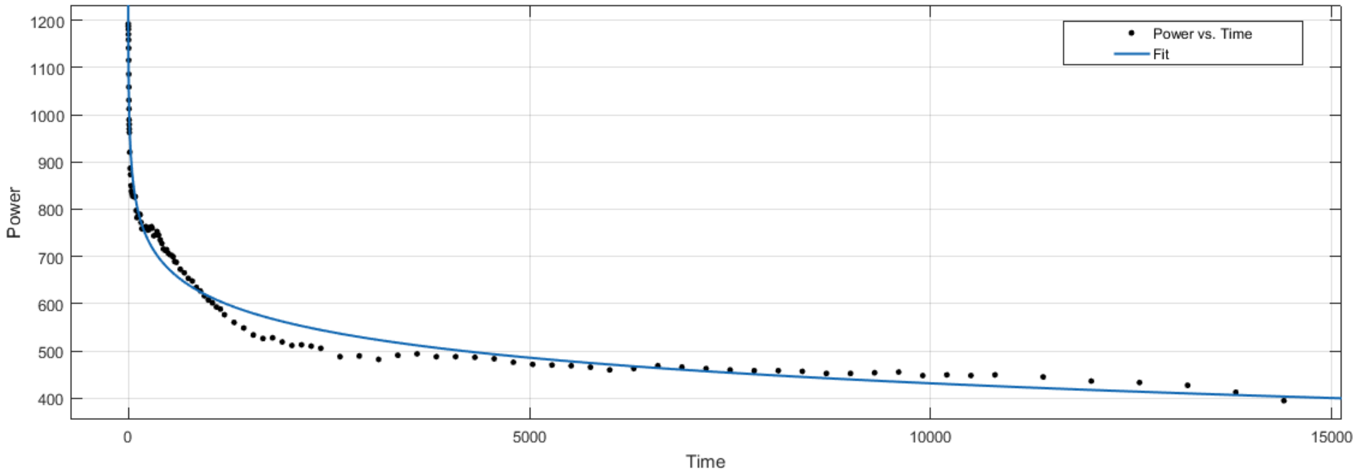

Figure 7 Power curve of time trial cyclists

Through curve fitting, we obtain the functional expressions of these two analytical expressions：

$$\begin{cases} f_{Sprinters(male)} = 58170 \cdot t^{-0.002553} - 56470 \\ f_{Time\ trial\ speacialist(male)} = 3269 \cdot t^{-0.03163} - 2011 \end{cases}$$

We analyzed the course data and their *FTP* of world-class female athletes and found that their energy curves tended to be nearly identical to their male counterparts.

The energy output of female athletes was approximately 88.6% of that of men, based on data analysis of their power-to-weight ratio ratios to that of male athletes. So we get:

$$\begin{cases} f_{Sprinters(female)} = 0.886 \cdot (58170 \cdot t^{-0.002553} - 56470) \\ f_{Time\ trial\ speacialist(female)} = 0.886 \cdot (3269 \cdot t^{-0.03163} - 2011) \end{cases}$$

Therefore, we defined the energy curves of two kinds of athletes through the Fatigue levels measurement model and the *CP* of athletes.

5. The application of the model in the time trial

5.1 Physics model

We define Power loss and divide it into P_{air} , P_{roll} and P_{G}

$$P_{\text{output}} \cdot \eta = P_{\text{air}} + P_{\text{roll}} + P_{\text{G}}$$

According to *Effects of Frictional Loss on Bicycle Chain Drive Efficiency*[20], we know that when a TT bike is well preserved, it's $\eta \approx 97.2\%$.

In this formula:

$$P_{air} = \frac{1}{2}\rho_{air} A C_d v^3$$

where p is the air density. A is the total frontal area of the cyclist and bicycle. Cd is the drag coefficient. v is the speed of the cyclist relative to the wind. The air resistance power loss is therefore defined by:

$$\rho_{air} = 1.293\mathrm{kg/m^3} \cdot \frac{P}{1\ atm} \cdot \frac{273.15\ K}{T}$$

When the temperature is 25℃ and the atmospheric pressure is 1atm, $\rho_{air} = 1.185\ \mathrm{kg/m^3}$.

