Simple Linear Regression

Prediction

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Topics

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- Predict the response given a value of the predictor variable

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- Use intervals to quantify the uncertainty in the predicted values

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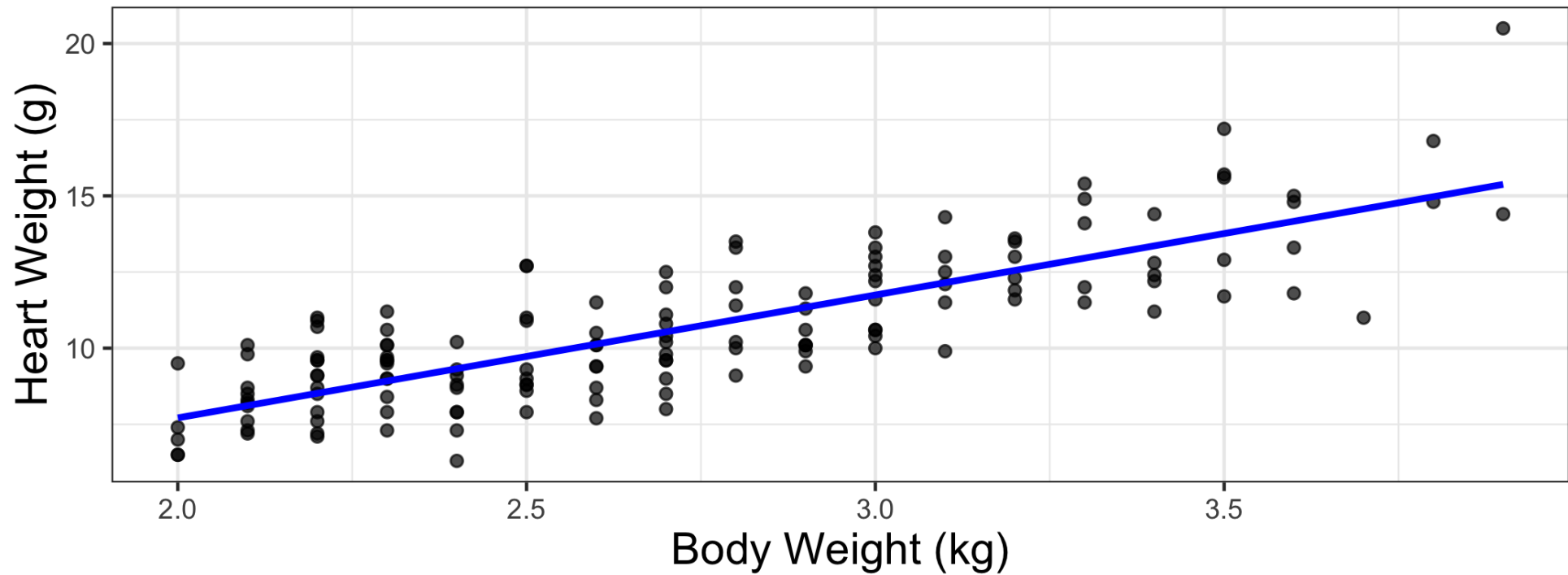- Define *extrapolation* and why we should avoid it

