Inference Review

Confidence Intervals

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Announcements

- Fill out the **Getting To Know You Survey on Sakai** - due TODAY at 11:59p
- Lab 01 due **Wednesday at 11:59p**

Today's Agenda

- Confidence intervals
- Understanding hypothesis tests (time permitting)

Sesame Street

- *Sesame Street* is a television series designed to teach children ages 3-5 skills such as reading and math.
- The show originally had a particular focus on reaching economically disadvantaged children. In the early 1970s, the Educational Testing Service (ETS) conducted a study to determine the show's effectiveness in helping this group of children develop the skills needed to be successful in school.

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Sesame Street

- A study was conducted to test whether the show was effective in helping children improve their reading and math skills. The 240 children who participated in the study were split into two groups:
  - **Group 1**: Those who were encouraged to watch the show (assume watched regularly)
  - **Group 2**: Those who didn't get encouragement to watch the show (assume didn't watch regularly)
- Each child was given a test before and after the study to measure their knowledge of basic math, reading, etc.
- We will focus on the change in reading (identifying letters) scores

Sesame Street Data - Full Description Original Study: *Ann Bogatz, Gerry & Ball, Samuel. (1971). The Second Year of Sesame Street: A Continuing Evaluation. Volume 1. vols. 1 & 2.*

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Let's look at the data

```
sesame_street %>%
  slice(1:10)
```

```
##          treatment prelet postlet change
## 1       Encouraged     23      30      7
## 2       Encouraged     26      37     11
## 3   Not Encouraged     14      46     32
## 4   Not Encouraged     11      14      3
## 5   Not Encouraged     47      63     16
## 6   Not Encouraged     26      36     10
## 7   Not Encouraged     12      45     33
## 8       Encouraged     48      47     -1
## 9       Encouraged     44      50      6
## 10      Encouraged     38      52     14
```

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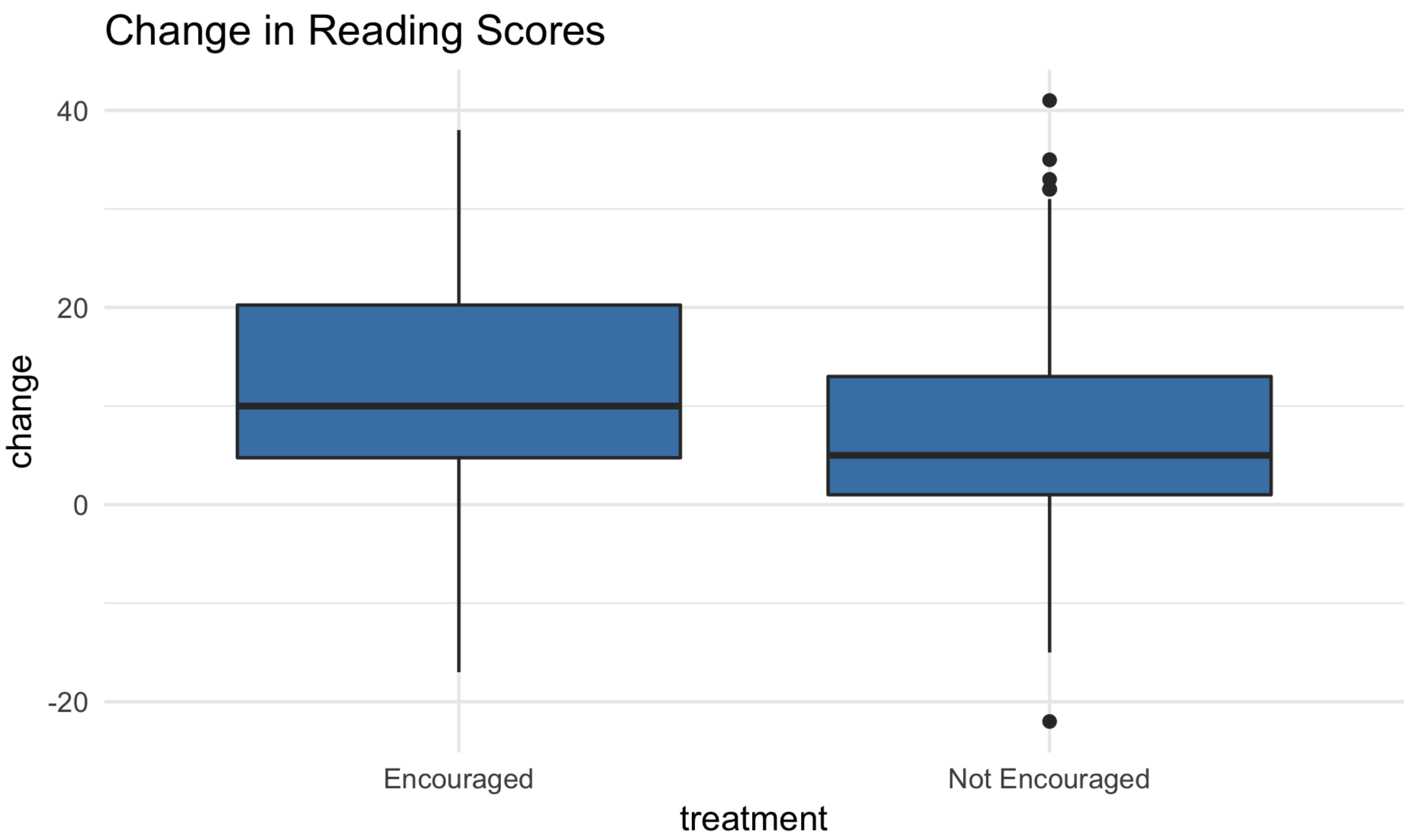
```r
ggplot(data = sesame_street, mapping = aes(y = change, x = treatme
  geom_boxplot(fill = "steelblue") +
  labs(title = "Change in Reading Scores") +
  theme_minimal()
```



```r
sesame_street %>%
  group_by(treatment) %>%
  summarise(n = n(), mean = mean(change), sd = sd(change))
```

```
## # A tibble: 2 x 4
##   treatment          n  mean    sd
##   <chr>          <int> <dbl> <dbl>
## 1 Encouraged       152 12.5   10.7
## 2 Not Encouraged    88  7.88  11.4
```