According to *Aerodynamic performance and riding posture in road cycling and triathlon*[18], when it is in TT posture, $A = 0.400\ m\hat{}2$.

According to *Riding against the wind: a review of competition cycling aerodynamics*[14], $C_d = 0.6$.

The roll resistance power loss is equal to:

$$P_{\mathrm{roll}} = \mu \mathrm{mg} \cdot v$$

where μ is the roll friction coefficient, m refers to the mass of the bike and cyclist. And g is the acceleration due to gravity, which approximately equals $9.81 m/s^2$.

From *Selecting cycling equipment*[13] *in*. We get $\mu = 0.002$.

Also:

$$P_G = \mathrm{mg}\sin\theta \cdot v$$

where θ represents the gradient angle (it can be either positive or negative) and G represents the gravitational force on the cyclists.

Therefore:

$$P_{\mathrm{output}} = \frac{\frac{1}{2}\rho_{air} A C_d \cdot v^3 + \mu \mathrm{mg} \cdot v + \mathrm{mg}\sin\theta \cdot v}{\eta}$$

Substitute the data mentioned above, we get:

$$P_{\mathrm{output}} = 0.1463\ v^3 + 1.6148\ v + 807.4 \cdot \sin\theta \cdot v$$

5.2 Course section analysis

5.2.1 Methodology of the analysis

Lemma: **On a section with a constant gradient, completing the section at the same time and using a constant power output will cause the least fatigue.**

First, we divide the total distance into $s_1, s_2, \ldots, s_k, \ldots, s_N$. Then consider a certain distance s_k where the average power P is constant. From the fatigue formula $Fatigue^{(i)} = v_{fatigue}{}^{(i)} t_i$, the total movement time *t* of this section of the journey.

From the power-time curve we can know that:

$$P = At^{-r} + c, \qquad r, c \in R^+, t = t_{fatigue}$$

$$t_{fatigue} = \frac{1}{A^r}(P - c)^r$$

Then we can get:

$$Fatigue^{(i)} = \frac{A^r t_i}{(P-c)^r}$$

Order $f(x) = \frac{1}{(x)^r}, x, r \in R^+,\ r < 1,\ \frac{d^2 f}{dx^2} = r(r+1)x^{-r-2} > 0$

$$w_i = \frac{t_i}{t}, \quad \sum w_i = 1$$

By the weighted Jenson inequality,

$$\sum w_i f(P_i - c) > f(P - c)$$

$$\sum \frac{A^r t_i}{(P-c)^r} > \frac{A^r t}{(P-c)^r},$$

More generally, when the driver's power varies continuously, there is:

$$\int_0^t \frac{A^r d\tau}{(P_\tau - c)^r} > \frac{A^r t}{(\bar{P} - c)^r}$$

In which:

$$\bar{P} = \frac{1}{t}\int_0^t P_\tau d\tau$$

$$\int Fatigue^{(\tau)} > Fatigue$$

Therefore, for $\sum Fatigue = 1$ is increased, the cyclist will be more fatigued. Proof is completed.

The Lemma serves as the theoretical basis for subsequent optimization strategies

- Data preprocessing:

  We use GetData Graph Digitizer to obtain altitude data from the 2020 Summer Olympics men's individual time trial course, and then perform differential operations on the data to obtain the gradient of the course. In order to reduce the time complexity, we divide the gradient into different grades and derive the road length for different grades of gradients.

- Conversion of relationship:

  Then, we calculate the relationship between the Fatigue value and the speed based on the physical model given before.

