Simple Linear Regression

Foundation

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STA 210

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General form of model

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Y: response variable

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f: fixed but unknown function

ϵ: random error

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Simple linear regression

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$$
\begin{aligned}
Y &= \mathbf{Model} + \text{Error} \\
&= \mathbf{f(X)} + \epsilon \\
&= \boldsymbol{\mu_{Y|X}} + \epsilon \\
&= \boldsymbol{\beta_0 + \beta_1 X} + \epsilon
\end{aligned}
$$

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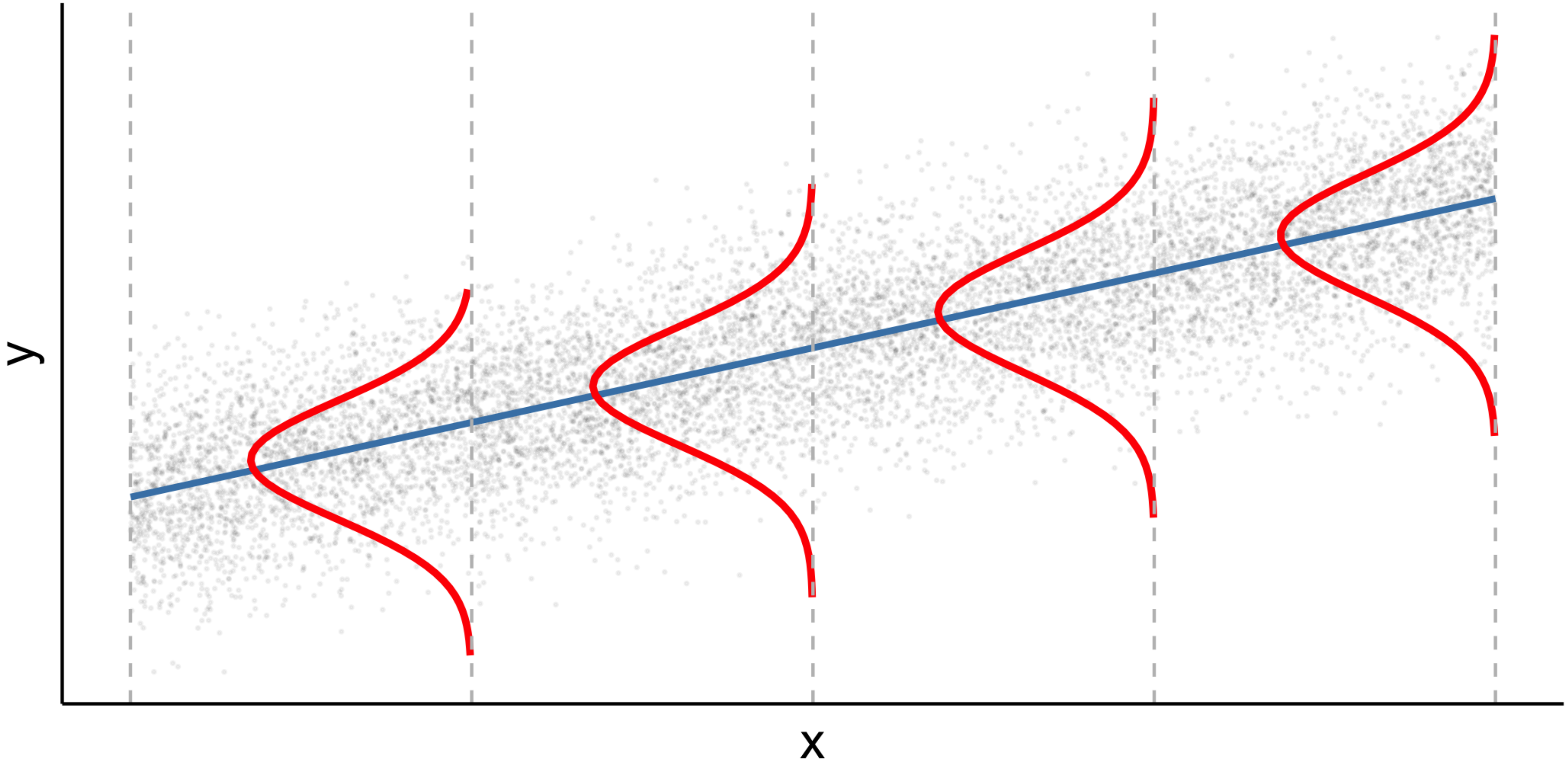
$$Y = \boldsymbol{\beta_0} + \boldsymbol{\beta_1} \boldsymbol{X} + \epsilon$$

