Intro to Probability

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What we've done so far...

- Use visualization techniques to *visualize* data
- Use descriptive statistics to *describe* and *summarize* data
- Use data wrangling tools to *manipulate* data
- ...all using the reproducible, shareable tools of R and git

That's all great, but what we eventually want to do is to *quantify uncertainty* in order to make **principled conclusions** about the data

The statistical process

Statistics is a process that converts data into useful information, where practitioners

1. form a question of interest,
2. collect and summarize data,
3. and interpret the results.

The population of interest

The **population** is the group we'd like to learn something about. For example:

- What is the prevalence of diabetes among **U.S. adults**, and has it changed over time?
- Does the average amount of caffeine vary by vendor in **12 oz. cups of coffee at Duke coffee shops**?
- Is there a relationship between tumor type and five-year mortality among **breast cancer patients**?

The **research question of interest** is what we want to answer - often relating one or more numerical quantities or summary statistics.

If we had data from every unit in the population, we could just calculate what we wanted and be done!

Sampling from the population

Unfortunately, we (usually) have to settle with a **sample** from the population.

Ideally, the sample is **representative** (has similar characteristics as the population), allowing us to make conclusions that are **generalizable** (i.e. applicable) to the broader population of interest.

We'll use probability and statistical inference (more on this later!) to draw conclusions about the population based on our sample.

Interpreting probabilities

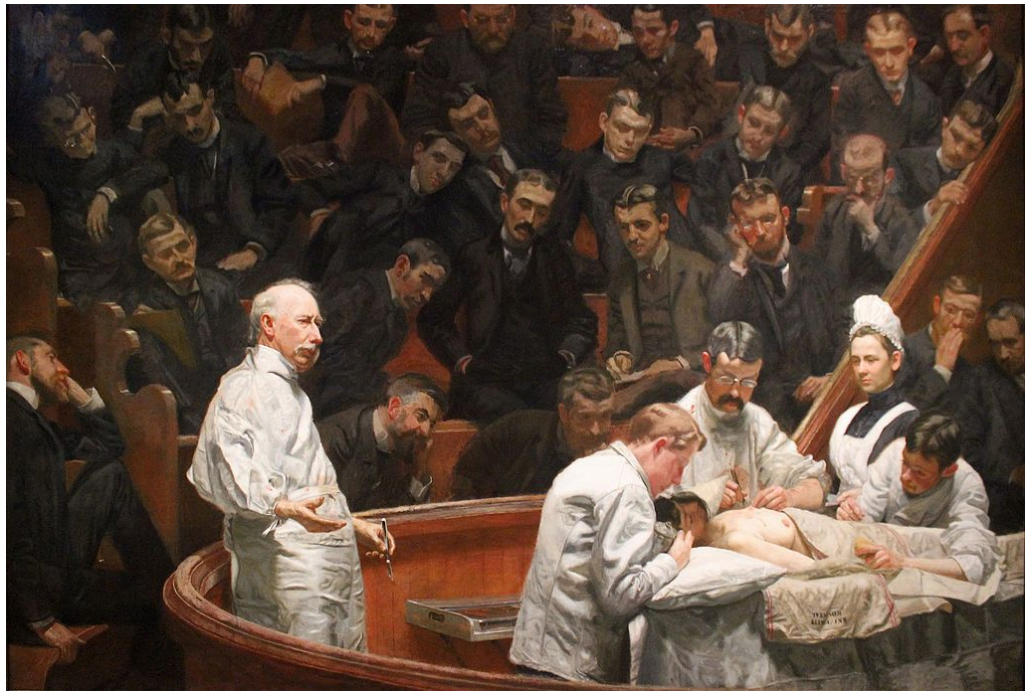
Interpretations of probability

"There is a 1 in 3 chance of selecting a white ball"

