Random slopes

Recap: a model with random slopes

$$Price_{ij} = \beta_0 + u_i + (\beta_1 + v_i)Satisfaction_{ij} + \varepsilon_{ij}$$

> How would I interpret each part of this model?

- β_0 =
- $\beta_0 + u_i$ =
- β_1 =
- $\beta_1 + v_i$ =

A model with random slopes

$$Price_{ij} = \beta_0 + u_i + (\beta_1 + v_i) Satisfaction_{ij} + \varepsilon_{ij}$$

- β_0 = mean price when satisfaction is 0 (average across neighborhoods)
- $\beta_0 + u_i$ = mean price when satisfaction is 0 in neighborhood i
- β_1 = average change in price for a one-unit increase in satisfaction (average across neighborhoods)
- $\beta_1 + v_i$ = average change in price for a one-unit increase in satisfaction in neighborhood i

Class activity

https://sta214-f22.github.io/class_activities/ca_lecture_29.html

Class activity

Mixed effects models are useful when there are group effects in our data.

> What are the groups in the data, and what are the observations within each group?

Class activity

The researchers hypothesize that anxiety levels depend on the type of performance (large or small ensembles), and that the difference in anxiety levels between large and small ensembles varies from person to person.

> What mixed effects model should the researchers use to investigate their hypothesis?

Class activity

$$Anxiety_{ij} = \beta_0 + u_i + (\beta_1 + v_i)LargeEnsemble_{ij} + \varepsilon_{ij}$$

> Interpret the fixed effects and random effects in the model.

Class activity

$$Anxiety_{ij} = \beta_0 + u_i + (\beta_1 + v_i)LargeEnsemble_{ij} + \varepsilon_{ij}$$

> Interpret the fixed effects and random effects in the model.

- β_0 = average performance anxiety before small ensemble and solo performances (average across musicians)
- $\beta_0 + u_i$ = average performance anxiety before small ensemble and solo performances for musician i
- β_1 = average difference in anxiety before large ensemble performances (compared to small/solo performances) (average across musicians)
- $\beta_1 + v_i$ = average difference in anxiety before large ensemble performances for musician i

Fitting a model with random slopes

```
m1 <- lmer(na ~ large + (large|id),
           data = music)
```

- This means we include `large` as a fixed effect, *and* we allow the coefficient on `large` to vary from individual to individual

Fitting a model with random slopes

```
m1 <- lmer(na ~ large + (large|id),
           data = music)
summary(m1)
```

```
...
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  16.7297     0.4908   34.09
## large        -1.6762     0.5425   -3.09
##
...
```

$\widehat{\beta}_0 = 16.73 \quad \widehat{\beta}_1 = -1.68$

> How would I interpret $\widehat{\beta}_0$ and $\widehat{\beta}_1$?

Fitting a model with random slopes

$$Anxiety_{ij} = \beta_0 + u_i + (\beta_1 + v_i)LargeEnsemble_{ij} + \varepsilon_{ij}$$

$$\widehat{\beta}_0 = 16.73 \quad \widehat{\beta}_1 = -1.68$$

- The average anxiety before a small/solo performance is 16.73
- On average, student anxiety decreases by 1.68 before a large performance (compared to a small/solo performance)

Fitting a model with random slopes

```
summary(m1)
```

```
...
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  id       (Intercept)  6.3330  2.5165
##           large        0.7429  0.8619   -0.76
##  Residual             21.7712  4.6660
## Number of obs: 497, groups:  id, 37
...
```

> What does this output tell us about the random effects and the noise?

Fitting a model with random slopes

$$Anxiety_{ij} = \beta_0 + u_i + (\beta_1 + v_i)LargeEnsemble_{ij} + \varepsilon_{ij}$$

```
...
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  id       (Intercept)  6.3330  2.5165
##           large        0.7429  0.8619   -0.76
##  Residual             21.7712  4.6660
...
```

- $\widehat{\sigma}_u^2 = 6.333$ (variability in anxiety before small performances, between students)
- $\widehat{\sigma}_v^2 = 0.743$ (variability in difference in anxiety before large performances, between students)
- $\widehat{\sigma}_\varepsilon^2 = 21.77$ (variability in anxiety between performances, within a student)

Correlation between slopes and intercepts

$$Anxiety_{ij} = \beta_0 + u_i + (\beta_1 + v_i)LargeEnsemble_{ij} + \varepsilon_{ij}$$

```
...
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  id       (Intercept)  6.3330  2.5165
##           large        0.7429  0.8619   -0.76
##  Residual             21.7712  4.6660
...
```

- $\hat{\rho}_{uv} = -0.76$ (estimated correlation between the random slope and random intercept for an individual)
  - Subjects with higher performance anxiety scores for solos and small ensembles tend to have greater decreases in performance anxiety for large ensemble performances

Writing down the model

$$Anxiety_{ij} = \beta_0 + u_i + (\beta_1 + v_i)LargeEnsemble_{ij} + \varepsilon_{ij}$$

$$\varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2) \qquad \begin{bmatrix} u_i \\ v_i \end{bmatrix} \overset{iid}{\sim} N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix} \right)$$

> Anybody know the name of this new thing?

Writing down the model

$$Anxiety_{ij} = \beta_0 + u_i + (\beta_1 + v_i)LargeEnsemble_{ij} + \varepsilon_{ij}$$

$$\varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2) \qquad \begin{bmatrix} u_i \\ v_i \end{bmatrix} \overset{iid}{\sim} N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix} \right)$$

- This just says that both u_i and v_i come from a normal distribution
  - the variance of u_i is σ_u^2
  - the variance of v_i is σ_v^2
  - the correlation between u_i and v_i is ρ_{uv}
- *Note:* the population model includes the distribution of the random effects and noise