Multinomial Logistic Regression

Prediction + model selection + conditions

Prof. Maria Tackett

STA 210

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Topics

- Predictions
- Model selection
- Checking conditions

NHANES Data

- National Health and Nutrition Examination Survey is conducted by the National Center for Health Statistics (NCHS)
- The goal is to *"assess the health and nutritional status of adults and children in the United States"*
- This survey includes an interview and a physical examination

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Health Rating vs. Age & Physical Activity

- **Question:** Can we use a person's age and whether they do regular physical activity to predict their self-reported health rating?

- We will analyze the following variables:

  - **`HealthGen`:** Self-reported rating of participant's health in general. Excellent, Vgood, Good, Fair, or Poor.

  - **`Age`:** Age at time of screening (in years). Participants 80 or older were recorded as 80.

  - **`PhysActive`:** Participant does moderate to vigorous-intensity sports, fitness or recreational activities

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Model in R

y.level	term	estimate	std.error	statistic	p.value
Vgood	(Intercept)	1.205	0.145	8.325	0.000
Vgood	Age	0.001	0.002	0.369	0.712
Vgood	PhysActiveYes	-0.321	0.093	-3.454	0.001
Good	(Intercept)	1.948	0.141	13.844	0.000
Good	Age	-0.002	0.002	-0.977	0.329
Good	PhysActiveYes	-1.001	0.090	-11.120	0.000
Fair	(Intercept)	0.915	0.164	5.566	0.000
Fair	Age	0.003	0.003	1.058	0.290
Fair	PhysActiveYes	-1.645	0.107	-15.319	0.000
Poor	(Intercept)	-1.521	0.290	-5.238	0.000
Poor	Age	0.022	0.005	4.522	0.000

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Predictions

Calculating probabilities

For categories $2, \dots, K$, the probability that the i^{th} observation is in the j^{th} category is

$$\hat{\pi}_{ij} = \frac{\exp\{\hat{\beta}_{0j} + \hat{\beta}_{1j}x_{i1} + \dots + \hat{\beta}_{pj}x_{ip}\}}{1 + \sum_{k=2}^{K} \exp\{\hat{\beta}_{0k} + \hat{\beta}_{1k}x_{i1} + \dots \hat{\beta}_{pk}x_{ip}\}}$$

For the baseline category, $k = 1$, we calculate the probability $\hat{\pi}_{i1}$ as

$$\hat{\pi}_{i1} = 1 - \sum_{k=2}^{K} \hat{\pi}_{ik}$$

NHANES: Predicted probabilities

```r
#calculate predicted probabilities
pred_probs <- as_tibble(predict(health_m, type = "probs")) %>%
                        mutate(obs_num = 1:n())
```

```
## # A tibble: 5 x 6
##   Excellent Vgood  Good   Fair    Poor obs_num
##       <dbl> <dbl> <dbl>  <dbl>   <dbl>   <int>
## 1    0.0705 0.244 0.451 0.198  0.0366     101
## 2    0.0702 0.244 0.441 0.202  0.0426     102
## 3    0.0696 0.244 0.427 0.206  0.0527     103
## 4    0.0696 0.244 0.427 0.206  0.0527     104
## 5    0.155  0.393 0.359 0.0861 0.00662    105
```

Add predictions to original data

```
health_m_aug <- inner_join(nhanes_adult, pred_probs,
                           by = "obs_num") %>%
  select(obs_num, everything())
```

```
## Rows: 6,710
## Columns: 10
## $ obs_num    <int> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1
## $ HealthGen  <fct> Good, Good, Good, Good, Vgood, Vgood, Vg
## $ Age        <int> 34, 34, 34, 49, 45, 45, 45, 66, 58, 54,
## $ PhysActive <fct> No, No, No, No, Yes, Yes, Yes, Yes, Yes,
## $ Education  <fct> High School, High School, High School, S
## $ Excellent  <dbl> 0.07069715, 0.07069715, 0.07069715, 0.07
## $ Vgood      <dbl> 0.2433979, 0.2433979, 0.2433979, 0.24442
## $ Good       <dbl> 0.4573727, 0.4573727, 0.4573727, 0.43725
## $ Fair       <dbl> 0.19568909, 0.19568909, 0.19568909, 0.20
```

