Inference Review

Hypothesis Testing

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01.15.20

STA 210

[Click for PDF of slides]

Announcements

- Complete surveys and consent form by TODAY at 11:59p
- Reading for next Wednesday
- Labs start tomorrow!
- No class Monday - Martin Luther King, Jr. Holiday
- Find more info about statistics related events on Sakai
  - Florida Blue visting Jan 22. Submit resume by today to be considered for interview during their visit

Today's Agenda

- Calculating & interpreting hypothesis tests
- Drawing conclusions using hypothesis tests and confidence intervals

Sesame Street

- *Sesame Street* is a television series designed to teach children ages 3-5 basic education skills such as reading (e.g. the alphabet) and math (e.g. counting)
- Today we are going to analyze data from an study conducted by the Educational Testing Service in the early 1970s to test the effectiveness of the program.

Sesame Street study

- Children from 6 locations around the United States (including Durham!) participated in the 26-week study. The children were split into two groups (`treatment`):
  - **Group 1**: Those who were encouraged to watch the show (assume watched regularly)
  - **Group 2**: Those who didn't get encouragement to watch the show (assume didn't watch regularly)
- Each child was given a test before and after the study to measure their knowledge of basic math, reading, etc.
- We will focus on the change in reading (identifying letters) scores (`change`)

Sesame Street Data - Full Description Original Study: *Ann Bogatz, Gerry & Ball, Samuel. (1971). The Second Year of Sesame Street: A Continuing Evaluation. Volume 1. vols. 1 & 2.*