where the errors are independent and normally distributed $\epsilon \sim N(0, \sigma_\epsilon^2)$

$$Y|X \sim N(\beta_0 + \beta_1 X, \sigma_\epsilon^2)$$



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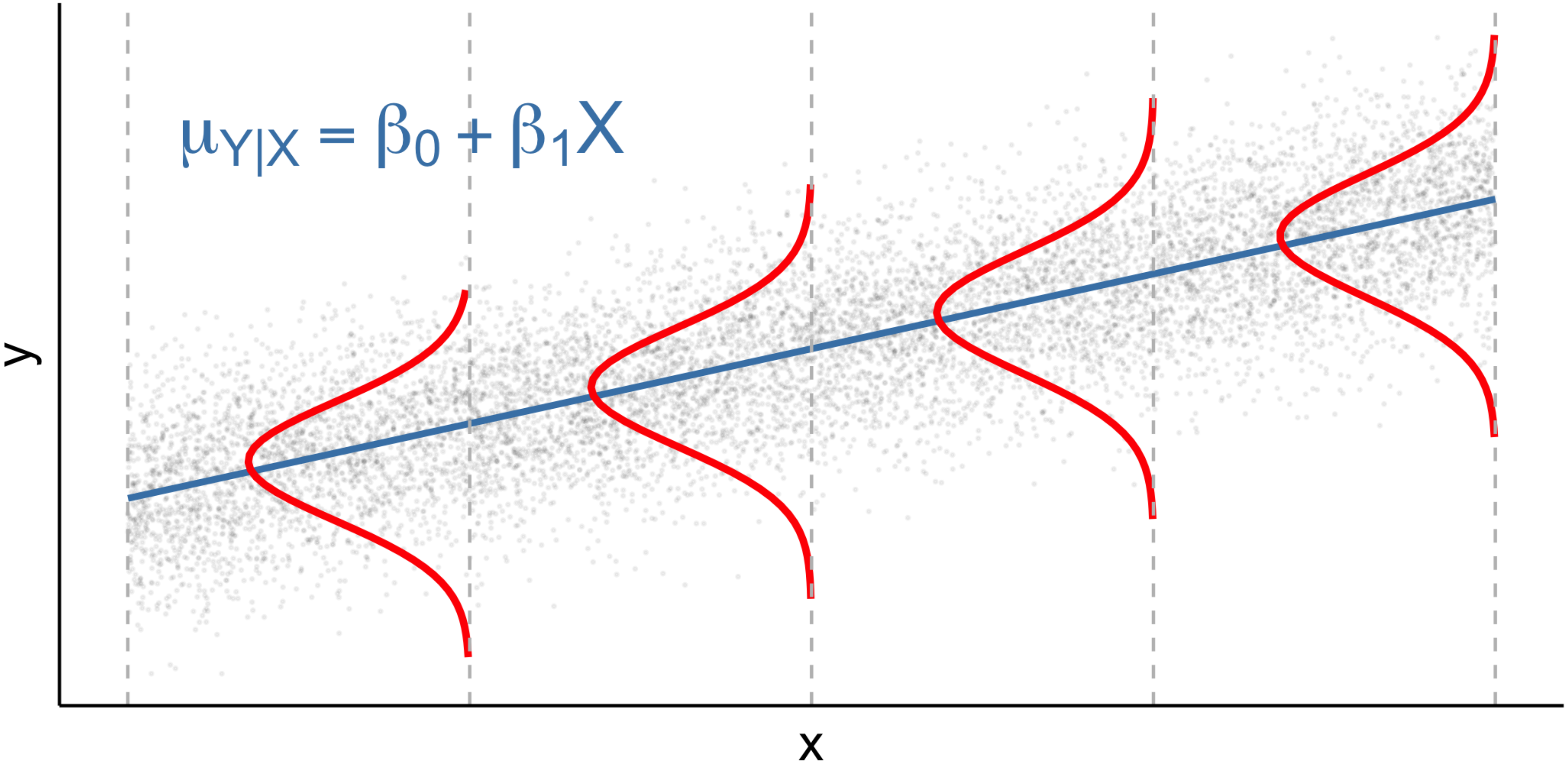

