SJMMA 2021　E 题

最佳击球策略

　　在台球运动中,击球是取胜的关键。玩家需要使用球杆击打母球的不同位置,从而控制母球以不同的轨迹运动,与不同的目标球碰撞使其入袋。尽管台球本身的碰撞物理机制较为简单,但是不同的击球方式可以改变母球的运动路线,进而决定是否犯规并影响其他球的位置、以不同的方式完成击打目标球的任务,因此可以影响球局的形势以及双方的策略。这也使得一场职业台球比赛有很高的可观赏性。

　　本问题希望你的团队用数学建模的方法分析不同的击球技巧以及它们分别产生的效果与完成的难度。同时,由于专业与非专业玩家控球能力不同,你的团队还需要分析二者在击球方面的区别,并且为非专业玩家提高控球水平提供练习方案。

任务 1:建立模型分析一名选手击打母球时的行为,以及击打效果与预期的偏离。重要的参数有哪些？从你的模型出发,定性或定量地讨论专业和非专业玩家控球能力的不同。

任务 2:忽略进攻策略导致的不同,你需要分析讨论恰当实现各种击球技巧的难易程度。在可以自由放置母球和目标球的情况下,你能否简单地为一位台球爱好者设计训练击球技巧的次序？

任务 3: 对于只有一颗母球和目标球,但在二者在台上不同位置的情况,基于简单的击球技巧(例如只考虑加塞,即击打母球的不同位置使之旋转)和简单的进攻策略(例如只考虑直接击球和母球或目标球撞击一次库边)和之前已经得到的结果,建立模型分析击球入袋的难易程度。请定性或定量地讨论专业和非专业玩家在这方面的不同。

任务 4：一种练习台球的方式是，在球台的正中央放置一颗目标球，并在通过该目标球、平行于球台长边的直线上等间距对称放置共 5-7 颗目标球。母球的位置任意，但要求每一次击球都使得某一颗目标球落袋(母球始终不可落袋)，否则就恢复原状重新开始。基于你之前的分析，你能否为一位台球爱好者设计每一次击球的技巧和策略，使得他能最轻松地完成一次练习？

提交：你的团队所提交的报告应包含一页"总结摘要"、一页"标题"、一页"目录"，正文内容不超过 20 页，（总页数限于 23 页）。附录与参考文献应置于正文之后，不计入 23 页之限。

SJMMA2021　Problem E

Optimal Strategies for Striking Billiards

Striking the cue ball is critical in the game of billiards. Players need to strike different parts of the cue ball to set it into motion in different trajectories, eventually colliding with different object balls and pocketing them. Despite the relatively simple physical mechanisms governing the motion of billiards, different ways of striking the cue ball can manipulate its trajectory and consequently determine fouls, displace other balls, and pocket object balls in different ways, ultimately impacting the entire game and strategies of both players. Hence the charm of professional billiard games.

This problem requires your team to exert mathematical modeling to analyze different techniques for striking cue balls and the corresponding effects and difficulty of each technique. In addition, given the difference in ball control between professionals and amateurs, your team should also analyze the differences between the two in striking the ball and offer practice schemes for amateurs to improve their skills.

Task 1: Devise a model to analyze the action of striking the cue ball and the deviation of the actual effect from the intended effect. What parameters are important? Qualitatively or quantitatively discuss the difference between the ball control of professionals and amateurs based on your model.

Task 2: Disregarding the differences caused by the strategies of offense, discuss the difficulty in realizing each technique of striking the cue ball. Where the cue ball and object balls are placed at will, can you determine the order of training different striking techniques for a billiard enthusiast?

Task 3: Where there are only one cue ball and one object ball placed at different

positions, devise a model to analyze the difficulty of pocketing the object ball based on simple striking techniques (e.g. only considering swirls, i.e. striking different parts of the cue ball to set it spinning), simple offense strategies (e.g. only considering striking the object ball directly or cushioning the cue ball or the object ball once), and results from previous tasks.

Task 4: One way of practicing billiards is to place an object ball at the center of the aren and place a total of 5-7 object balls at regular distances along the long axis of the aren. Randomly place the cue ball. Each strike should pocket an object ball (the cue ball must not be pocketed) or the practice should be restarted. Based on your previous analysis, can you design the technique and strategy of each strike for a billiard enthusiast so that he can complete an exercise the most easily?

Submission: Your solution paper should contain a one-page Summary, a title page, and a one-page table of contents. The body should not exceed 20 pages for a maximum of 23 pages in total. Appendices and references should be appended to the end of the paper and do not count towards the 23-page limit.