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Actual vs. Predicted Health Rating

- We can use our model to predict a person's perceived health rating given their age and whether they exercise

- For each observation, the predicted perceived health rating is the category with the highest predicted probability

```
health_m_aug <- health_m_aug %>%
  mutate(pred_health = predict(health_m,
                               type = "class"))
```

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Actual vs. Predicted Health Rating

```
health_m_aug %>%
  count(HealthGen, pred_health, .drop = FALSE) %>%
  pivot_wider(names_from = pred_health, values_from = n)
```

```
## # A tibble: 5 x 6
##   HealthGen Excellent Vgood  Good  Fair  Poor
##   <fct>         <int> <int> <int> <int> <int>
## 1 Excellent         0   550   223     0     0
## 2 Vgood             0  1376   785     0     0
## 3 Good              0  1255  1399     0     0
## 4 Fair              0   300   642     0     0
## 5 Poor              0    24   156     0     0
```

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Why do you think no observations were predicted to have a rating of "Excellent", "Fair", or "Poor"?

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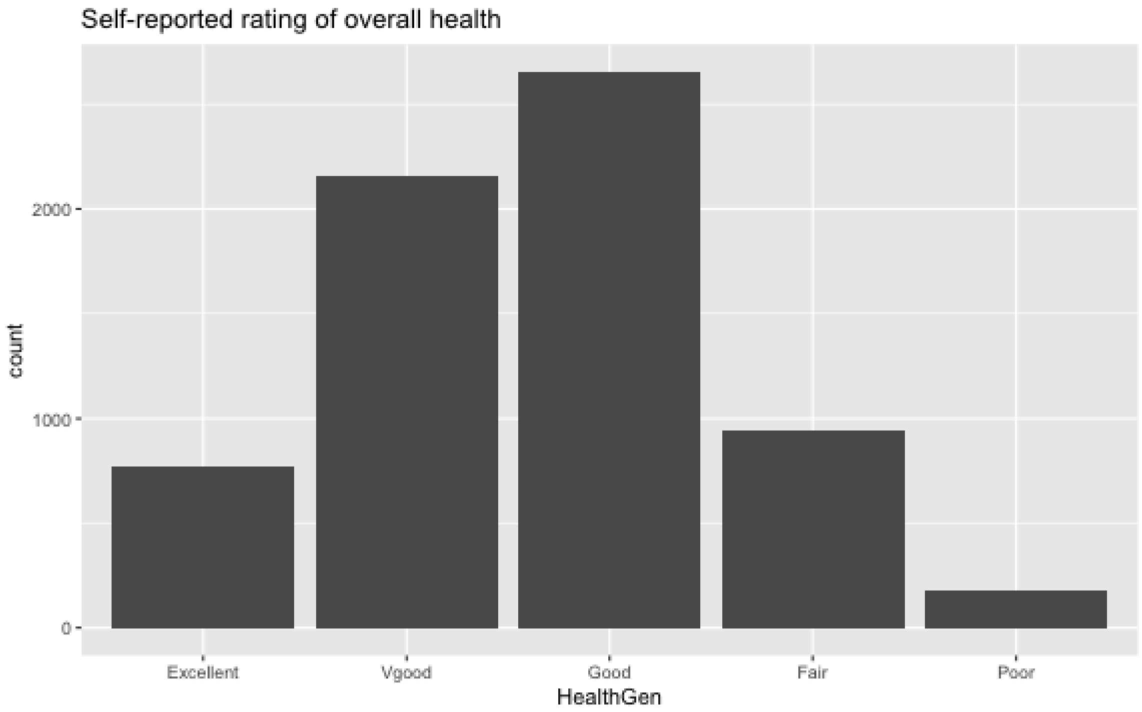

Model selection

Comparing Nested Models

- Suppose there are two models:
  - Reduced Model includes predictors $x_1, \ldots, x_q$
  - Full Model includes predictors $x_1, \ldots, x_q, x_{q+1}, \ldots, x_p$
- We want to test the hypotheses

$$
\begin{aligned}
H_0 &: \beta_{q+1} = \cdots = \beta_p = 0 \\
H_a &: \text{ at least 1 } \beta_j \text{ is not} 0
\end{aligned}
$$

- To do so, we will use the **Drop-in-Deviance test** (very similar to logistic regression)

Add `Education` to the model?