From the previous fitted equation,

$$P = 3269 \cdot t_{Fatigue}{}^{-0.03163} - 2011$$

We can get:

$$t_{Fatigue} = \left(\frac{P + 2011}{3269}\right)^{-31.6156}$$

$$F = \frac{t}{t_{Fatigue}} = \frac{t}{\left(\frac{P + 2011}{3269}\right)^{-31.6156}}$$

Substitute it into:

$$P_{\text{output}} = 0.1463\ kg \cdot m^{-1} \cdot v^3 + 1.6148\ kg \cdot \text{m} \cdot s^{-2} \cdot v + 807.4\ kg \cdot \text{m} \cdot s^{-2} \cdot \sin\theta \cdot v$$

Where:

$$v = \frac{D}{t}$$

F & t 's relationship can be expressed as:

$$F = t \cdot \left(\frac{0.1463\ kg \cdot m^{-1} \cdot \left(\frac{D}{t}\right)^3 + 1.6148\ kg \cdot \text{m} \cdot s^{-2} \cdot \frac{D}{t} + 807.4\ kg \cdot \text{m} \cdot s^{-2} \cdot \sin\theta \cdot \frac{D}{t} + 2011}{3269}\right)^{31.6156}$$

- Find the minimum value:

  Finally, we applied the Depth First Search algorithm to the usage of the Fatigue value on each gradient level (granularity = 0.01) to find the shortest time and the ideal power distribution in the case.

  In the next section, we apply this particular methodology to the cycling course.

5.2.2 2020 Summer Olympics men's individual time trial course in Tokyo

Figure 8 The course map & the elevation map

We digitized the elevation data of the course and made the gradient graph:

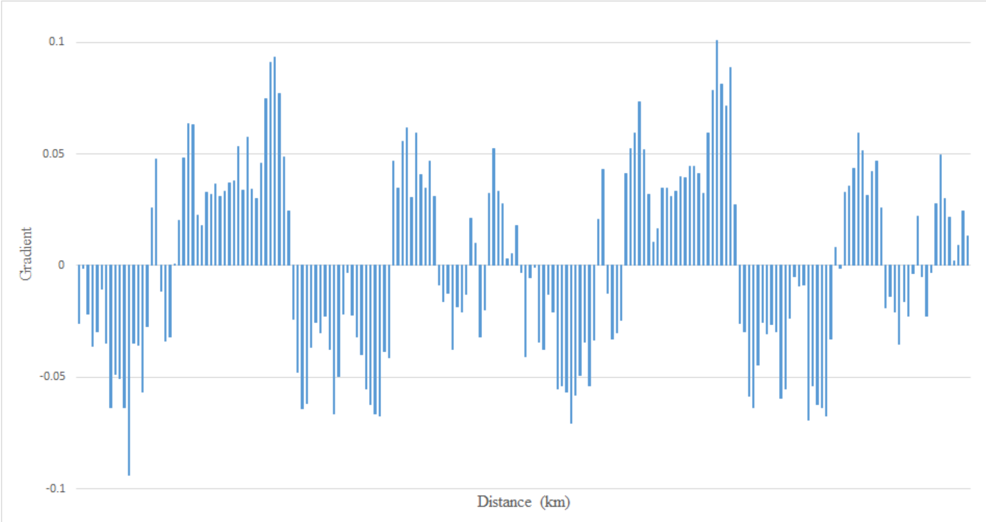

Figure 9 Gradient of the course

Then, we apply the methodology and get the shortest time $t_{min} = 55.67\ min$
Also, the ideal power distribution is shown:

