Simple Linear Regression

Prediction

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STA 210

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Topics

- Predict the response given a value of the predictor variable
- Use intervals to quantify the uncertainty in the predicted values
- Define *extrapolation* and why we should avoid it

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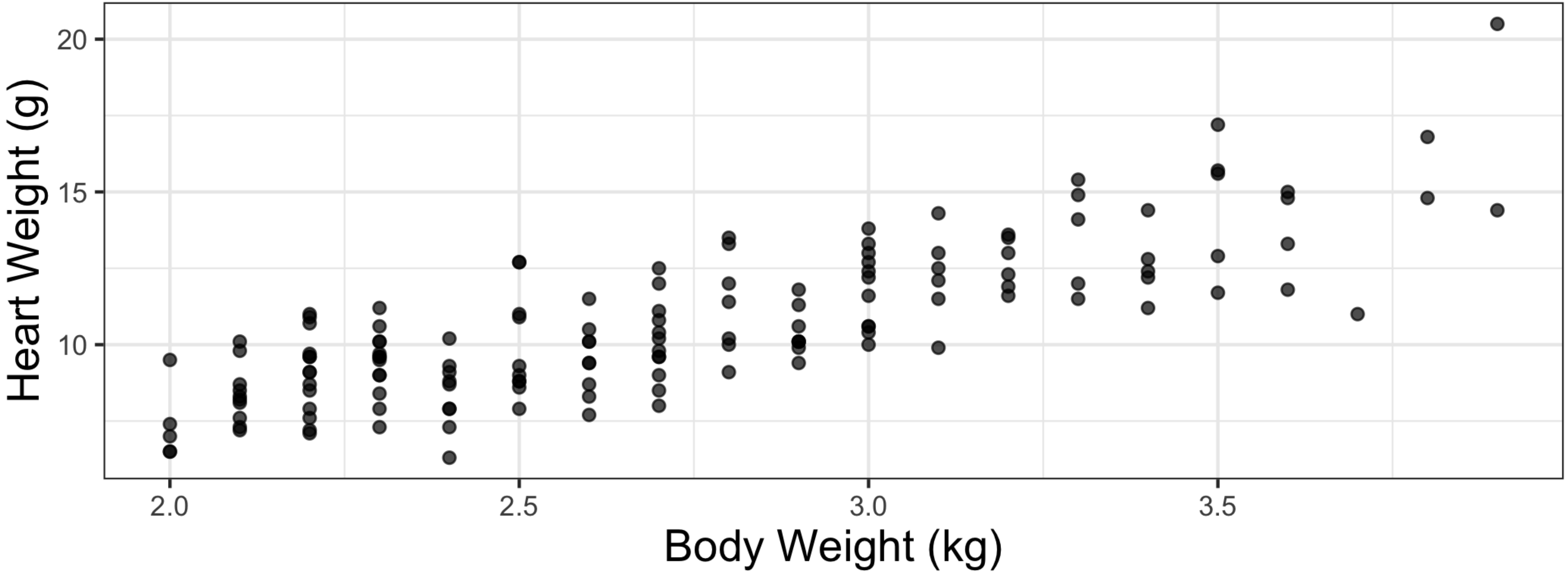
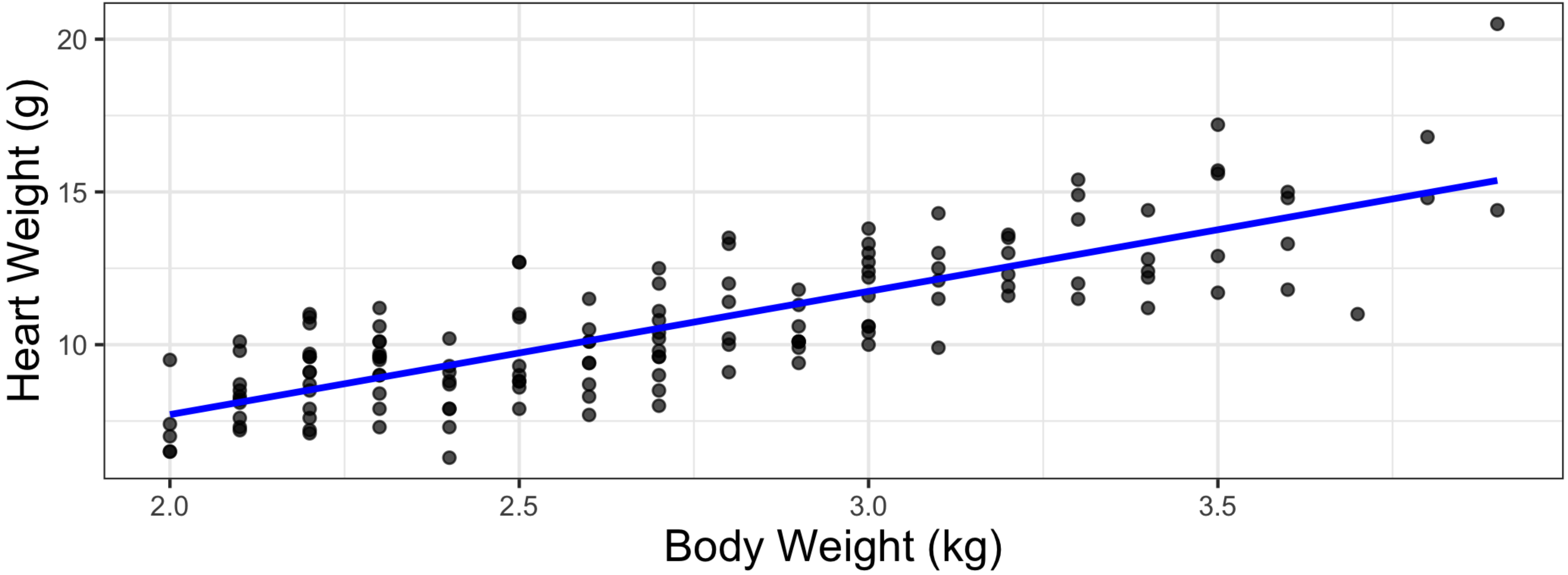
Cats data