- **Parameter**: $\mu_e - \mu_{ne}$
- **Statistic**: $\bar{x}_e - \bar{x}_{ne}$
- In the last class, we conducted a hypothesis test and came to the conclusion that children who watched *Sesame Street* regularly showed greater improvement in reading scores, on average, than children who didn't want the show regularly.

Today we will estimate the difference in average improvement between the two groups, i.e. estimate $\mu_e - \mu_{ne}$.

Recall: Statistical inference

- **Statistical inference** is the process of using sample data to make conclusions about the underlying population from which the sample was taken

- Types of inference: testing and estimation

  - **Confidence Intervals:** Estimate the parameter of interest
  - **Hypothesis Tests:** Test a specified claim or hypothesis

- Today, we will focus on confidence intervals

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Confidence Intervals

- Developed by Jerzy Neyman (in the 1930s)
- What: Plausible range of values for a population parameter
  - Assuming sample data is a random sample from the population
- Why: Because the statistic is a random variable, its value is subject to chance error, i.e. random variability
  - We want to take that variability into account by reporting a range of plausible values the parameter can take rather than solely relying on a single statistic

Recall: Central Limit Theorem

- Using the Central Limit Theorem (CLT) we know the form of the sampling distribution for certain statistics such as the mean, proportion, difference in means, etc.
    - CLT does not apply to all statistics (e.g. the median)
- By the Central Limit Theorem, when the conditions are met, we know the sampling distribution of the sample statistic will..
    - be approximately Normal
    - have a mean equal to the unknown population parameter
    - have a standard error proportional to the inverse of the square root of the sample size.

Deriving the confidence interval

- In the *Sesame Street* example, the parameter of interest is the difference in means, $\mu_1 - \mu_2$. Let's look at the confidence interval for $\mu_1 - \mu_2$ based on the CLT

- The statistic is the difference in sample means $\bar{x}_1 - \bar{x}_2$

- Assuming the conditions for the CLT are met (independent observations and large n), the sampling distribution for $\bar{x}_1 - \bar{x}_2$ is

$$\bar{x}_1 - \bar{x}_2 \sim N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

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Deriving the confidence interval

