Simple Linear Regression

Conditions

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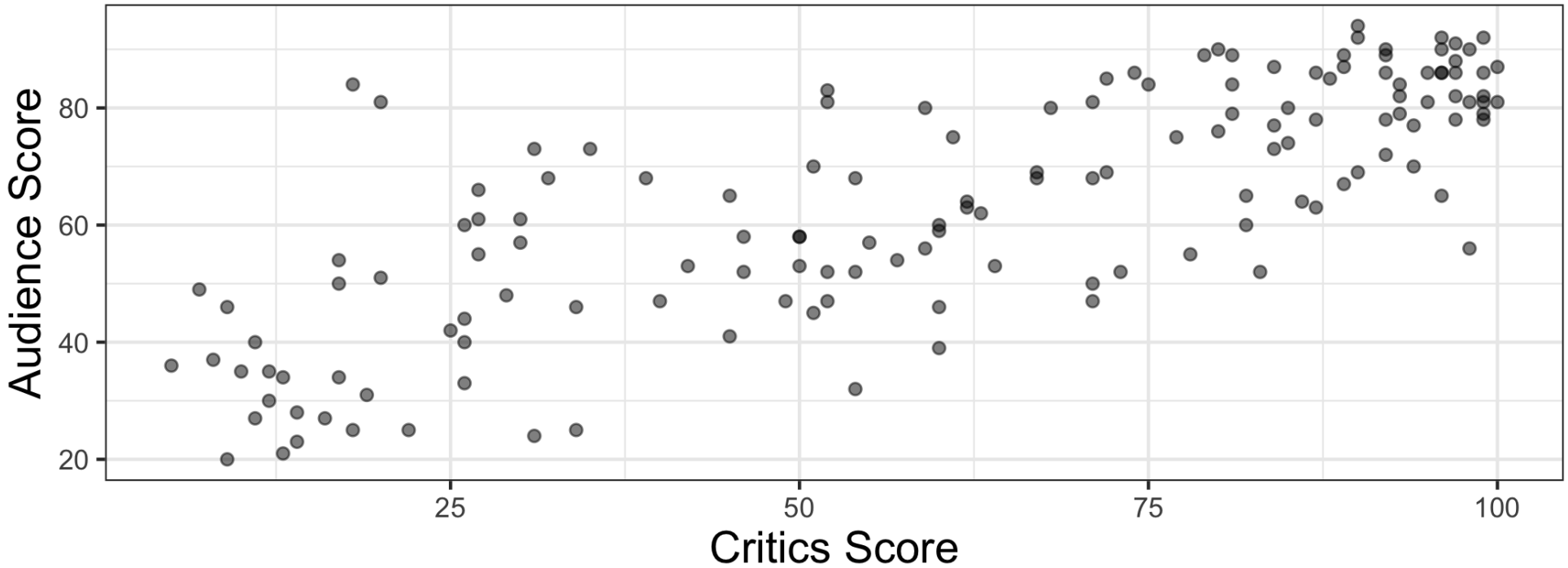
Topics

- List the conditions for simple linear regression
- Use plots of the residuals to check the conditions

Movie ratings data

The data set contains the "Tomatometer" score (**`critics`**) and audience score (**`audience`**) for 146 movies rated on rottentomatoes.com.



The model

$$\hat{\text{audience}} = 32.316 + 0.519 \times \text{critics}$$

term	estimate	std.error	statistic	p.value
(Intercept)	32.316	2.343	13.795	0
critics	0.519	0.035	15.028	0

$$Y|X \sim N(\beta_0 + \beta_1 X, \sigma_\epsilon^2)$$

Model conditions

1. **Linearity:** There is a linear relationship between the response and predictor variable.