Interpretations of probability

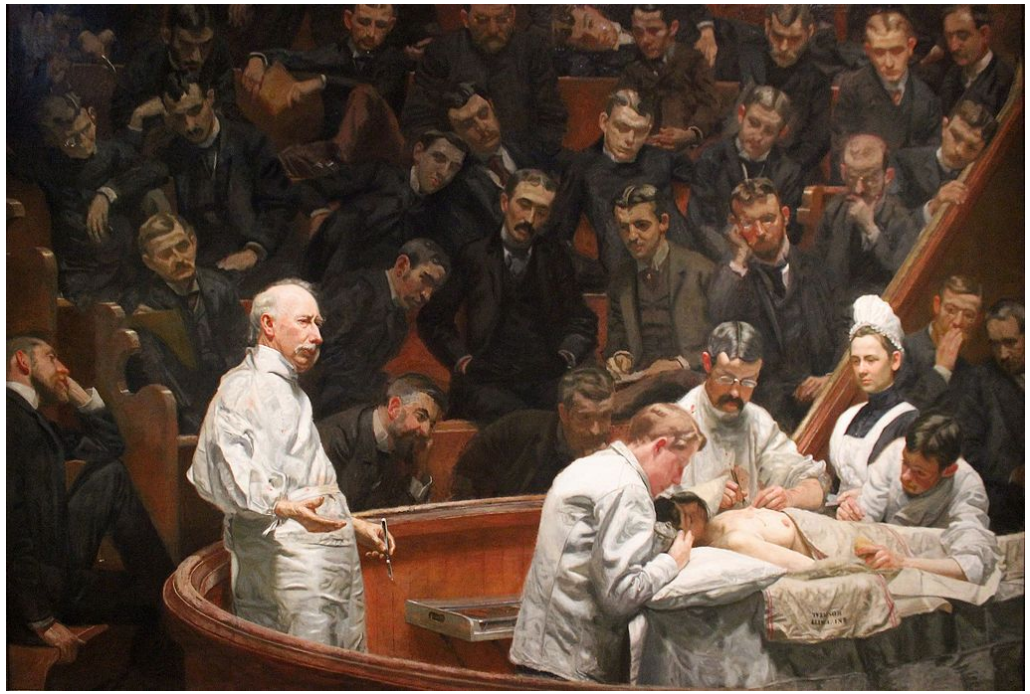
"There is a 75% chance of rain tomorrow"

Interpretations of probability



"The surgery has a 50% probability of success"

Interpretations of probability



Long-run frequencies vs. degree of belief

Formalizing probabilities

What do we need?

We can think of probabilities as objects that model random phenomena. We'll use three components to talk about probabilities:

1️⃣ **Sample space**: the set of all possible **outcomes**

2️⃣ **Events**: Subsets of the sample space, comprise any number of possible outcomes (including none of them!)

3️⃣ **Probability**: Proportion of times an event would occur if we observed the random phenomenon an infinite number of times.

Sample spaces

Sample spaces depend on the random phenomenon in question

- Tossing a single fair coin
- Sum of rolling two fair six-sided dice
- Guessing the answer on a multiple choice question with choices *a, b, c, d*.

What are the sample spaces for the random experiments above?

Events

Events are subsets of the sample space that comprise all possible outcomes from that event. These are the "plausibly reasonable" outcomes we may want to calculate the probabilities for*

- Tossing a single fair coin
- Sum of rolling two fair six-sided dice
- Guessing the answer on a multiple choice question with choices *a, b, c, d*.

What are some examples of events for the random experiments above?

Probabilities

Consider the following possible events and their corresponding probabilities:

- Getting a head from a single fair coin toss: **0.5**
- Getting a prime number sum from rolling two fair six-sided dice: **5/12**
- Guessing the correct answer: **1/4**

We'll talk more about how we calculated these probabilities, but for now remember that probabilities are numbers describing the likelihood of each event's occurrence, which map events to a number between 0 and 1, inclusive.