- We consider adding the participants' **`Education`** level to the model.
  - Education takes values **`8thGrade`**, **`9-11thGrade`**, **`HighSchool`**, **`SomeCollege`**, and **`CollegeGrad`**
- Models we're testing:
  - Reduced Model: **`Age`**, **`PhysActive`**
  - Full Model: **`Age`**, **`PhysActive`**, **`Education`**

$$
\begin{aligned}
H_0 &: \beta_{9-11thGrade} = \beta_{HighSchool} = \beta_{SomeCollege} = \beta_{CollegeGrad} = 0 \\
H_a &: \text{ at least one } \beta_j \text{ is not equal to } 0
\end{aligned}
$$

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Add `Education` to the model?

$$H_0: \beta_{9-11thGrade} = \beta_{HighSchool} = \beta_{SomeCollege} = \beta_{CollegeGrad} = 0$$
$$H_a: \text{ at least one } \beta_j \text{ is not equal to } 0$$

```
model_red <- multinom(HealthGen ~ Age + PhysActive,
               data = nhanes_adult)
model_full <- multinom(HealthGen ~ Age + PhysActive +
                         Education,
               data = nhanes_adult)
```

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Add `Education` to the model?

```r
anova(model_red, model_full, test = "Chisq") %>%
  kable(format = "markdown")
```

Model	Resid. df	Resid. Dev	Test	Df	LR stat.	Pr(Chi)
Age + PhysActive	25848	16994.23		NA	NA	NA
Age + PhysActive + Education	25832	16505.10	1 vs 2	16	489.1319	0

Add **Education** to the model?

```r
anova(model_red, model_full, test = "Chisq") %>%
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Model	Resid. df	Resid. Dev	Test	Df	LR stat.	Pr(Chi)
Age + PhysActive	25848	16994.23		NA	NA	NA
Age + PhysActive + Education	25832	16505.10	1 vs 2	16	489.1319	0

At least one coefficient associated with **Education** is non-zero. Therefore, we will include **Education** in the model.

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Model with `Education`

y.level	term	estimate	std.error	statistic	p.value	conf.low	conf.high
Vgood	(Intercept)	0.582	0.301	1.930	0.054	-0.009	1.173
Vgood	Age	0.001	0.003	0.419	0.675	-0.004	0.006
Vgood	PhysActiveYes	-0.264	0.099	-2.681	0.007	-0.457	-0.071
Vgood	Education9 - 11th Grade	0.768	0.308	2.493	0.013	0.164	1.372
Vgood	EducationHigh School	0.701	0.280	2.509	0.012	0.153	1.249
Vgood	EducationSome College	0.788	0.271	2.901	0.004	0.256	1.320
Vgood	EducationCollege Grad	0.408	0.268	1.522	0.128	-0.117	0.933
Good	(Intercept)	2.041	0.272	7.513	0.000	1.508	2.573

Compare NHANES models using AIC