- By the CLT and properties of the Normal distribution, in 95% of random samples,

$$(\mu_1 - \mu_2) - 1.96 \times \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \bar{x}_1 - \bar{x}_2 \leq (\mu_1 - \mu_2) + 1.96 \times \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

- Now, let's center the inequality around the parameter $\mu_1 - \mu_2$

$$(\bar{x}_1 - \bar{x}_2) - 1.96 \times \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + 1.96 \times \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Range of plausible values for $\mu_1 - \mu_2$ (using 95% confidence)

$$(\bar{x}_1 - \bar{x}_2) \pm 1.96 \times \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

General form of the CI

- Generalizing the equations on the previous slide, all confidence intervals take the form $[LB, UB]$

$$\text{Lower Bound (LB)} = \text{ Estimate} - \text{ (critical value) } \times SE$$

$$\text{Upper Bound (UB)} = \text{ Estimate} + \text{ (critical value) } \times SE$$

- Let's talk about the ***SE*** and the critical value

Standard Error of $\bar{x}_1 - \bar{x}_2$

- In practice, we don't know the population standard deviations σ_1 and σ_2
- We will use the sample standard deviations s_1 and s_2 to estimate σ_1 and σ_2
- Thus, the **standard error of $\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2$** is

$$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

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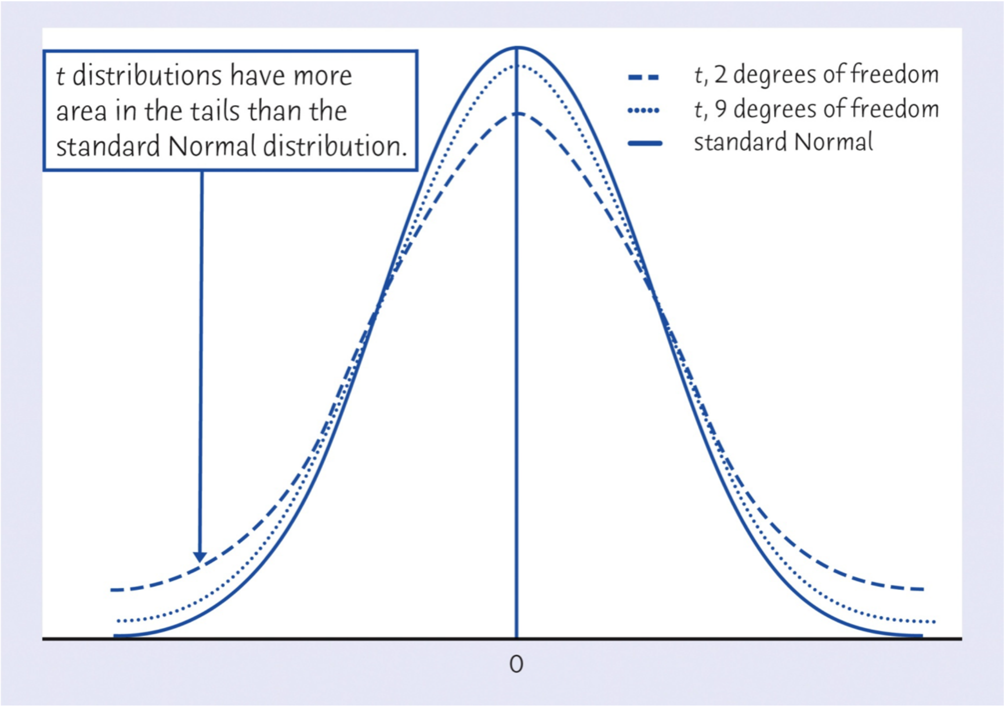
t-distribution vs. Normal

- We need to account for the extra variability that comes from using s_1 and s_2 (instead of σ_1 and σ_2). Therefore, we will use the t distribution for sampling distribution of $\bar{x}_1 - \bar{x}_2$



Picture from *The Basic Practice of Statistics (7th edition)*

Confidence interval for the difference in means

The C confidence interval to estimate $\bar{\mu}_1 - \bar{\mu}_2$ is

$$(\bar{x}_1 - \bar{x}_2) \pm t^*_{df} \times \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where t^*_{df} is the critical value calculated from the *t* distribution with *df* degrees of freedom

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Calculating the critical value

The critical value, t^*, follows a t distribution with degrees of freedom given by the formula:

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2} \approx min\{n_1 - 1, n_2 - 1\}$$

In practice, we will use R to calculate the degrees of freedom.