Working with probabilities

Set operations

Remember that events are (sub)sets of the outcome space. For two sets (in this case events) A and B, the most common relationships are:

- Intersection (A and B): A **and** B both occur
- Union (A or B): A **or** B occurs (including when both occur)
- Complement (A^c): A does **not** occur

Two sets A and B are said to be disjoint or mutually exclusive if they cannot happen at the same time, i.e. A and $B = \emptyset$.

Combining set operations

DeMorgan's laws

- Complement of union:
  $(A \text{ or } B)^c = A^c \text{ and } B^c$
- Complement of intersection:
  $(A \text{ and } B)^c = A^c \text{ or } B^c$

These can be straightforwardly extended to more than two events

How do probabilities work?

Kolmogorov axioms

✅ The probability of any event is real number that's ≥ 0

✅ The probability of the entire sample space is 1

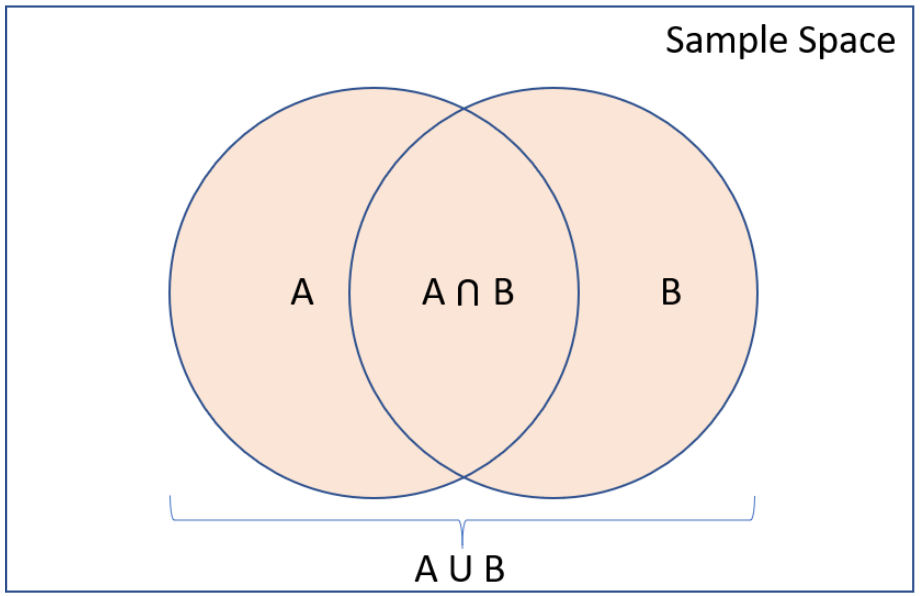
✅ If A and B are disjoint events, then $P(A \text{ or } B) = P(A) + P(B)$

The Kolmogorov axioms lead to all probabilities being between 0 and 1 inclusive, and also lead to important rules...

Two important rules

Suppose we have events A and B, with probabilities $P(A)$ and $P(B)$ of occurring. Based on the Kolmogorov axioms:

- Complement Rule: $P(A^c) = 1 - P(A)$
- Inclusion-Exclusion: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$



Practicing with probabilities

ORIGINAL RESEARCH

Annals of Internal Medicine

Coffee Drinking and Mortality in 10 European Countries
A Multinational Cohort Study

	Did not die	Died
Does not drink coffee	5438	1039
Drinks coffee occasionally	29712	4440
Drinks coffee regularly	24934	3601

Source: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5788283/

Practicing with probabilities

	Did not die	Died
Does not drink coffee	5438	1039
Drinks coffee occasionally	29712	4440
Drinks coffee regularly	24934	3601

Define events A = died and B = non-coffee drinker. Calculate the following for a randomly selected person in the cohort:

- $P(A)$
- $P(B)$
- $P(A \text{ and } B)$
- $P(A \text{ or } B)$
- $P(A \text{ or } B^c)$