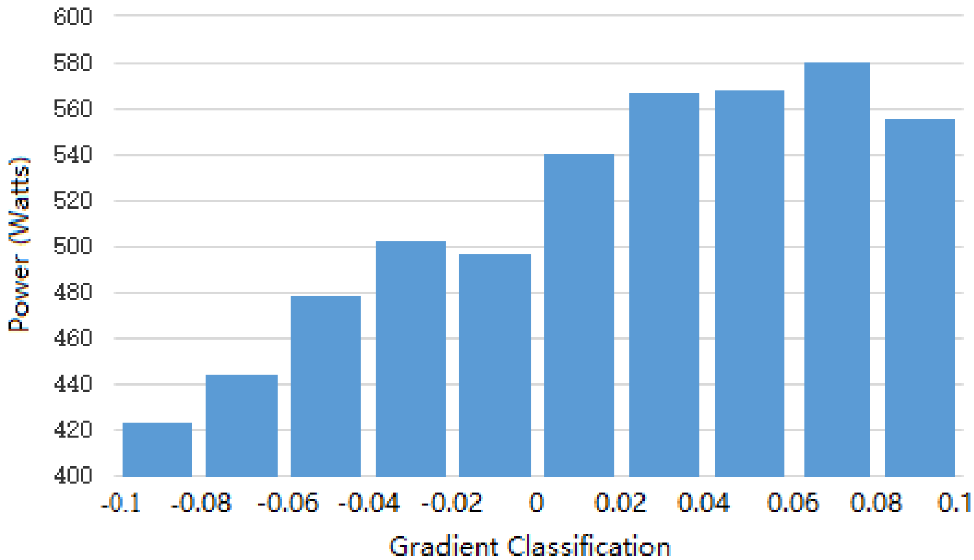

Figure 10 the ideal power distribution

5.2.3 2021 UCI World Championship men's elite individual Time Trial course in Flanders, Belgium

Figure 11 The course map & the elevation map

We analyzed the elevation data of the course and got the gradient graph:

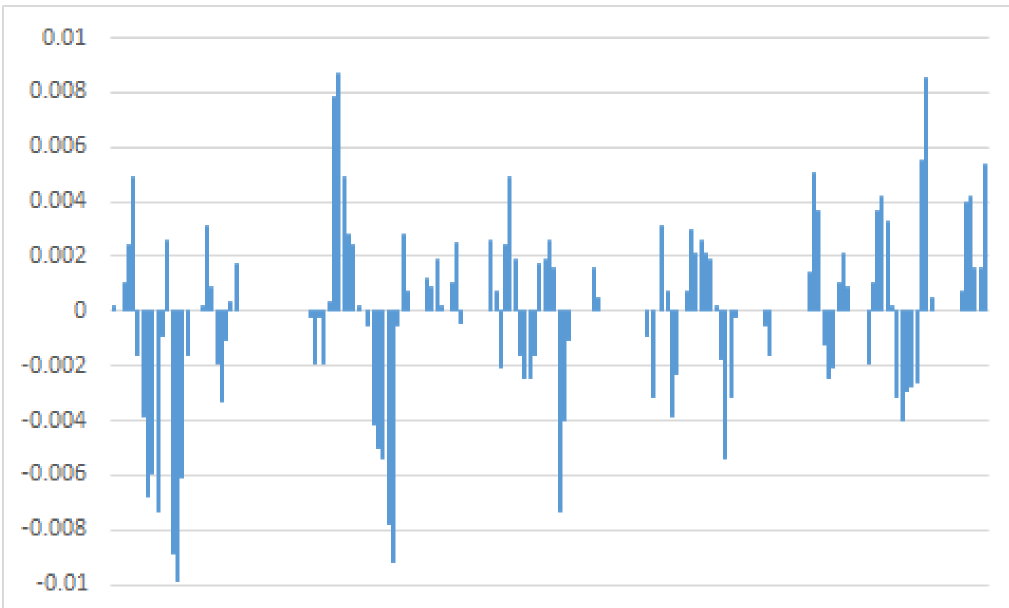

Figure 12 Gradient of the course

Then, we apply the methodology, and get the shortest time $t_{min} = 49.26\ min$

Also, the ideal power distribution is shown below:

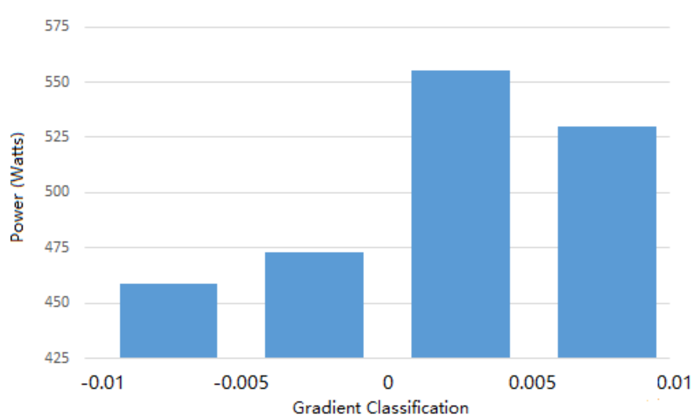

Figure 13 the ideal power distribution

5.2.4 The course of our own design

As requested, we need a loop course with at least four sharp turns and a non-trivial road grade. Because sharp turns only get involved when the rider has a considerable amount of speed, we assume that the sharp turns are in the downhill section. This reminds us of the hairpin turns in Alpe d'Huez, which is famous for its horrible gradient and spectacular hairpin turns. So, we designed a similar but theoretical course using the 13.2km section of the Alpe d'Huez from the summit as a prototype.