```
glance(model_red)$AIC
```

```
## [1] 17018.23
```

```
glance(model_full)$AIC
```

```
## [1] 16561.1
```

Compare NHANES models using AIC

```
glance(model_red)$AIC
```

```
## [1] 17018.23
```

```
glance(model_full)$AIC
```

```
## [1] 16561.1
```

Use the **`step()`** function to do model selection with AIC as the selection criteria

Checking conditions

Assumptions for multinomial logistic regression

We want to check the following assumptions for the multinomial logistic regression model:

1. **Linearity**: Is there a linear relationship between the log-odds and the predictor variables?

2. **Randomness**: Was the sample randomly selected? Or can we reasonably treat it as random?

3. **Independence**: There is no obvious relationship between observations

Checking linearity

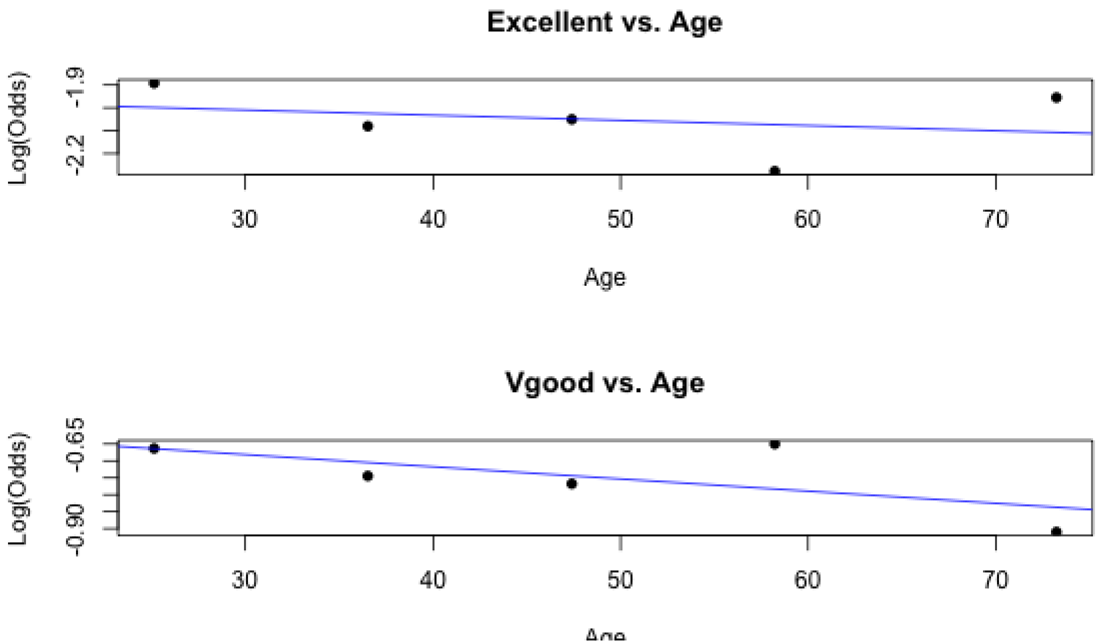
Similar to logistic regression, we will check linearity by examining empirical logit plots between each level of the predictor and the quantitative predictor variables.

```
nhanes_adult <- nhanes_adult %>%
  mutate(Excellent = factor(if_else(HealthGen == "Excellent", "1", "0
         Vgood = factor(if_else(HealthGen == "Vgood", "1", "0")),
         Good = factor(if_else(HealthGen == "Good", "1", "0")),
         Fair = factor(if_else(HealthGen == "Fair", "1", "0")),
         Poor = factor(if_else(HealthGen == "Poor", "1", "0"))
  )
```

Checking linearity