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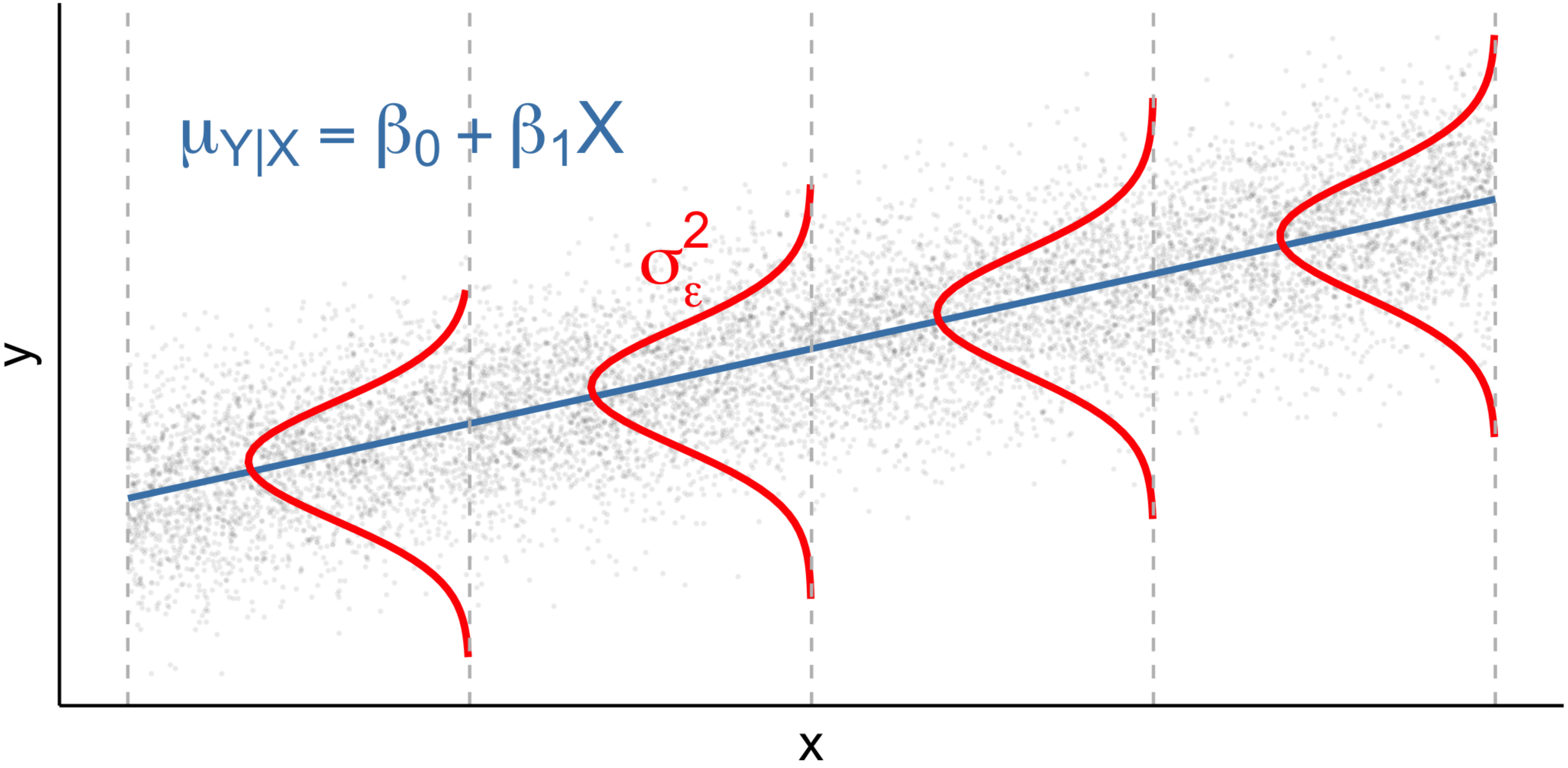