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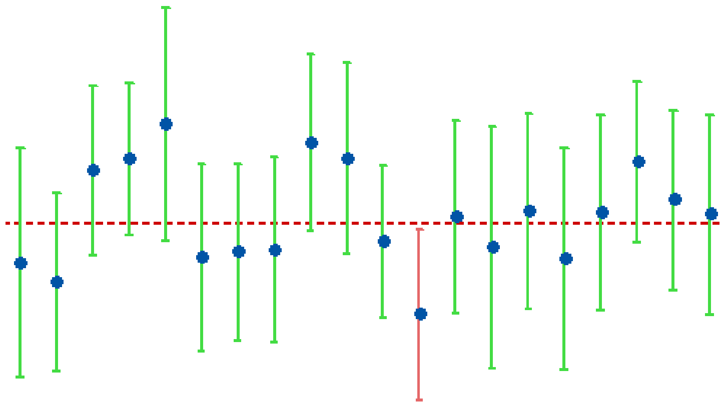
Understanding a 95% Confidence Interval



- The goal is to produce an interval for the parameter of interest using statistics calculated from a random sample
- If we repeated this process thousands of times, we would expect about 95% of the intervals to contain the true parameter of interest
- Note this is **not** the same as saying there's a 95% probability that the parameter is in a given interval

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Sesame Street Example

The 95% confidence interval to estimate the mean difference in reading score improvement between children who watched *Sesame Street* regularly versus those who didn't is

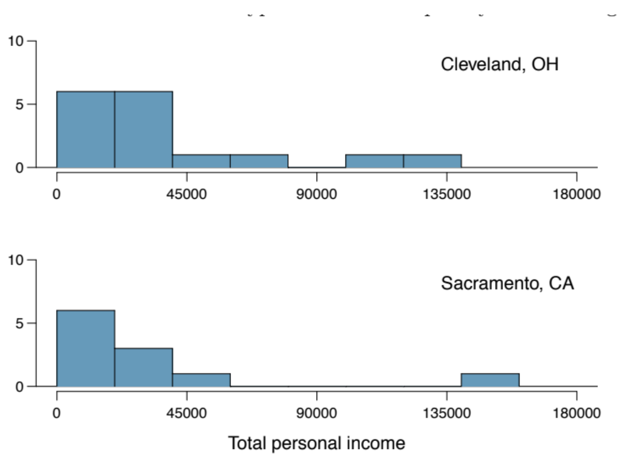
[1.682, 7.568]

1. Interpret this interval in context.
2. Based on this interval, what do you conclude about the effectiveness of *Sesame Street*?

Practice (OpenIntro 5.25)

Average income varies from one region of the country to another, and it often reflects both lifestyles and regional living expenses.

Suppose a new graduate is considering a job in two locations, Cleveland, OH and Sacramento, CA, and he wants to see whether the average income in one of these cities is higher than the other. The summary data is show below:



city	mean	sd	n
Cleveland, OH	35749	39421	21
Sacramento, CA	35500	41512	17

Practice: (OpenIntro 5.25)

The 95% confidence interval for the difference in the mean income between Cleveland and Sacramento is

LB	UB
-278.8564	776.8564

1. Interpret the interval iin context.

2. How would the interval change if we increased the confidence level to 99%?

3. Why might any conclusions based on this interval be unreliable?

Confidence intervals and hypothesis tests

- Confidence intervals can be used to assess a hypothesis or claim about a population parameter
- Suppose μ is the parameter of interest and you calculate a 95\% confidence interval
- Let's also suppose that the hypotheses are $H_0 : \mu = 1$ vs. $H_a : \mu \neq 1$
    - If the 95% confidence interval contains 1, then this two-sided hypothesis test will result in a p-value that is greater than 0.05
    - If the 95% confidence interval does not contain 1, then this two-sided hypothesis test will result in a p-value that is less than 0.05

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Practice

Determine if each state is true or false. If it is false, rewrite the statement so it is true.

1. If you increase sample size, the width of confidence intervals will increase.

2. For a given standard error, higher confidence levels (e.g. 99% vs. 95%) result in wider confidence intervals.

1. The statement, "the p-value is .003", is equivalent to the statement, "there is a 0.3% probability that the null hypothesis is true".