The course, just like Alpe d'Huez, is 13.2 km long, with 19 hairpins left. We define every part between two hairpins as equal in length, which is 600 m. The equivalent radius of every hairpin is 3 meters.

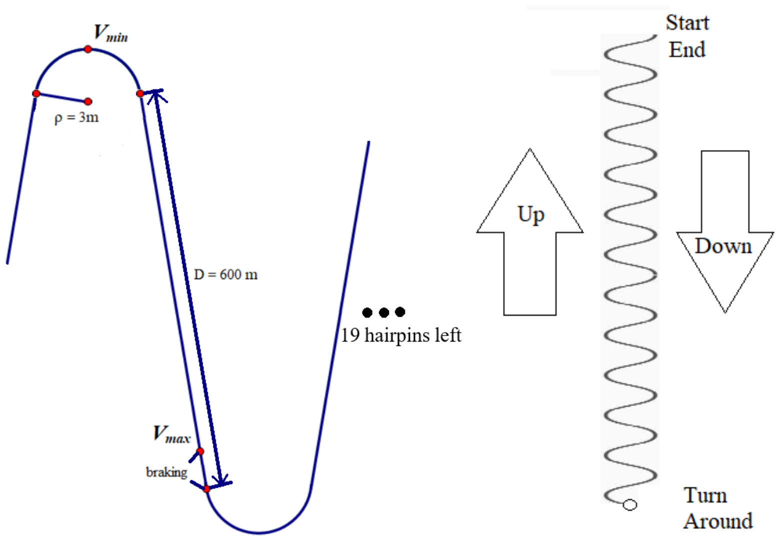

Figure 14 The source we design

Considering the racer's hairpin bending motion on the hillside plane, the radius of curvature at the inflection point is R, the tangential acceleration $a = a_r = \frac{v_M - v_m}{t}$, and the horizontal angle ϕ between the center of the curvature circle and after a short time dt, ϕ becomes ϕ_1, f_x, f_y is the component of f_{roll} in the plane.

We have;

$$\int W_{f_x} = \int (\mu g cos\theta + a)m(sin\phi_1 - sin\phi) \cdot R d\phi = 0$$

$$\int W_{f_y} = \int (\mu g cos\theta + a)m(cos\phi_1 - cos\phi) \cdot R d\phi$$

The second one equals:

$$\int_0^{\pi} -\mu mg cos\theta sin\phi \cdot R d\phi = -\mu mg cos\theta \cdot R - ma \cdot R \triangleq E_k$$

According to the kinetic energy theorem and the momentum conservation law:

$$\begin{cases} \frac{1}{2}m\cdot(v_M^2 - v_m^2) = -\Delta E_k \\ m(v_M + v_m) = 2mt(\mu g cos\theta + a) \end{cases}$$

$$\begin{cases} v_M + v_m = 2t(\mu g cos\theta + a) \\ v_M - v_m = \frac{\mu g cos\theta t}{t^2(\mu g cos\theta + a) - 1} \end{cases}$$

We substitute $a = a_r = \frac{v_M - v_m}{t}$ into the formula. Hence:

$$v_M - v_m = \frac{R - t^2\mu g\cos\ \theta \pm \sqrt{(R - t^2\mu g\cos\ \theta)^2 + 4Rt^2\mu g\cos\ \theta}}{2t}$$

$$\begin{cases} v_M = \frac{2R}{t} + t\mu g\cos\ \theta \\ v_m = t\mu g\cos\ \theta \end{cases}$$

Here, $\theta = \arctan(Average\ gradient)\ , Average\ gradient \approx 7.9\%$.