```
library(Stat2Data)
```

```
par(mfrow = c(2,1))
emplogitplot1(Excellent ~ Age, data = nhanes_adult, ngroups = 5, mair
emplogitplot1(Vgood ~ Age, data = nhanes_adult, ngroups = 5, main = '
```



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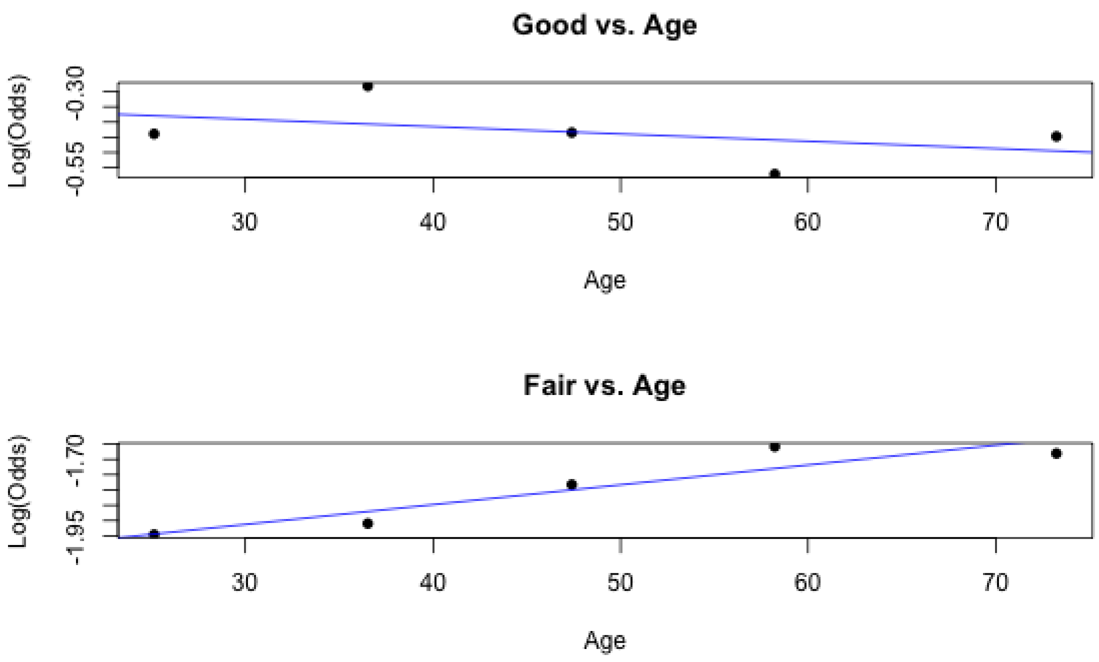
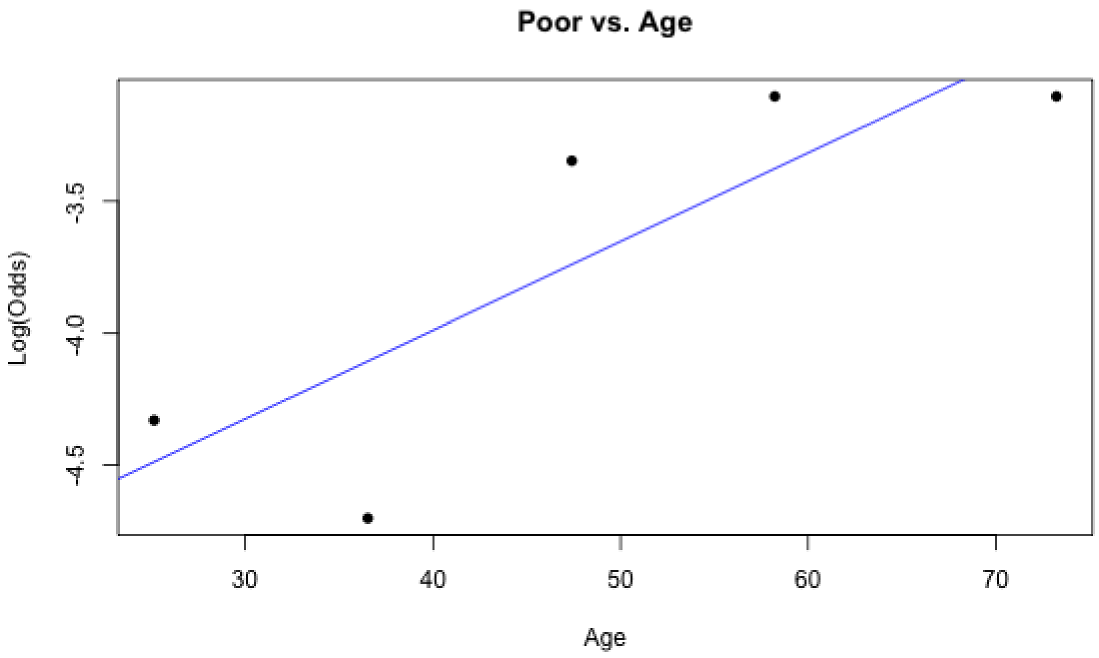
Checking linearity