Regression standard error

Once we fit the model, we can use the residuals to calculate the **regression standard error**

$$\hat{\sigma}_\epsilon = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-2}} = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n-2}}$$

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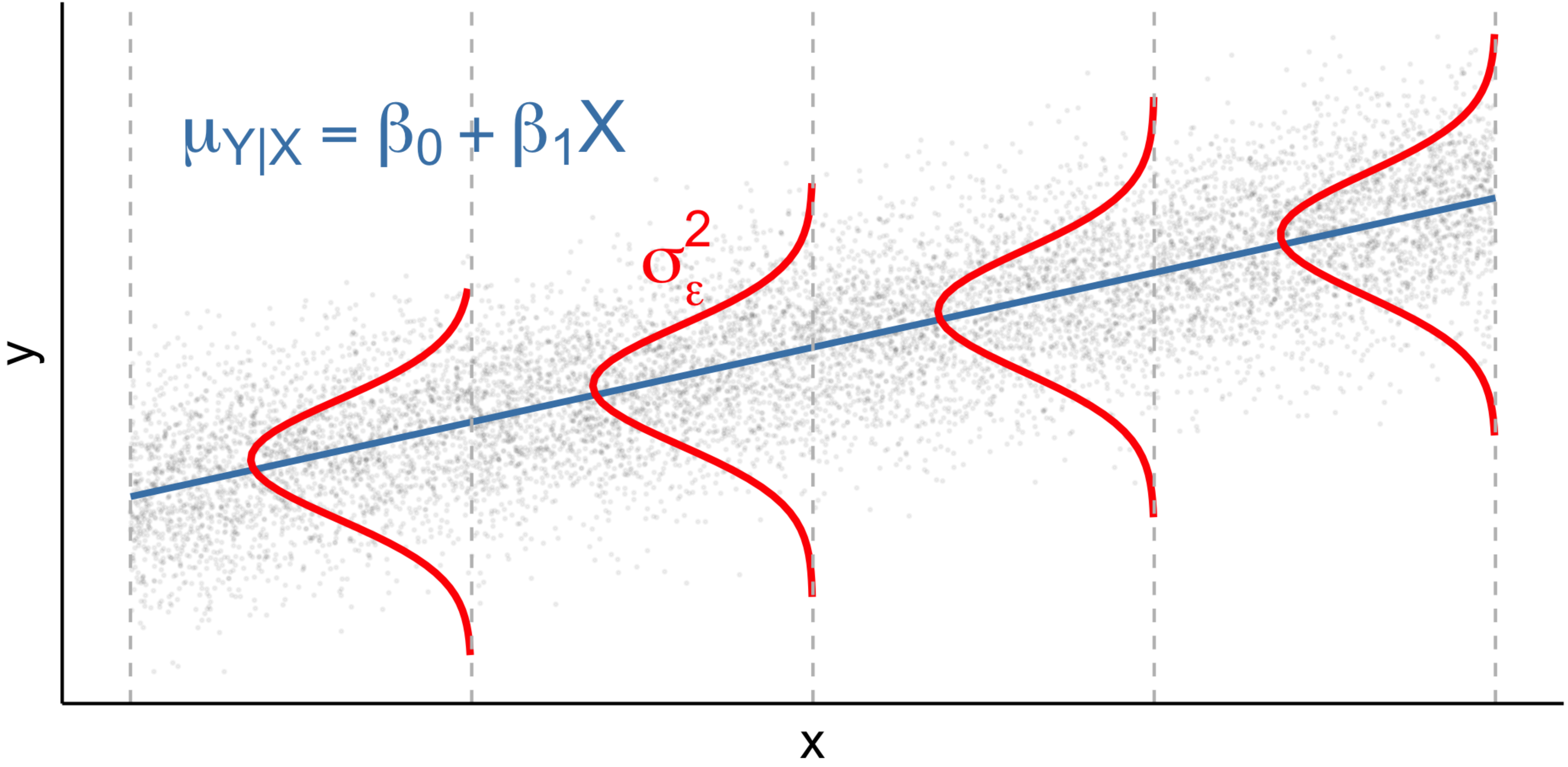
Standard error of $\hat{\beta}_1$

$$SE_{\hat{\beta}_1} = \hat{\sigma}_\epsilon \sqrt{\frac{1}{(n-1)s_X^2}}$$



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