6. Sensitivity analysis

6.1 The air resistance and the power loss

6.1.1 Sensitivity analysis for idealized wind resistance model

From the formula:

$$P_{\text{output}} = \frac{\frac{1}{2}\rho_{air}AC_d\cdot v^3 + \mu \text{mg}\cdot v + \text{mg}\sin\theta\cdot v}{\eta}$$

We can get:

$$\frac{\eta dP_{output}}{dv} = \frac{3}{2}\rho_{air}AC_d\cdot v^2 + \mu\text{mg} + \text{mg}\sin\theta$$

The formula equals to $\frac{dP_{output}}{dv} = 0.4778v^2 + 329.3212$ when assumed that $m = 65kg, \theta = \frac{\pi}{6}$

6.1.2 Sensitivity analysis of wind resistance model

We can regard a certain section of s_i as a conservative field of wind.

Hence:

$$\oint_{A\to B} f_{air}ds = \frac{1}{t}\int f_{air}ds$$

$$f_{air}\cdot|A_1B_1| = f_{air}\cdot\vec{e}\cdot\overrightarrow{AB}$$

$$f_{air} = \frac{1}{2}\rho_{air}AC_d v^2$$

To some extent, wind resistance can be assessed before the competition:

$$\sum_{Sum\ over\ all\ paths} v^2 \cdot \overrightarrow{AB} \approx \bar{v}^2 \cdot \vec{s}$$

$$P_{average} \approx \frac{1}{2}\rho_{air}AC_d\bar{v}^2 \cdot (\vec{e} \cdot \vec{s}) \lesssim \frac{1}{2}\rho_{air}AC_d\bar{v}^2 \cdot s \cdot cos\langle s, wind\rangle$$

Here, $\vec{s}$ refers to vectors that represent the beginning and end of the journey.

6.2 Rider deviations from the target power distribution

We used the Monte Carlo algorithm to simulate the finish time of the rider when his execution accuracy is a given value ($n = 100$) and obtained the following figure:

Figure 15 Rider deviations

7. Team time trial energy usage model

Drafting and slipstreaming are a kind of techniques where two bicycles or other objects are caused to align in a close group. Therefore, it can reduce the overall effect of drag due to exploiting the lead object's slipstream. Especially when high speeds are involved, for example, in a cycling time trial, drafting can reduce the paceline's average energy expenditure required to maintain a certain speed. It can also slightly reduce the energy expenditure of the lead cyclist.

Besides, the rider who is being drafted also benefits from this situation. Because the "vacuum" which is filled by the rear bike does not have to immediately gather the air being changed, thus reducing energy consumption.

Figure 16 Body posture of time trial runners

Based on *Aerodynamic drag in cycling team time trials*[16], we can know that:

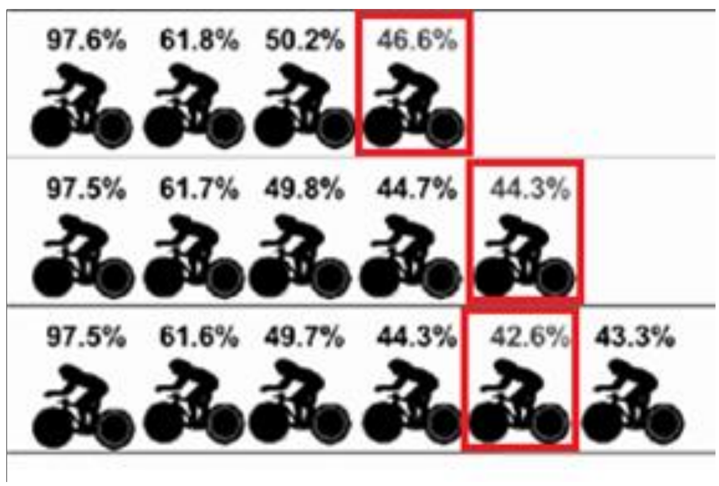

Figure 17 Wind resistance for each athlete

It can be seen from this that the first athlete bears the greatest wind resistance, which decreases in turn. And the number of athletes is different, each athlete bears different wind resistance. The red box outlines the fourth athlete we need to study.

Because the total consumption of the team's fatigue level is conserved, we need to consume the 5th and 6th cyclists as soon as possible to ensure the power output of the last four cyclists in the last stint.