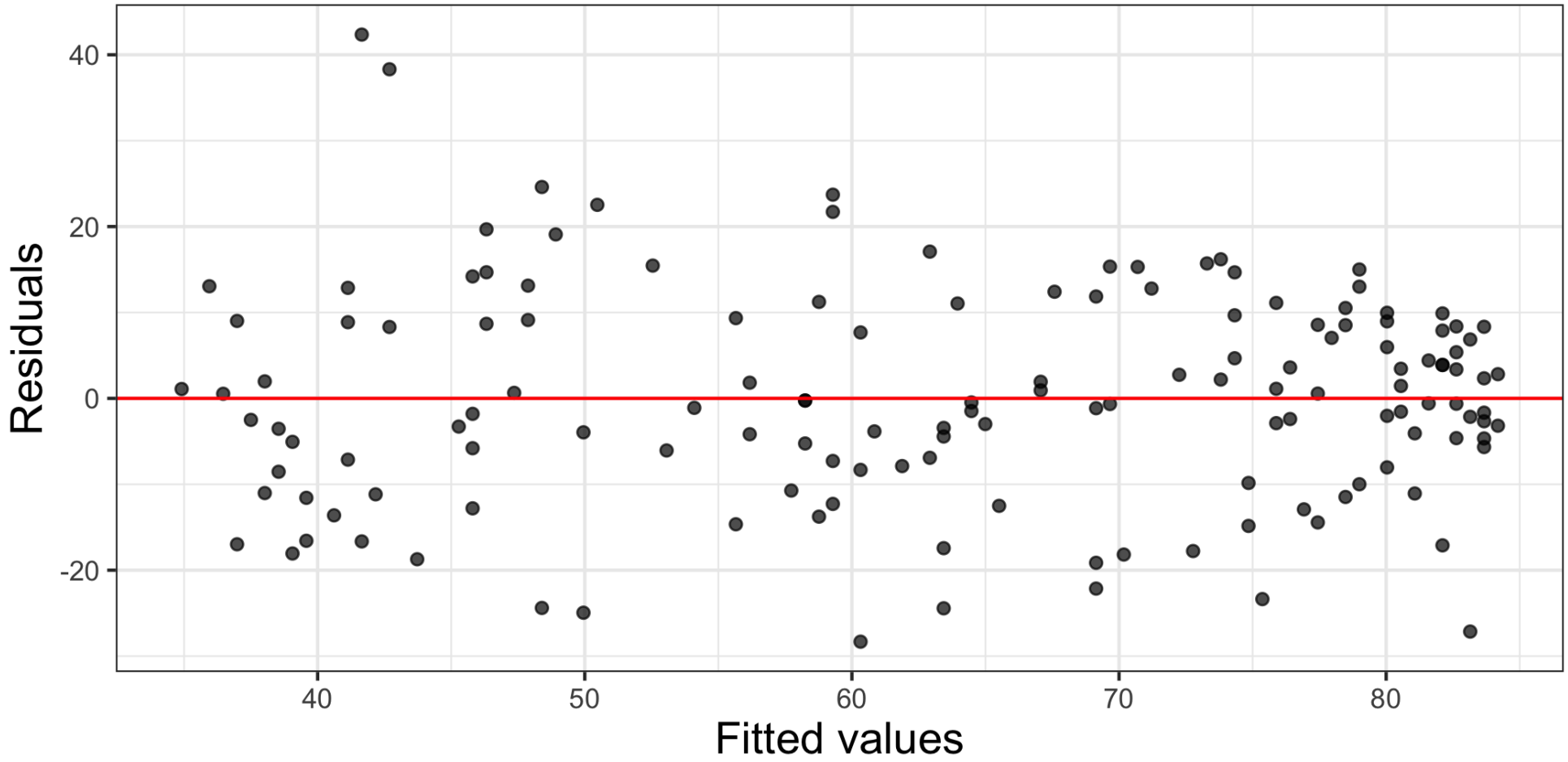
2. **Constant Variance:** The variability of the errors is equal for all values of the predictor variable.

3. **Normality:** The errors follow a normal distribution.