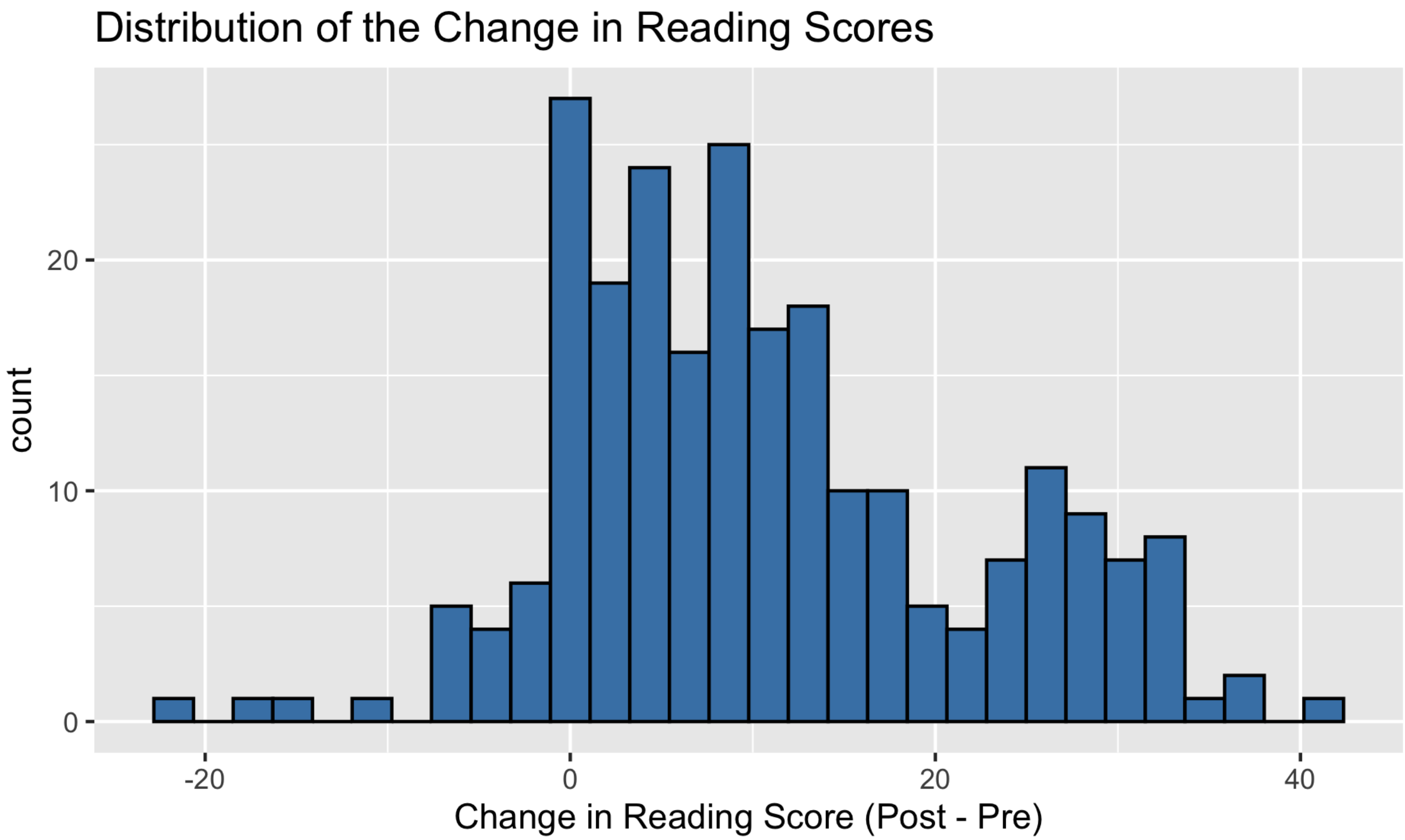
Let's look at the data

`sesame_street.csv` is available in the datasets repo on GitHub.

```
sesame_street %>%
  slice(1:10)
```

```
## # A tibble: 10 x 4
##    treatment      prelet postlet change
##    <chr>           <dbl>   <dbl>  <dbl>
##  1 Encouraged         23      30      7
##  2 Encouraged         26      37     11
##  3 Not Encouraged     14      46     32
##  4 Not Encouraged     11      14      3
##  5 Not Encouraged     47      63     16
##  6 Not Encouraged     26      36     10
##  7 Not Encouraged     12      45     33
##  8 Encouraged         48      47     -1
##  9 Encouraged         44      50      6
## 10 Encouraged         38      52     14
```

Exploratory Data Analysis - Univariate



Exploratory Data Analysis - Univariate

- Calculate summary statistics for change

```
sesame_street %>%
  summarise(n = n(), min = min(change), median = median(change), m
            IQR = IQR(change),
            mean = mean(change), std_dev = sd(change))
```

```
## # A tibble: 1 x 7
##       n   min median   max   IQR  mean std_dev
##   <int> <dbl>  <dbl> <dbl> <dbl> <dbl>   <dbl>
## 1   240   -22      9    41    15  10.8    11.2
```

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95% CI for mean change in reading score

The 95% confidence interval for the mean change in reading score is

$$[9.384, 12.224]$$

- Interpret the interval at http://bit.ly/sta210-sp20-CI-2
- Use **NetId@duke.edu** for your email address.
- You are welcome (and encouraged!) to discuss these questions with 1 - 2 people around you, but **each person** must submit a response.

03:00

Confidence Interval Recap

Hypothesis Tests

Question we want to answer

- Let's focus on the children who were encouraged to watch *Sesame Street*
- In general, those children watched the show regularly, so let's see if the show impacted their reading skills

> The mean change in reading scores after 26 weeks for children ages 3 - 5 is 11.
>
> Is there evidence that mean change in reading scores for children encouraged to watch *Sesame Street* is "significantly" different from the mean change in reading score for all children?

Let's look at the data


```
## # A tibble: 1 x 3
##       n  mean std_dev
##   <int> <dbl>   <dbl>
## 1   152  12.5    10.7
```

Outline of a Hypothesis Test

- Identify the parameter of interest.
- Identify a null hypothesis, H_0, that represents the baseline
- Set an alternative hypothesis, H_a, that represents the research question, i.e. what you're testing
- Conduct a hypothesis test under the assumption that the null hypothesis is true and calculate a p-value
  - The **p-value** is the probability of getting the observed outcome or a more extreme outcome given the null hypothesis is true

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Outline of a Hypothesis Test

- Assess the p-value. A small p-value means...

  a. The assumed (null) hypothesis is incorrect

  b. The assumed (null) hypothesis is correct and a rare event has occurred

- State a conclusion about the hypothesis based on the assessment of the p-value
  - Since event (b) is by definition rare, we will conclude a "small" p-value indicates that there is sufficient evidence to claim that the assumed hypothesis is false.
    - In other words, the data are not consistent with the assumed hypothesis

- When the p-value is "not small", we will conclude that there is not sufficient evidence to claim the assumed hypothesis is false.