Cats data

The data set contains the **heart weight (Hwt)** and **body weight (Bwt)** for 144 domestic cats.
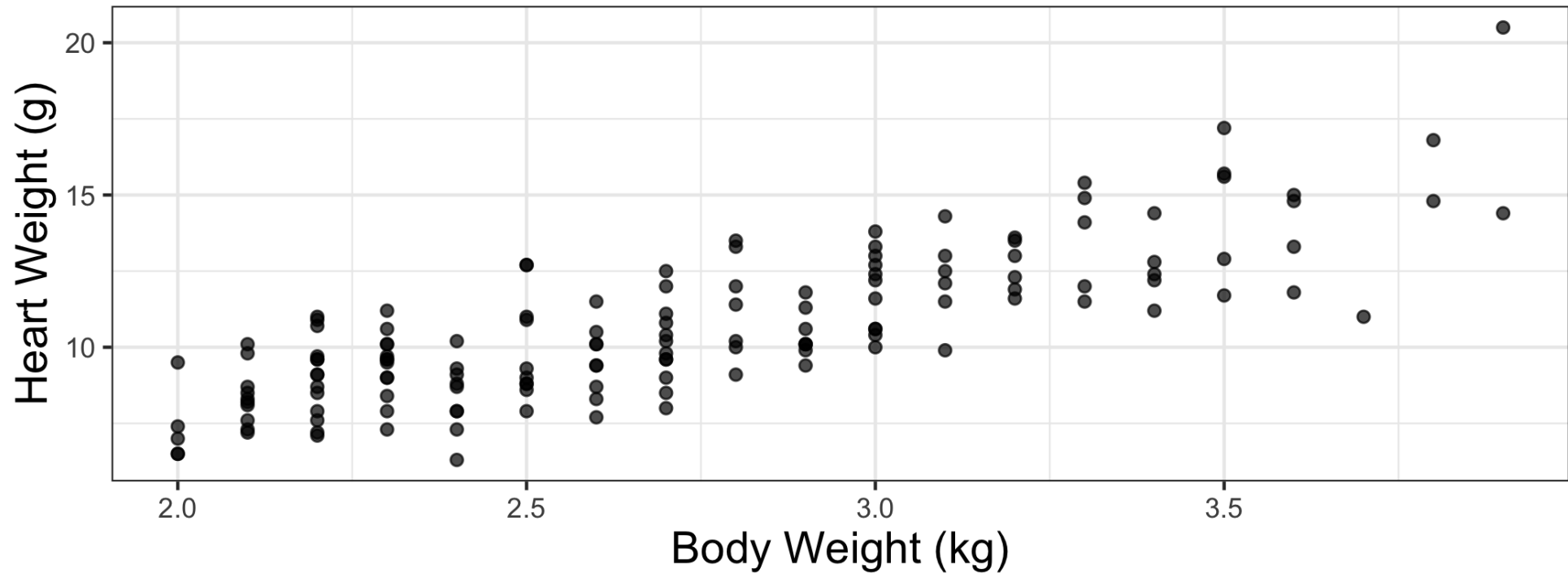
Cats data

We want to fit a model so we can use a cat's body weight to predict how much its heart weighs.

The model

$$\hat{\mathrm{Hwt}} = -0.357 + 4.034 \times \mathrm{Bwt}$$

term	estimate	std.error	statistic	p.value
(Intercept)	-0.357	0.692	-0.515	0.607
Bwt	4.034	0.250	16.119	0.000

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Estimate the **mean** response when the predictor variable is equal to a value x_0

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Predict the response for an **individual** observation with a value of the predictor equal to x_0

Calculating a predicted value

My cat Mindy weighs about 3.18 kg (7 lbs).

Based on this model, about how much does her heart weigh?

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$$\hat{\text{Hwt}} = -0.357 + 4.034 \times \textbf{3.18}$$
$$= \textbf{12.471} \text{ g}$$

Uncertainty in predictions

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Confidence interval for the mean response

$$\hat{y} \pm t^*_{n-2} \times \mathbf{SE}_{\hat{\mu}}$$

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Prediction interval for an individual observation

$$\hat{y} \pm t^*_{n-2} \times \mathbf{SE}_{\hat{\mathbf{y}}}$$

Standard errors

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We are 95% confident that mean heart weight for the subset of cats that weigh 3.18 kg is between 12.143 g and 12.801 g.

Prediction interval

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The 95% **prediction interval** for an *individual* cat (Mindy) that weighs 3.18 kg is

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We can predict with 95% confidence that Mindy's heart weighs between 9.582 g and 15.362 g.