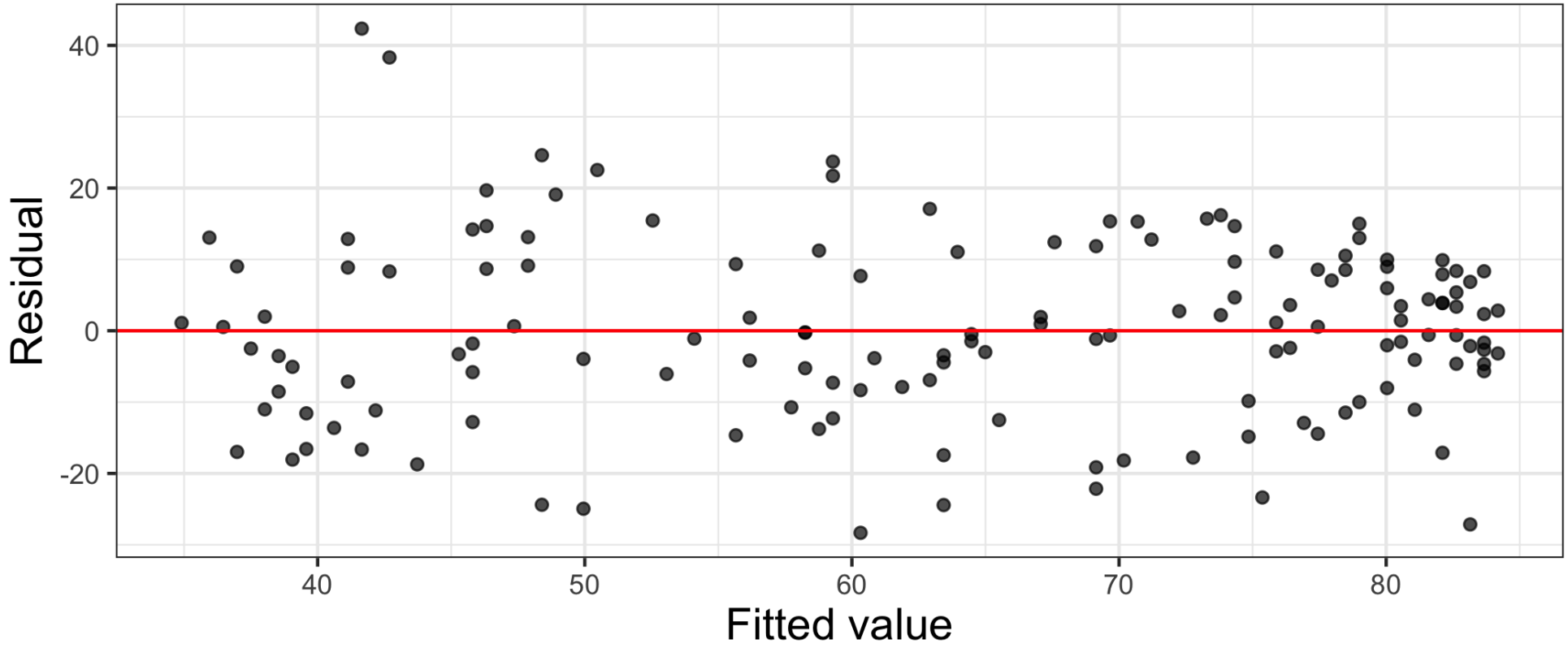
4. **Independence:** The errors are independent from each other.