```
par(mfrow = c(2,1))
emplogitplot1(Good ~ Age, data = nhanes_adult, ngroups = 5, main = "G
emplogitplot1(Fair ~ Age, data = nhanes_adult, ngroups = 5, main = "F
```



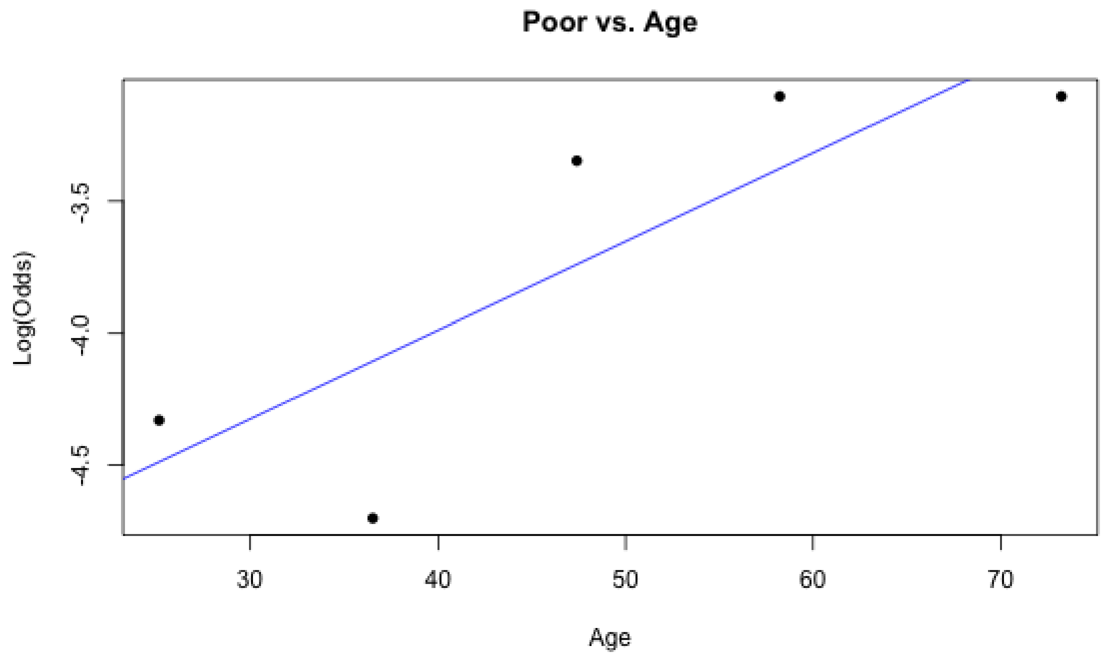
Checking linearity

```
emplogitplot1(Poor ~ Age, data = nhanes_adult, ngroups = 5, main = "P
```



Checking linearity

```
emplogitplot1(Poor ~ Age, data = nhanes_adult, ngroups = 5, main = "P
```



✅ The linearity condition is satisfied. There is a linear relationship between the empirical logit and the quantitative predictor variable,

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Checking randomness

We can check the randomness condition based on the context of the data and how the observations were collected.

- Was the sample randomly selected?
- If the sample was not randomly selected, ask whether there is reason to believe the observations in the sample differ systematically from the population of interest.

Checking randomness

We can check the randomness condition based on the context of the data and how the observations were collected.

- Was the sample randomly selected?
- If the sample was not randomly selected, ask whether there is reason to believe the observations in the sample differ systematically from the population of interest.

✅ The randomness condition is satisfied. We do not have reason to believe that the participants in this study differ systematically from adults in the U.S..

Checking independence

We can check the independence condition based on the context of the data and how the observations were collected.

Independence is most often violated if the data were collected over time or there is a strong spatial relationship between the observations.

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We can check the independence condition based on the context of the data and how the observations were collected.

Independence is most often violated if the data were collected over time or there is a strong spatial relationship between the observations.

✅ The independence condition is satisfied. It is reasonable to conclude that the participants' health and behavior characteristics are independent of one another.

Recap

- Predictions
- Model selection
- Checking conditions