Then the 6th cyclist needs to be in the first place in the first stint, and the 5th in the second place. After that, the 6th is dropped, and then the 5th is on top of the first stint, starting the second stint. Finally, 5th is dropped, and the remaining four enter the last stint.

It is worth mentioning that the remaining four people have been switching positions to ensure that the consumption is even so that the four people cross the line at the same time (almost at the same time) to finish the race in the shortest time.

Thus, what needs to be calculated is the team speed of the 1st and the 2nd stint led by the 6th and 5th.

The following figure shows the team strategy, the numbers represent the air resistance:

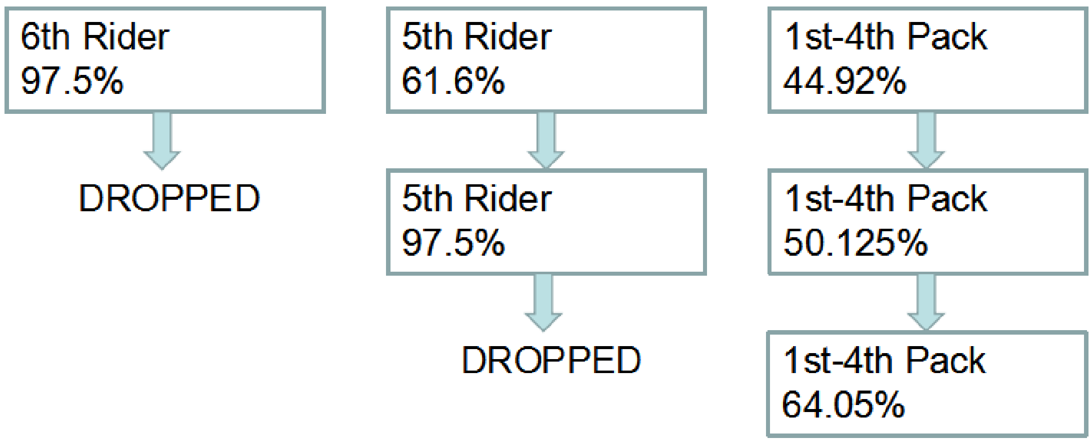

Figure 18 Air resistance

In order to focus on the impact of the TTT strategy on the finishing time, we ignore the changes in the terrain, assuming that all roads are completely flat. We write the program using the Depth First Search algorithm, and get the result:

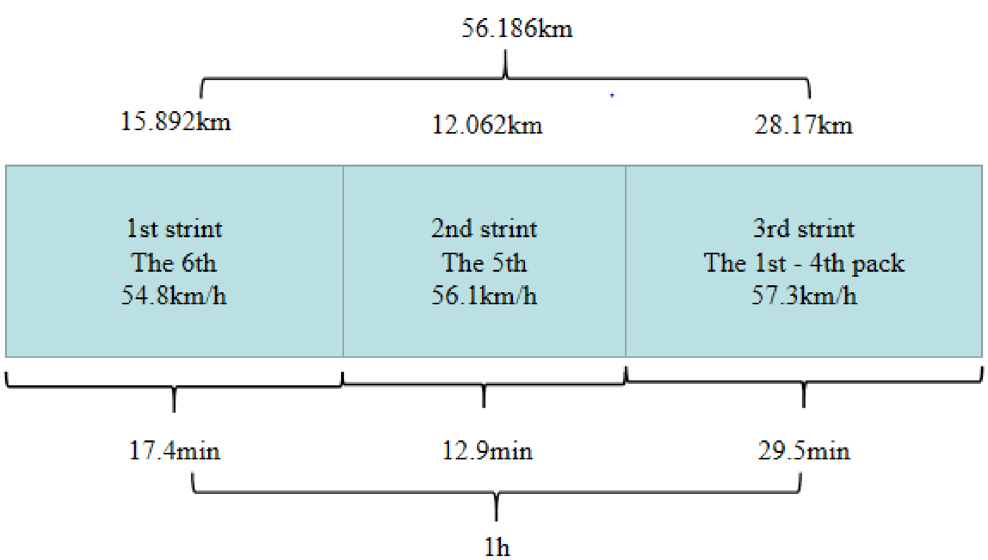

Figure 19 The TTT strategy

8. A rider's race guidance for the Directeur Sportif

Dear Directeur Sportif:

Hello!