2. A p-value of .04 is more evidence against the null hypothesis than a p-value of .08.

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Understanding the Hypothesis Test

Calculating the p-value

- **p-value:** probability of getting a test statistic as extreme or more extreme than the calculated test statistic, assuming the null hypothesis is true

- When the alternative has a $>$, the p-value is calculated using the area to the right of the test statistic

- When the alternative has a $<$, the p-value is calculated using the area to the left of the test statistic

- When the alternative has $\neq$, the p-value is calculated as the area to the left of $-|\text{test statistic}|$ and to the right of $|\text{test statistic}|$

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Interpreting the p-value

What the p-value is NOT:

- It is *not* the probability the null hypothesis is true
  - The null hypothesis is either true or not true
- (1 - *p-value*) is *not* the probability that the alternative hypothesis is true
  - The alternative hypothesis is either true or not true

The p-value **IS**

The probability of getting a test statistic as extreme or more extreme than the calculated test statistic, *assuming the null hypothesis is true.*

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Interpreting the p-value

Magnitude of p-value	Interpretation
p-value < 0.01	strong evidence against H_0
0.01 < p-value < 0.05	moderate evidence against H_0
0.05 < p-value < 0.1	weak evidence against H_0
p-value > 0.1	effectively no evidence against H_0

Note: These are general guidelines. The strength of evidence depends on the context of the problem.

Statistical Significance

- A threshold can be used to decide whether or not to reject H_0.
- This threshold is called the **significance level** and is usually denoted by α
- When H_0 is rejected, we use the term **statistically significant** to describe the outcome of the test.
- *Example*: When $\alpha = 0.05$, results are statistically significance when the p-value is < 0.05

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Statistical Significance

- Do not rely strictly on the significance level to make a conclusion!
- Suppose the significance level is 0.05
  - If the p-value is 0.05001, we do not reject H_0
  - If the p-value is 0.04999, we do reject H_0
- p-values of 0.05001 and 0.04999 are practically the same, yet they lead to different conclusions.
- Always state the p-value when reporting results and assess it's magnitude in the context of your problem.

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Results that Aren't Statistically Significant

- An outcome of failing to reject H_0 is *not* a failed study/experiment
- Obtaining an outcome of "no significant effect" or "no significant difference" is still valid
- It is often just as important to learn that the H_0 can't be refuted

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Type I & Type II Errors

		Truth about the population	
		H_0 true	H_a true
Conclusion based on sample	Reject H_0	Type I error	Correct conclusion
	Fail to reject H_0	Correct conclusion	Type II error

Image: *The Basic Practice of Statistics (7th Ed.)*

- **Type I Error**: Reject H_0 when H_0 is true
- **Type II Error**: Fail to reject H_0 when H_1 is true
- Replicate study when possible to reduce these errors

Reducing Error

- Probability of Type I error is the significance level, i.e the threshold for rejecting H_0
- Probability of Type II error decreases as the sample size increases
  - When designing a study, it is good practice to conduct a power analyses to determine the sample size required to minimize the chance of Type II error

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Practice (OpenIntro 5.29)

A food safety inspector is called upon to investigate a restaurant with a few customer reports of poor sanitation practices. The food safety inspector uses a hypothesis testing framework to evaluate whether regulations are not being met. If he decides the restaurant is in gross violation, its license to serve food will be revoked.

a. What are the null and alternative hypotheses (in words)?

a. What is a Type 1 Error in this context?

b. What is a Type 2 Error in this context?

d. Which error is more problematic for the diners? Why?

Before Next Class

- Fill out the **Getting To Know You Survey on Sakai** - due TODAY at 11:59p
- **New to R or need a refresher?**
  - Duke Libraries Rfun - Intro to R Workshop: Data Transformations, Data Structures, and the Tidyverse
    - September 12 1p - 3p
    - To register: https://duke.libcal.com/event/5497129
  - *Work with Data* primer on RStudio Cloud: https://rstudio.cloud/learn/primers/2
  - "Data Visualization" in *R for Data Science*: https://r4ds.had.co.nz/data-visualisation.html
- **More on statistical inference**
  - OpenIntro Statistics Chapter 5: Inference for numerical data