$$\text{residual}_i = e_i = y_i - \hat{y}_i$$

Residuals vs. fitted values



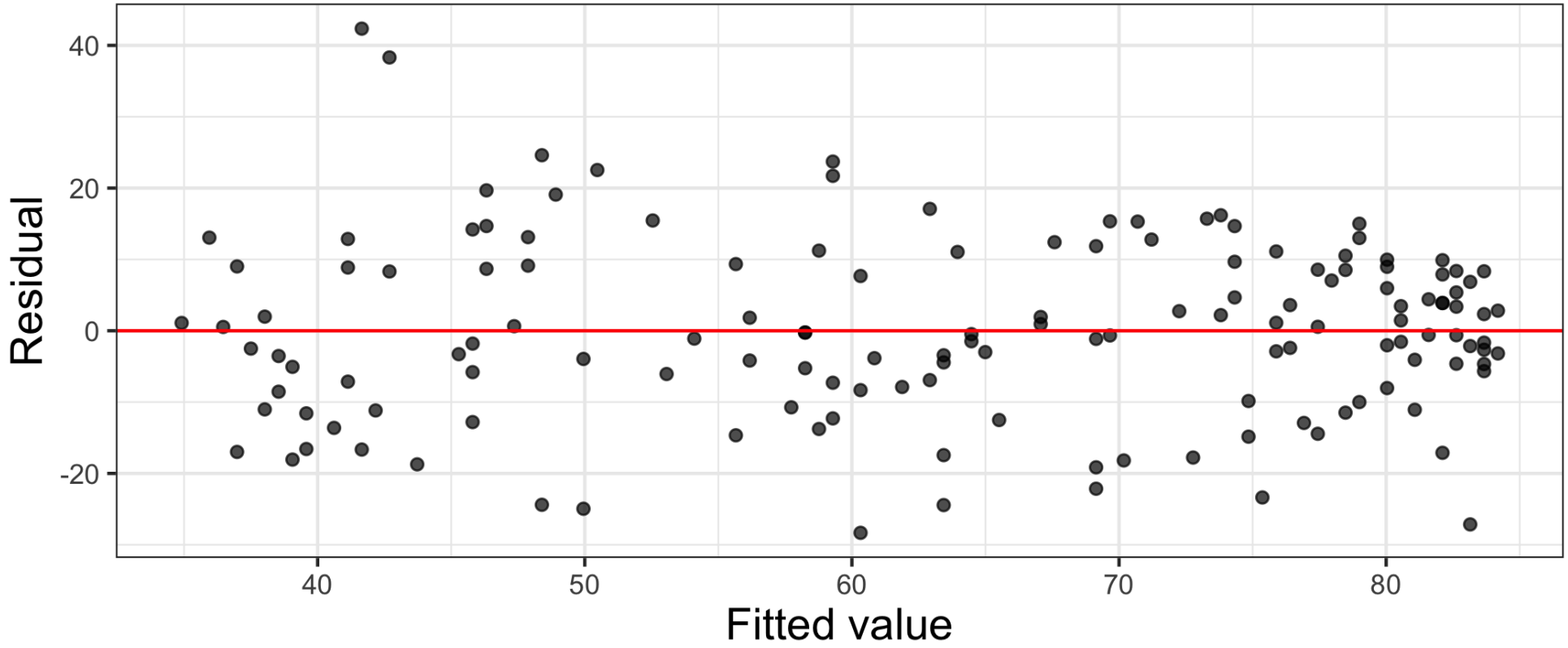
Checking linearity



✅ There is no distinguishable pattern or structure. The residuals are randomly scattered.