The data set contains the **heart weight** (`Hwt`) and **body weight** (`Bwt`) for 144 domestic cats.



Cats data

We want to fit a model so we can use a cat's body weight to predict how much its heart weighs.



The model

$$\widehat{\text{Hwt}} = -0.357 + 4.034 \times \text{Bwt}$$

term	estimate	std.error	statistic	p.value
(Intercept)	-0.357	0.692	-0.515	0.607
Bwt	4.034	0.250	16.119	0.000

Prediction

We can use the regression model to

Estimate the <u>mean</u> response when the predictor variable is equal to a value x_0

Predict the response for an <u>individual</u> observation with a value of the predictor equal to x_0

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Calculating a predicted value

My cat Mindy weighs about 3.18 kg (7 lbs).

Based on this model, about how much does her heart weigh?

$$\begin{aligned}\widehat{\text{Hwt}} &= -0.357 + 4.034 \times \mathbf{3.18} \\ &= \mathbf{12.471}\ \text{g}\end{aligned}$$

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Uncertainty in predictions

Confidence interval for the mean response

$$\hat{y} \pm t^*_{n-2} \times \mathbf{SE}_{\hat{\mu}}$$

Prediction interval for an individual observation

$$\hat{y} \pm t^*_{n-2} \times \mathbf{SE}_{\hat{\mathbf{y}}}$$

Standard errors

$$SE_{\hat{\mu}} = \hat{\sigma}_\epsilon \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

$$SE_{\hat{y}} = \hat{\sigma}_\epsilon \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

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Standard errors

$$SE_{\hat{\mu}} = \hat{\sigma}_\epsilon \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

$$SE_{\hat{y}} = \hat{\sigma}_\epsilon \sqrt{\mathbf{1} + \frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

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Confidence interval

The 95% **confidence interval** for the *mean* heart weight of cats that weigh 3.18 kg is

fit	lwr	upr
12.472	12.143	12.801

> We are 95% confident that mean heart weight for the subset of cats that weigh 3.18 kg is between 12.143 g and 12.801 g.