Identify parameter & hypotheses

- **Null hypothesis, H_0**: This is the baseline hypothesis, i.e. the "there is nothing going on" hypothesis.
  - The mean change in reading score for children encouraged to watch the show is 11 (same as the mean for all children)

- **Alternative hypothesis, H_a**: This is typically what you want to show, i.e. the "there is something going on" hypothesis
  - The mean change in reading score for children encouraged to watch the show not 11 (different from the mean for all children)

$$H_0 : \mu = 11$$
$$H_a : \mu \neq 11$$

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Distribution $\bar{x}$ under H_0

- We want to draw conclusions about μ, so we'll use our best guess $\bar{x}$
- Recall from the Central Limit Theorem, when certain conditions are met (they are!), we know that

$$\bar{x} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

- We conduct a hypothesis test under the assumption that H_0 is true, so for this test

$$\bar{x} \sim N\left(11, \frac{\sigma}{\sqrt{n}}\right)$$

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Distribution $\bar{x}$ under H_0

- We don't know σ, so we can use the standard error $s/\sqrt{n}$ to approximate $\sigma/\sqrt{n}$.
- Thus, putting it all together, we know

$$\bar{x} \approx N\left(11, \frac{10.7}{\sqrt{152}}\right)$$

Given $\bar{x} \approx N\left(11, \frac{10.7}{\sqrt{152}}\right)$, what is the probability of observing a mean change in score at least 1.5 points away from the center (11) in a random sample of 152 children ages 3 - 5?

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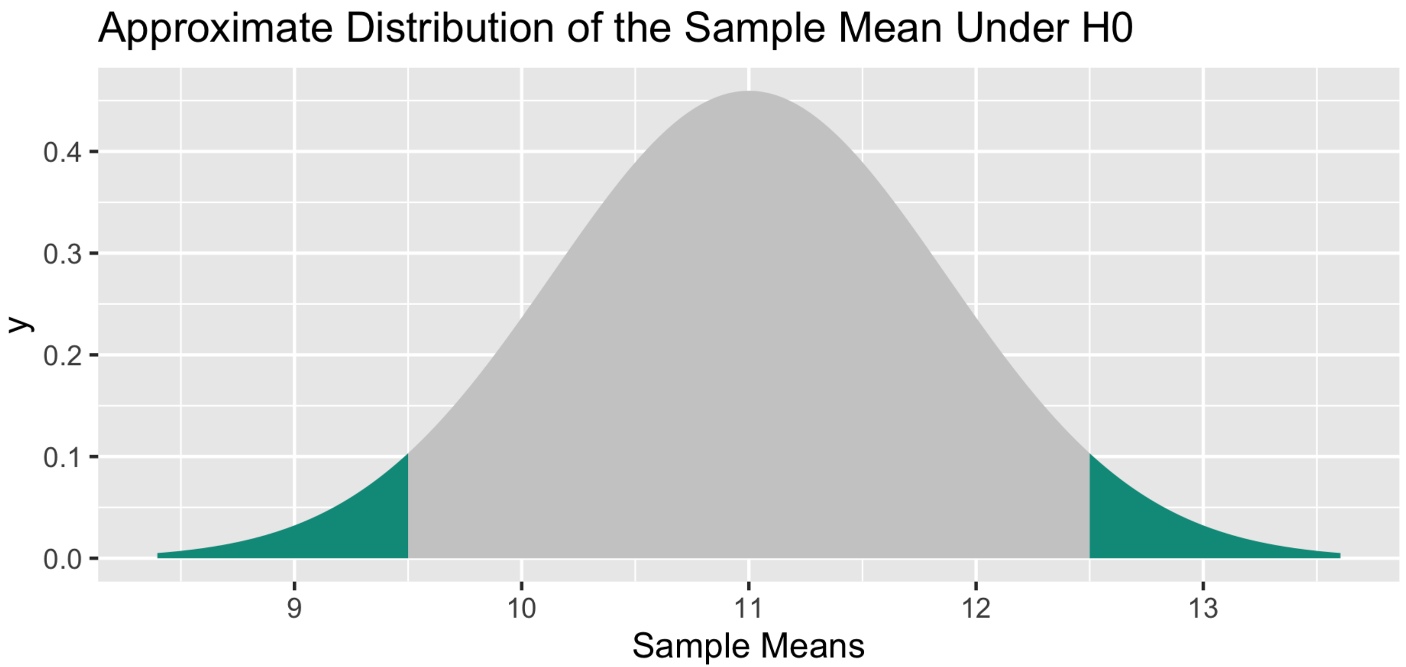
Visualize



The shaded area represents the (approximate) probability of obtaining a sample mean at least as far away from the center as the one we observed given the true mean change is 11.

Test Statistic

- Let's quantify how "unusual" our observed sample mean is given $H_0 : \mu = 11$ is true
- We'll begin by calculating how "far away" the observed mean is from the center of the distribution under H_0
- The **test statistic** is the number of standard errors the observed value is from the hypothesized value. The general form of the test statistic is

$$\frac{\text{observed value} - \text{hypothesized value}}{SE}$$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{12.5 - 11}{\frac{10.7}{\sqrt{152}}} \approx 1.728$$

where the test statistic follows the t distribution with $n - 1$ df

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Motivating the p-value

- We got a test statistic of 1.728. In other words...