❌ Violation: distinguishable pattern

Checking constant variance

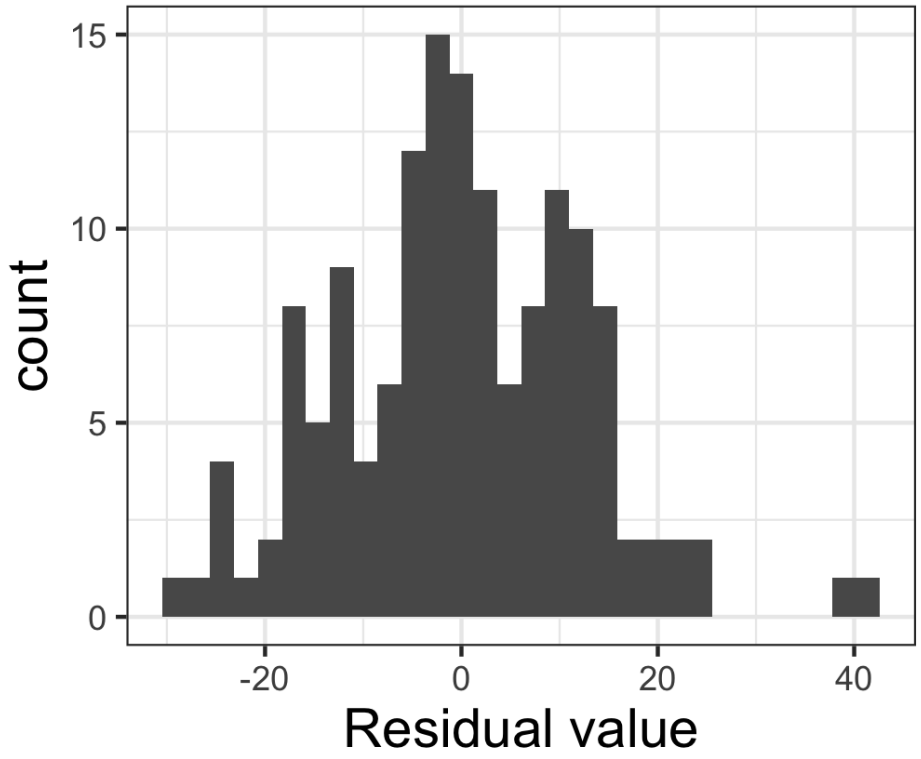


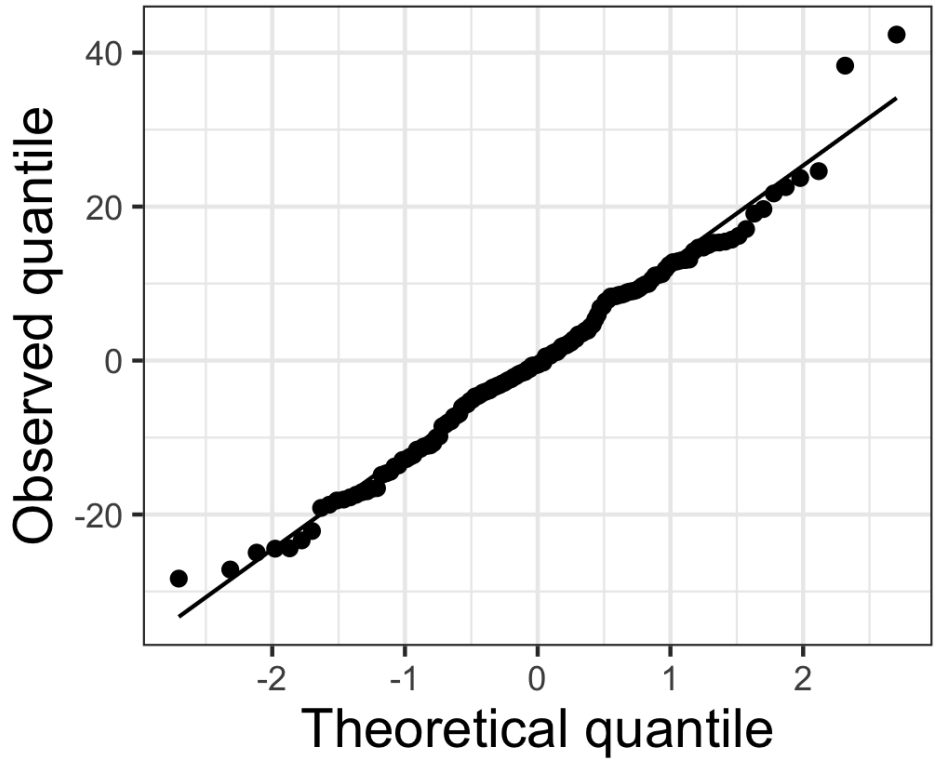
✅ The vertical spread of the residuals is relatively constant across the plot.

❌ Violation: non-constant variance

Normal quantile plot

Checking normality





✅ Points fall along a straight diagonal line on the normal quantile plot.

Checking independence

- We can often check the independence assumption based on the context of the data and how the observations were collected.
- If the data were collected in a particular order, examine a scatterplot of the residuals versus order in which the data were collected.

✅ Based on available information, the error for one movie does not tell us anything about the error for another movie.

In practice

As you check the model conditions, ask if any observed deviation from the model conditions are so great that

1️⃣ a different model should be proposed.

2️⃣ conclusions drawn from the model should be used with caution.

✅ If not, the conditions are sufficiently met and we can proceed with the current model.

Recap

- Used plots of the residuals to check conditions for simple linear regression:
  - **Linearity**
  - **Constant Variance**
  - **Normality**
  - **Independence**