We have developed this guide to assist team directors in completing ITT safely and efficiently. This guide explains the information necessary to participate in the time trial, the driver's race instructions and time trial advice, etc.

Before the race

During the week before the race, the cyclist should exercise at a moderate intensity and consume a lot of carbohydrates and fat. The racer should eat something light the night before the race and avoid chocolate before the race as it can make blood sugar rise and make the cyclist feel tired.

In pre-race training, the racer should be fully familiar with the race site, fully implement the optimal power distribution plan and form muscle memory. Before a race, the cyclist can do moderate leg muscle stretching to relieve fatigue.

Equipment should check whether there is damage to the outer tire and whether the chain is lubricated and tightened. The tires should be inflated. As a bicycle's tires move on the road, they flatten and expand as they roll. Narrower tires have a smaller contact area with the ground and a smaller rolling resistance coefficient, but they have a greater energy loss when facing the rough ground. Wider tires are generally more aerodynamic and have more grip, but will have more rolling resistance. Therefore, extensive attention should be paid to the terrain, weather, and other factors before the race to prepare more suitable bike accessories for the source. Proper rolling resistance can also increase the rider's comfort and reduce fatigue during a race.

During the race

The cyclist can warm-up and stretch moderately, which is very effective for improving race conditions and relieving muscle stiffness caused by mental tension. The cyclist should be sure to warm up before going to get the blood flowing in the body, get muscles in gear, and most importantly get the heart and lungs in gear.

Aerodynamics plays a crucial role in the time trial. According to our aerodynamics model, the racer should maintain the aerodynamic position during training, and adopting a more appropriate position will allow the racer to ride faster with little effort. To minimize the front windward area of the rider, flatten his back, and tuck his arms and elbows in. The cyclist shall make sure don't drop the posture too low to maintain it.

The biggest factor in a time trial is always the rider himself, no matter how good his bike or equipment is. The rider's body is subjected to about 75% of the air resistance, so adjusting the rider's position on the bike is an effective way to mitigate the effects of wind resistance. Also, an aerodynamic outfit can save three or four minutes on the road.

According to our optimal power distribution model, rhythm in power distribution is critical to maintaining peak performance. After the cyclist starts, he should leave the cushion and sprint within 10 seconds, which will allow the racer to achieve considerable speed. At the same time, the racer should maintain a steady pace, maintain the intensity of the exercise and not slow down suddenly for no reason, which can cause lactic acid to build up and greatly increase fatigue.

To achieve optimal power distribution and to control the racer's energy and fatigue, we recommend that the racer not risk relying on glycogen and fat reserves. During tough fights and long-distance races, riders quickly lose minerals, sugars, and electrolytes from their bodies. Loss of minerals and electrolytes can cause cramps, and failure to replenish sugar can cause fatigue, so the cyclists shall remember to replenish energy during the race and take it every 30km or 45 minutes.

In a large group race, we consider the effect of wind resistance on the racers. Teams should strictly implement the team alignment procedure, which will allow the first four sprinters to achieve minimum fatigue and save energy loss, thus achieving maximum power in the sprint.

As mentioned in the article, the 6^{th} cyclist needs to be in the first place in the first stint, and the 5^{th} in the second place. Then, drop the 6^{th} cyclist. After cycling for a period of time, the 5^{th} is dropped and the remaining four enter the last stint. The four racers should divide their energy expenditure equally so that the fourth racer has the best score.

Best wishes!

Sincerely, Team 2226873

9. Strength and weakness

9.1 Strength

- With data support, program support and a theoretical basis, the process of analyzing the power distribution planning is rigorous.
- Consider many factors such as wind resistance and rolling resistance.
- The model algorithm is optimized. The gradients are combined to facilitate the calculation of the allocation strategy.

9.2 Weakness

- We ignore the fact that players are taking energy gels to fuel up.
- We idealize all the players to simplify the calculation.
- The influence of biological factors other than HR, LA, etc. is neglected.
- The effect of road sequence on fatigue was ignored.

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