Multiple Comparisons

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Announcements

- HW 01 **due TODAY at 11:59p**
- Reading 03 for Monday
- HW 02 due Wednesday, 9/25 at 11:59p

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Today's Agenda

- Multiple comparisions
- Introducing multiple linear regression

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R Packages used in the notes

```
library(tidyverse)
library(knitr)
library(broom)
```

Multiple Comparisons

After ANOVA: Individual Group Means

- Suppose you conduct an ANOVA and conclude that at least one group mean has a different mean response value.
- The next question you want to answer is **which group?**
- One way to answer this question is to compare the estimated means for each group, accounting for the random variability we'd naturally expect
- Since we've assumed the variance is the same for all groups, we can use a pooled standard error with $n - K$ degrees of freedom to calculate the confidence

$$\bar{y}_i \pm t^* \times \frac{s_P}{\sqrt{n_i}}$$

where s_P is the pooled standard deviation

After ANOVA: Difference in Means

- We can also estimate the difference in two means, $\mu_1 - \mu_2$ for each pair of groups

$$(\bar{y}_1 - \bar{y}_2) \pm t^* \times s_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where s_P is the pooled standard deviation

- If we have K groups, we will make $\binom{K}{2} = K(K-1)/2$ such comparisons
  - Ex: If we have 6 groups, we'll make $\binom{6}{2} = 6(6-1)/2 = 15$ comparisons

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Multiple Comparisons

- When making multiple comparisons, there is a higher chance that a Type I error will occur, e.g. conclude that there is a significant difference between two groups even when there is not
- **At a Minimum**: When calculating multiple confidence intervals or conducting multiple hypothesis tests to compare means, you should clearly state how many CIs and/or tests you computed.
- **Good practice**: Account for the number of comparisons being made in the analysis
  - We will discuss one method: **Bonferroni correction**

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Confidence levels

- **Individual confidence level:** success rate of a procedure for calculating a single confidence interval
- **Familywise confidence level:** success rate of a procedure for calculating a family of confidence intervals
  - "success": all intervals in the family capture their parameters
- **Issue:** There is an increased chance of making at least one error when calculating multiple confidence intervals
  - The same is true when conducting multiple hypothesis tests

Bonferroni correction

- **Goal:** Achieve at least $100(1-\alpha)\%$ familywise confidence level for C confidence intervals
  - Where α is the significance level for the corresponding two-sided hypothesis test
- Calculate each of the k confidence intervals at a $100(1-\frac{\alpha}{C})\%$ confidence level
  - When there are K groups, there are $C=\frac{K(K-1)}{2}$ pairs of means being compared
- **Notes**:
  - The exact familywise confidence level is not easily predictable. This partially depends on the level of dependence between the intervals.
  - Bonferroni correction is sometimes too conservative, i.e don't reject H_0 as much as you should

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Population Density in the Midwest

- There are 5 groups (states) in the `midwest` data, so we will do $\binom{5}{2} = 10$ comparisons.
- If we want a familywise confidence level of 95%, then we should use a $(1 - 0.05/10) \times 100 = 99.5$ % confidence level for each pairwise comparison

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Pairwise CI

```
library(pairwiseCI)
pairwiseCI(log_popdensity ~ state, data = midwest,
           method = "Param.diff", conf.level = 0.995, var.equal =
```

```
##
## 99.5 %-confidence intervals
##  Method:  Difference of means assuming Normal distribution and equal var
##
##
##       estimate   lower   upper
## IN-IL   0.4089  0.0213  0.7966
## MI-IL   0.0315 -0.4564  0.5194
## OH-IL   0.8237  0.4050  1.2424
## WI-IL  -0.1959 -0.6745  0.2827
## MI-IN  -0.3774 -0.8457  0.0909
## OH-IN   0.4148  0.0246  0.8049
## WI-IN  -0.6048 -1.0547 -0.1550
## OH-MI   0.7922  0.2903  1.2940
## WI-MI  -0.2274 -0.7987  0.3438
## WI-OH  -1.0196 -1.5070 -0.5322
##
##
```

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Pairwise CI

estimate	lower	upper	comparison
0.409	0.021	0.797	IN-IL
0.032	-0.456	0.519	MI-IL
0.824	0.405	1.242	OH-IL
-0.196	-0.674	0.283	WI-IL
-0.377	-0.846	0.091	MI-IN
0.415	0.025	0.805	OH-IN
-0.605	-1.055	-0.155	WI-IN
0.792	0.290	1.294	OH-MI
-0.227	-0.799	0.344	WI-MI
-1.020	-1.507	-0.532	WI-OH