Comparing intervals

— Confidence interval for mean   — Prediction interval for individual
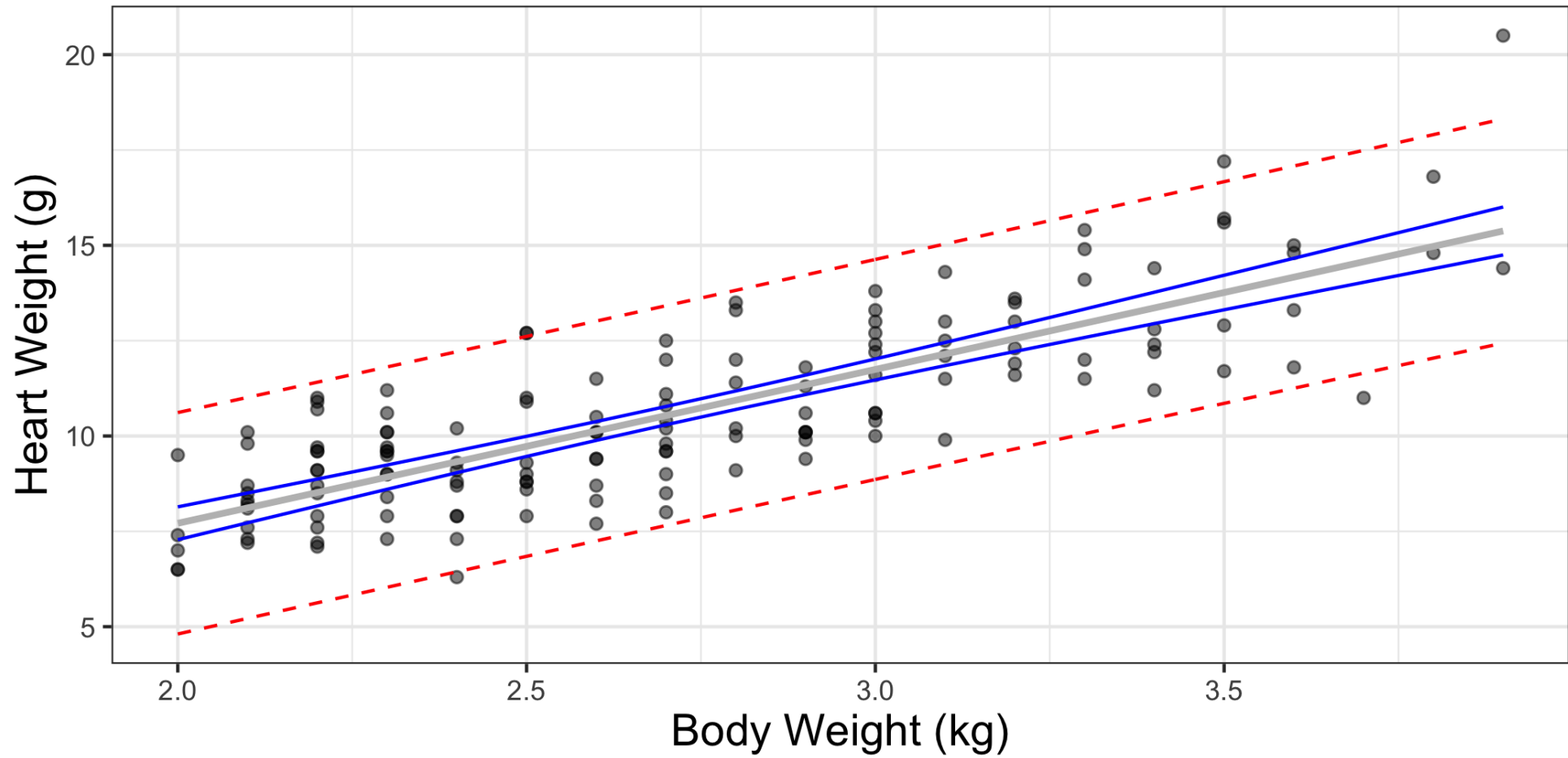
🛑 Caution! Extrapolation

We should **<u>not</u>** use the model to predict for values of X far outside the range of values used to fit the model.

This is called **extrapolation**.
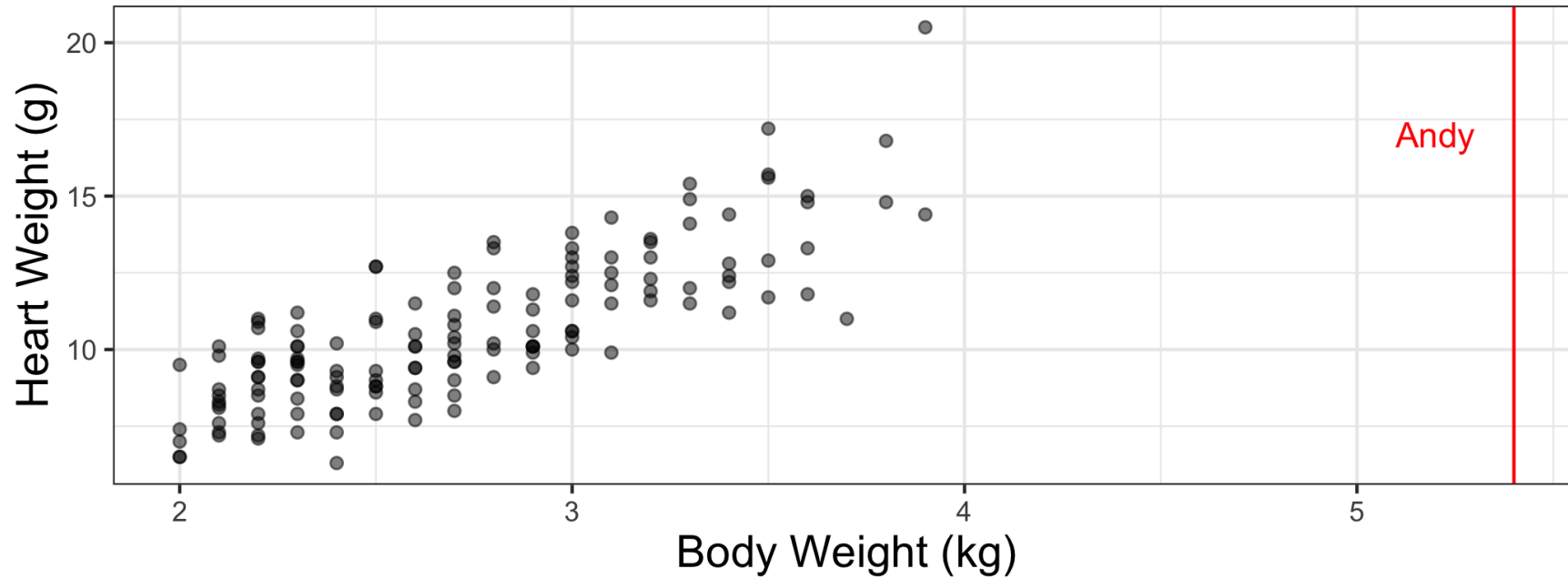
Predict Andy's heart weight?