Given $\bar{x} \approx N\left(11, \frac{10.7}{\sqrt{152}}\right)$, what is the probability of observing a mean change in score at least 1.5 points away from the center (11) in a random sample of 152 children ages 3 - 5?

$$\Downarrow$$

Given the t distribution with 151 degrees of freedom, what is the probability of observing a test statistic with magnitude 1.728 or larger?

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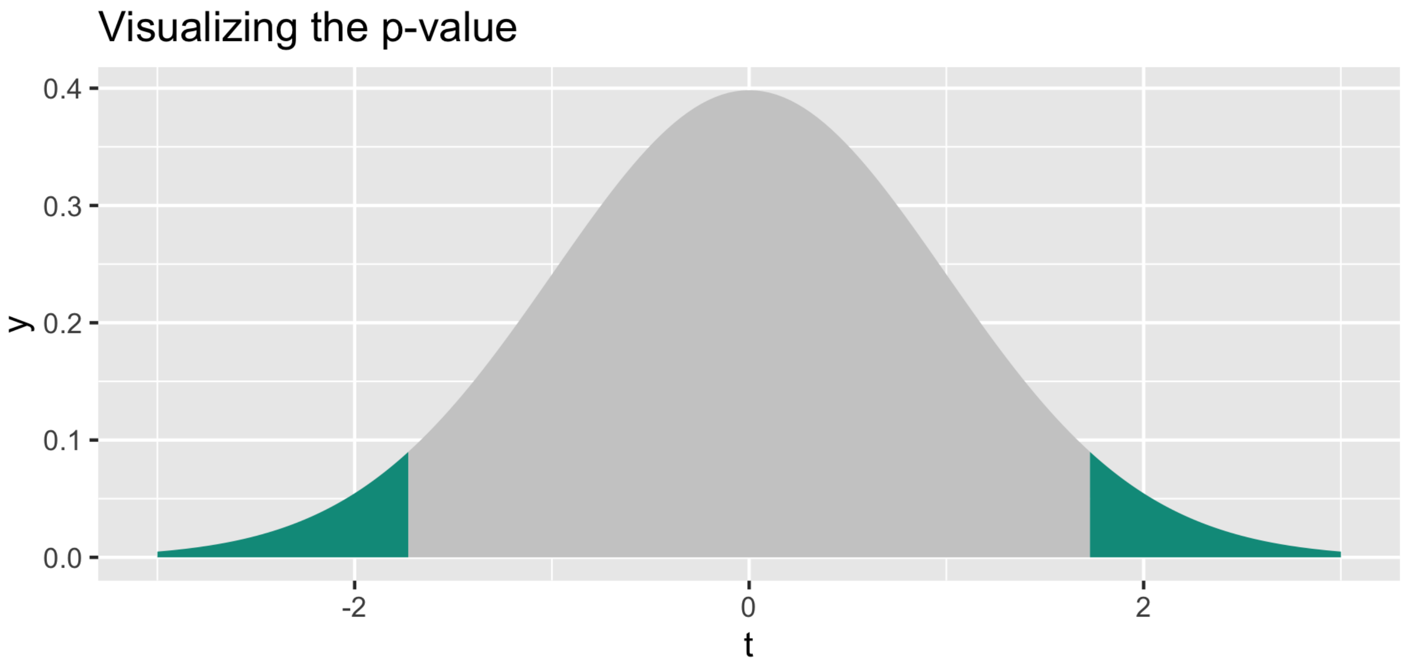
p-value

Given the t distribution with 151 degrees of freedom, what is the probability of observing a test statistic with magnitude 1.728 or larger?




```r
(p_value <- 2 * pt(-1.728, 151))
```

```
## [1] 0.08603259
```

General guide for interpreting the p-value

Magnitude of p-value	Interpretation
p-value < 0.01	strong evidence against H_0
0.01 < p-value < 0.05	moderate evidence against H_0
0.05 < p-value < 0.1	weak evidence against H_0
p-value > 0.1	effectively no evidence against H_0

Note: These are general guidelines. The strength of evidence depends on the context of the problem.

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Drawing the conclusion: Part 1

- A threshold can be used to decide whether or not to reject H_0 in favor of the alternative H_a
- This threshold is called the significance level and is usually denoted by α
- If the p-value is less than α, then we conclude there is sufficient evidence against H_0 and we **reject the null hypothesis**
- Otherwise, we conclude that there isn't sufficient evidence against H_0 and **fail to reject the null hypothesis**

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Don't just rely on p-values

- Do not rely strictly on the p-value and significance level to make a conclusion!
- Suppose the significance level is 0.05
  - If the p-value is 0.05001, we fail to reject H_0
  - If the p-value is 0.04999, we reject H_0
- 0.05001 and 0.04999 are practically the same, yet they led to different conclusions.