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Prediction interval

The 95% **prediction interval** for an *individual* cat (Mindy) that weighs 3.18 kg is

fit	lwr	upr
12.472	9.582	15.362

We can predict with 95% confidence that Mindy's heart weighs between 9.582 g and 15.362 g.

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Comparing intervals

🛑 Caution! Extrapolation

We should **not** use the model to predict for values of X far outside the range of values used to fit the model.

This is called **extrapolation**.

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Predict Andy's heart weight?

My cat Andy weighs about 5.44 kg (12 lbs).

Should we use this regression model to predict how much his heart weighs?

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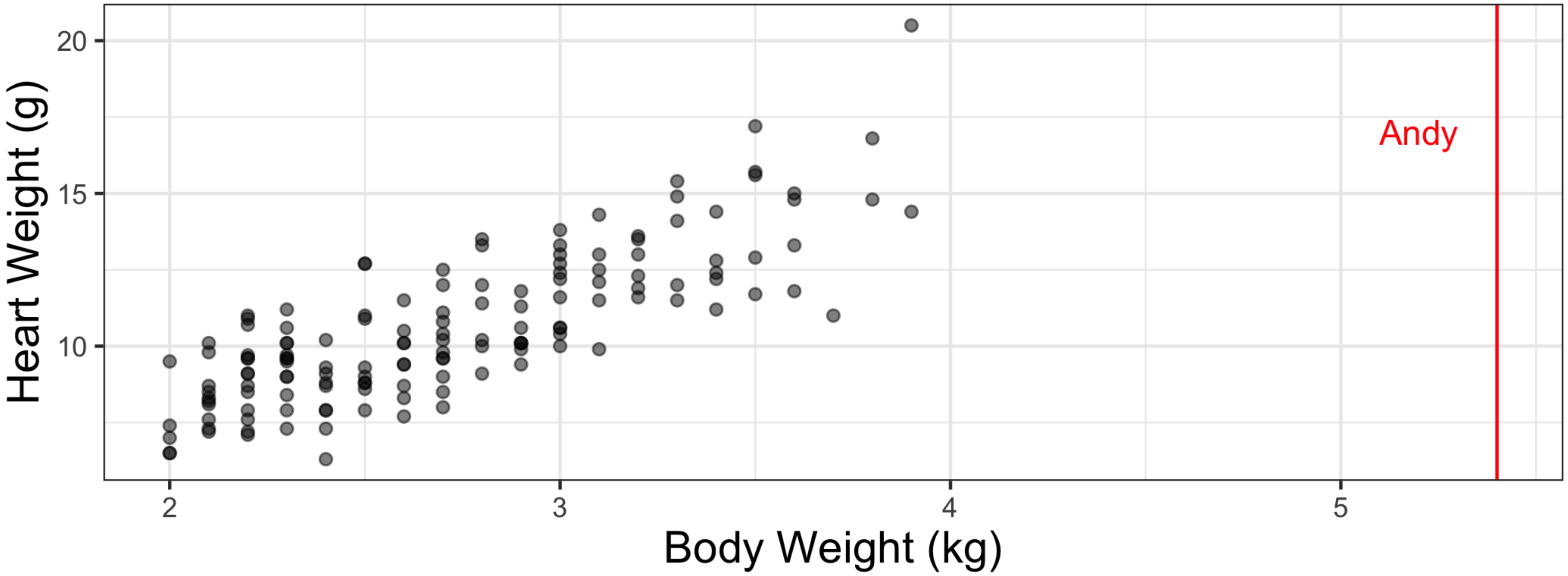
Predict Andy's heart weight?



We should **not** use this model to predict Andy's heart weight, since that would be **extrapolation**.

Recap

- Predicted the response given a value of the predictor variable
- Used intervals to quantify the uncertainty in the predicted values
  - Confidence interval for the mean response
  - Prediction interval for individual response
- Defined **extrapolation** and why we should avoid it