Random slopes

Recap: a model with random slopes

$$Price_{ij} = \beta_0 + u_i + \underbrace{(\beta_1 + v_i)}_{\text{different for each neighborhood } i} Satisfaction_{ij} + \varepsilon_{ij}$$

> How would I interpret each part of this model?

- β_0 = mean price when satisfaction is 0 (averaged across neighborhoods)
- $\beta_0 + u_i$ = mean price when satisfaction = 0 for neighborhood i
- β_1 = mean change in price for a one-unit increase is satisfaction (averaged across neighborhoods)
- $\beta_1 + v_i$ = mean change in price for a one-unit increase in satisfaction, in neighborhood i

Class activity

https://sta214-f22.github.io/class_activities/ca_lecture_29.html

Class activity

Mixed effects models are useful when there are group effects in our data.

> What are the groups in the data, and what are the observations within each group?

Groups: each musician is a group

observations w/in groups: performances for each musician bivariate normal distribution

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} \overset{iid}{\sim} N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix} \right)$$

Class activity

$\sigma_u^2 = Var(u_i)$, $\sigma_v^2 = Var(v_i)$, ρ_{uv} = correlation between u_i & v_i

The researchers hypothesize that anxiety levels depend on the type of performance (large or small ensembles), and that the difference in anxiety levels between large and small ensembles varies from person to person.

> What mixed effects model should the researchers use to investigate their hypothesis?

$$\underbrace{Anxiety_{ij}}_{\text{anxiety of musician } i \text{ before performance } j} = \underbrace{\beta_0 + u_i}_{\text{overall anxiety varies from person to person}} + \underbrace{(\beta_1 + v_i)}_{\text{effect of performance type can vary from person to person}} \underbrace{LargeEnsemble_{ij}}_{\text{ensemble size (performance type)}} + \underbrace{\varepsilon_{ij}}_{\text{variation from performance to performance for a musician}}$$

$\varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$

$u_i, v_i \sim ?$

Class activity

$$Anxiety_{ij} = \beta_0 + u_i + (\beta_1 + v_i)LargeEnsemble_{ij} + \varepsilon_{ij}$$

Interpret the fixed effects and random effects in the model.

(1|id) => add a random intercept for each musician

Fitting a model with random slopes

anxiety (response)

fixed effect

(large|id) => random intercept and random slope for each musician

```
m1 <- lmer(na ~ large + (large|id),
           data = music)
```

- This means we include `large` as a fixed effect, *and* we allow the coefficient on `large` to vary from individual to individual

$\hat{\beta}_0$ = average anxiety before a small/solo performance (averaged across musicians)

Fitting a model with random slopes

$\hat{\beta}_1$ = average change in anxiety for a large vs. small performance, averaged across musicians

```
m1 <- lmer(na ~ large + (large|id),
           data = music)
summary(m1)
```

```
...
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  16.7297     0.4908   34.09
## large        -1.6762     0.5425   -3.09
##
...
```

$$\widehat{\beta}_0 = 16.73 \quad \widehat{\beta}_1 = -1.68$$

> How would I interpret $\widehat{\beta}_0$ and $\widehat{\beta}_1$?

$\hat{\rho}_{uv} = -0.76$ => if u_i is ↑ positive, then v_i is ↓ negative

=> subjects with higher performance anxiety before solos / small performances tend to have a greater decrease in anxiety before large performances

Fitting a model with random slopes

```
summary(m1)
```

$\hat{\sigma}_u^2 = 6.33$ $\sigma_v^2 = 0.74$

$\hat{\rho}_{uv} = -0.76$

```
...
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  id       (Intercept)  6.3330  2.5165
##           large        0.7429  0.8619   -0.76
##  Residual             21.7712  4.6660
## Number of obs: 497, groups:  id, 37
...
```

$\hat{\sigma}_\varepsilon^2 = 21.77$

> What does this output tell us about the random effects and the noise?

Fitting a model with random slopes

$$Anxiety_{ij} = \beta_0 + u_i + (\beta_1 + v_i)LargeEnsemble_{ij} + \varepsilon_{ij}$$

```
...
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  id       (Intercept)  6.3330  2.5165
##           large        0.7429  0.8619   -0.76
##  Residual             21.7712  4.6660
...
```

- $\hat{\sigma}_u^2 = 6.333$ (variability in anxiety before small performances, between students)
- $\hat{\sigma}_v^2 = 0.743$ (variability in difference in anxiety before large performances, between students)
- $\hat{\sigma}_\varepsilon^2 = 21.77$ (variability in anxiety between performances, within a student)

Correlation between slopes and intercepts

$$Anxiety_{ij} = \beta_0 + u_i + (\beta_1 + v_i)LargeEnsemble_{ij} + \varepsilon_{ij}$$

```
...
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  id       (Intercept)  6.3330  2.5165
##           large        0.7429  0.8619   -0.76
##  Residual             21.7712  4.6660
...
```

- $\hat{\rho}_{uv} = -0.76$ (estimated correlation between the random slope and random intercept for an individual)
  - Subjects with higher performance anxiety scores for solos and small ensembles tend to have greater decreases in performance anxiety for large ensemble performances

Writing down the model

$$Anxiety_{ij} = \beta_0 + u_i + (\beta_1 + v_i)LargeEnsemble_{ij} + \varepsilon_{ij}$$

$$\varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2) \qquad \begin{bmatrix} u_i \\ v_i \end{bmatrix} \overset{iid}{\sim} N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix} \right)$$

> Anybody know the name of this new thing?

Writing down the model

$$Anxiety_{ij} = \beta_0 + u_i + (\beta_1 + v_i)LargeEnsemble_{ij} + \varepsilon_{ij}$$

$$\varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2) \qquad \begin{bmatrix} u_i \\ v_i \end{bmatrix} \overset{iid}{\sim} N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix} \right)$$

- This just says that both u_i and v_i come from a normal distribution
  - the variance of u_i is σ_u^2
  - the variance of v_i is σ_v^2
  - the correlation between u_i and v_i is ρ_{uv}
- *Note:* the population model includes the distribution of the random effects and noise