> Use confidence intervals and other statistical summaries to provide more context about the results.

t-test for *Sesame Street* data

```r
enc <- sesame_street %>%
  filter(treatment == "Encouraged")

t.test(enc$change, mu = 11, conf.level = 0.9,
       direction = "two.sided")
```

```
##
##      One Sample t-test
##
## data:  enc$change
## t = 1.7226, df = 151, p-value = 0.08701
## alternative hypothesis: true mean is not equal to 11
## 90 percent confidence interval:
##  11.05883 13.94117
## sample estimates:
## mean of x
##      12.5
```

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In-class exercise

- Answer the questions: http://bit.ly/sta210-sp20-ht
- Use **NetId@duke.edu** for your email address.
- You are welcome (and encouraged!) to discuss these questions with 1 - 2 people around you, but **each person** must submit a response.

04:00

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Conclusion

p-value: 0.087

90% confidence interval: [11.059, 13.941]

- Using a significance level of 0.1, what is your conclusion from the test?
  - **This is the "statistical" conclusion from our test based on the mechanics of hypothesis testing.**
  - In this case, we would reject H_0 and conclude there is sufficient evidence that the mean change in reading scores is not equal to 11 for those encouraged to watch *Sesame Street*.

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Conclusion

p-value: 0.087

90% confidence interval: [11.059, 13.941]

- Suppose you are advising a group of educators about whether they should spend additional time and money to encourage children to watch *Sesame Street*. Based on these results, would you advise the educators to spend the resources? Why or why not?
  - **This the "practical" conclusion from the test based on a combination of the results of the hypothesis test, the magnitude of the p-value, the confidence interval, business context, etc.**
  - There is no single correct answer to this question, but this is an example of the type of question we ultimately are trying to answer when we do statistical inference, i.e. this is statistical inference in practice.

Inference for difference in means $\mu_1 - \mu_2$

By the Central Limit Throem, when the conditions are met,

$$(\bar{x}_1 - \bar{x}_2) \sim N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

- We don't know σ_1 and σ_2 in practice, so we use the **standard error** in all calculations.

$$SE(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Inference for difference in means $\mu_1 - \mu_2$

Confidence Interval to estimate $\mu_1 - \mu_2$

$$(\bar{x}_1 - \bar{x}_2) \pm t^*_{df} \times \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

t^* follows a t distribution with [degrees of freedom](#) computed in R.

Inference for difference in means $\mu_1 - \mu_2$

Hypothesis Test: Is there a difference in the means between Group 1 and Group 2?

- Null hypothesis: $H_0 : \mu_1 - \mu_2 = 0$
- Test statistic:

$$\frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

- p-value: Calculated using the t distribution with degrees of freedom computed in R.

Muddiest point - Optional

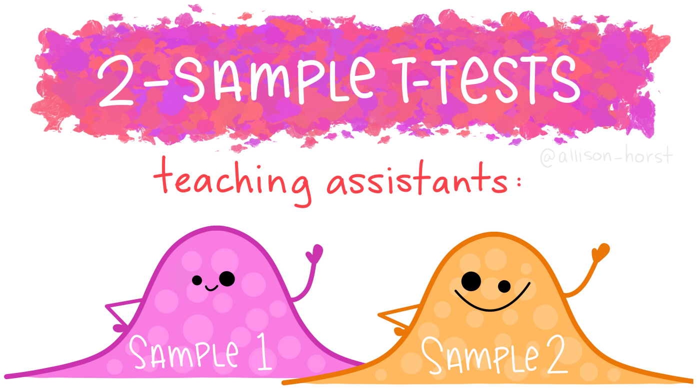
What is one question you have about hypohtesis testing and confidence intervals?

Submit your question here: http://bit.ly/sta210-sp20-review

- Use **NetId@duke.edu** for your email address.
- Type "none" only if you really, really, really don't have a question

Additional Resources

- Discussion in the scientific community about p-values: "Scientists rise up against statistical significance" in *Nature*
- Fun review of two-sample tests by @allison_horst: https://twitter.com/allison_horst/status/1216411185240690688



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By Thursday at noon

- Make sure you are a member of the course organization on GitHub
- Make sure you have access to RStudio
- If you are using RStudio on your local machine, make sure you have git configured and you can knit a PDF (need a Latex editor installed)