My cat Andy weighs about 5.44 kg (12 lbs).

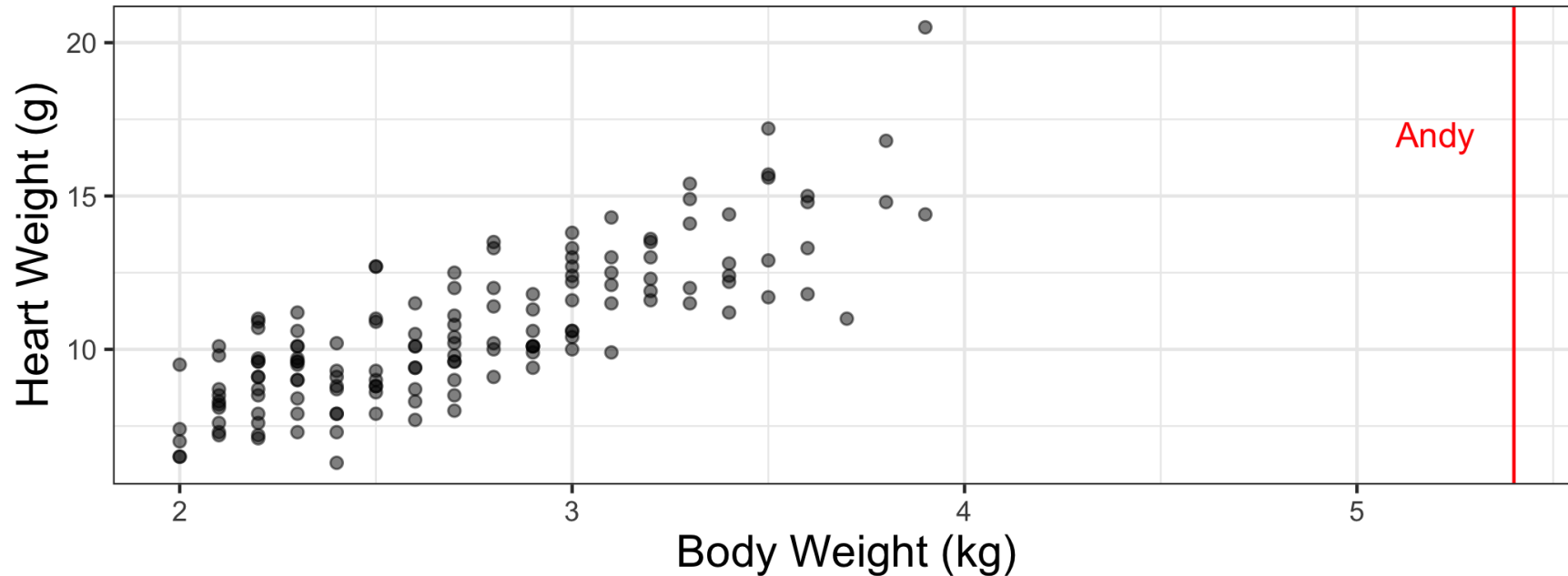Should we use this regression model to predict how much his heart weighs?

Predict Andy's heart weight?

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We should **<u>not</u>** use this model to predict Andy's heart weight, since that would be **extrapolation**.

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- Defined **extrapolation** and why we should avoid it