Logistic regression interpretation

Class activity, Part I

https://sta214-f22.github.io/class_activities/ca_lecture4.html

Class activity

$$\log\left(\frac{\widehat{\pi}_i}{1-\widehat{\pi}_i}\right) = -2.901 + 0.0036\ Score_i$$

Calculate the odds ratio comparing odds of acceptance for a student with a GRE score of 701 to a student with a GRE score of 700.

Class activity

$$\log\left(\frac{\widehat{\pi}_i}{1-\widehat{\pi}_i}\right) = -2.901 + 0.0036\ Score_i$$

> Calculate the odds ratio comparing odds of acceptance for a student with a GRE score of 701 to a student with a GRE score of 700.

$$\frac{e^{-2.901+0.0036\cdot 701}}{e^{-2.901+0.0036\cdot 700}} = 1.0036$$

> This is the same odds ratio as comparing a 601 GRE to a 600 GRE!

Class activity

$$\log\left(\frac{\widehat{\pi}_i}{1-\widehat{\pi}_i}\right) = -2.901 + 0.0036\ Score_i$$

Odds ratio for increasing GRE by one point:

$Score \to Score + 1$

odds ratio: $\dfrac{\exp\{-2.901 + 0.0036(Score+1)\}}{\exp\{-2.901 + 0.0036\,Score\}}$

$$\frac{\cancel{e^{-2.901}}\ \cancel{e^{0.0036Score}}\ e^{0.0036}}{\cancel{e^{-2.901}}\ \cancel{e^{0.0036Score}}} = e^{0.0036} \approx 1.0036$$

why is $e^{0.0036} \approx 1.0036$

$e^2 \neq 3$

<u>if</u> x is close to 0 $\implies e^x \approx 1+x$

why?

Taylor series for e^x (around 0):

$$e^x = 1 + x + \underbrace{\frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots}_{\text{if } x \text{ is close to } 0, \text{ this is really small}}$$

$$\approx 1 + x$$

Interpreting coefficients

$$\log\left(\frac{\widehat{\pi}_i}{1-\widehat{\pi}_i}\right) = \widehat{\beta}_0 + \widehat{\beta}_1 x_i$$

Interpretation of $\widehat{\beta}_1$:

- A one unit increase in x is associated with an increase of $\widehat{\beta}_1$ in the log odds
- A one unit increase in x is associated with an increase in the odds by a factor of $\exp\{\widehat{\beta}_1\}$

> How do you think we interpret $\widehat{\beta}_0$?

Interpreting coefficients

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Interpretation of $\widehat{\beta}_0$:

- The estimated log odds when $x = 0$ are $\widehat{\beta}_0$
- The estimated odds when $x = 0$ are $\exp\{\widehat{\beta}_0\}$

Fitting logistic regression in R

← generalized linear model

```
gre_glm <- glm(admit ~ gre, data = grad_app,
               family = binomial)
summary(gre_glm)
```

← specifies logistic regression (binary response)

same

z instead of t!

```
...
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -2.901344   0.606038  -4.787 1.69e-06 ***
## gre          0.003582   0.000986   3.633  0.00028 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 499.98  on 399  degrees of freedom
## Residual deviance: 486.06  on 398  degrees of freedom
## AIC: 490.06
##
## Number of Fisher Scoring iterations: 4
```

← ignore for now

instead of R^2 etc.

← new

Class activity, Part II

https://sta214-f22.github.io/class_activities/ca_lecture4.html

Class activity

$$\log\left(\frac{\widehat{\pi}_i}{1-\widehat{\pi}_i}\right) = -4.358 + 1.051\, GPA_i$$

> What is the change in the odds of acceptance associated with an increase of 1 point in GPA?

Class activity

$$\log\left(\frac{\widehat{\pi}_i}{1-\widehat{\pi}_i}\right) = -4.358 + 1.051\, GPA_i$$

> What is the change in the odds of acceptance associated with an increase of 1 point in GPA?

An increase of 1 point in GPA is associated with an increase in the odds of acceptance by a factor of $\exp\{1.051\} = 2.861$

Class activity

$$\log\left(\frac{\widehat{\pi}_i}{1-\widehat{\pi}_i}\right) = -4.358 + 1.051\, GPA_i$$

> What is the estimated probability that a student with a GPA of 3.5 is accepted?

Class activity

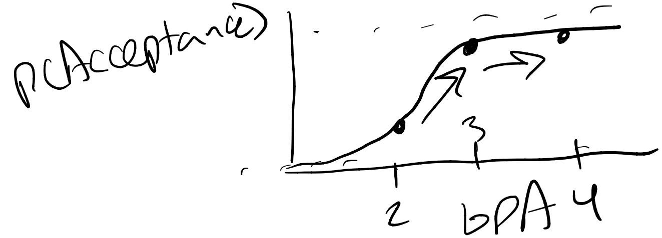
$$\log\left(\frac{\widehat{\pi}_i}{1-\widehat{\pi}_i}\right) = -4.358 + 1.051\, GPA_i$$

What is the estimated probability that a student with a GPA of 3.5 is accepted?

$$\frac{e^{-4.358+1.051(3.5)}}{1+e^{-4.358+1.051(3.5)}} \approx 0.336$$

Class activity

> The logistic regression model assumes that the change in odds associated with an increase of 1 point in GPA is constant. Is the change in *probability* also constant?



Class activity

> The logistic regression model assumes that the change in odds associated with an increase of 1 point in GPA is constant. Is the change in *probability* also constant?

No:

$$\frac{0.231}{0.095} \approx 2.43$$

- GPA = 2.0, estimated probability = 0.095
- GPA = 3.0, estimated probability = 0.231
- GPA = 4.0, estimated probability = 0.462

$$0.231 - 0.095 = 0.136$$

$$0.462 - 0.231 = 0.231$$

$$\frac{0.462}{0.231} = 2$$

- no constant additive change
- no constant multiplicative change