Beginning linear mixed effects models

Data: flipped classrooms?

- A *flipped classroom* involves students watching lectures at home, and doing activities during class time
- There is debate about the pros and cons of this teaching method
- Here we will look at simulated data from an experiment with flipped classrooms

Data: flipped classrooms?

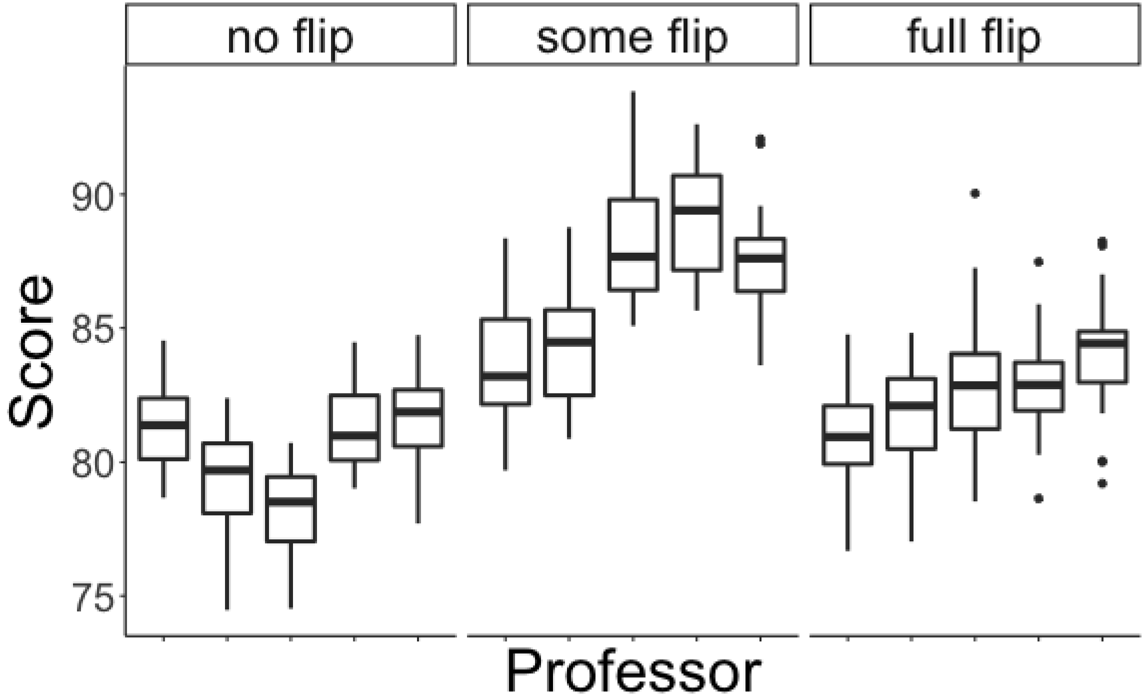
- 15 classes of introductory statistics
- 25 students in each class (so 375 students total)
- Each class taught by a different professor
- Each professor randomly assigned a teaching style: No flip, Some flip, and Fully flipped
- At the end of the semester, we give all the students in all the classes the same exam, and compare their results

Data: flipped classrooms?

Data set has 375 rows (one per student), and the following variables:

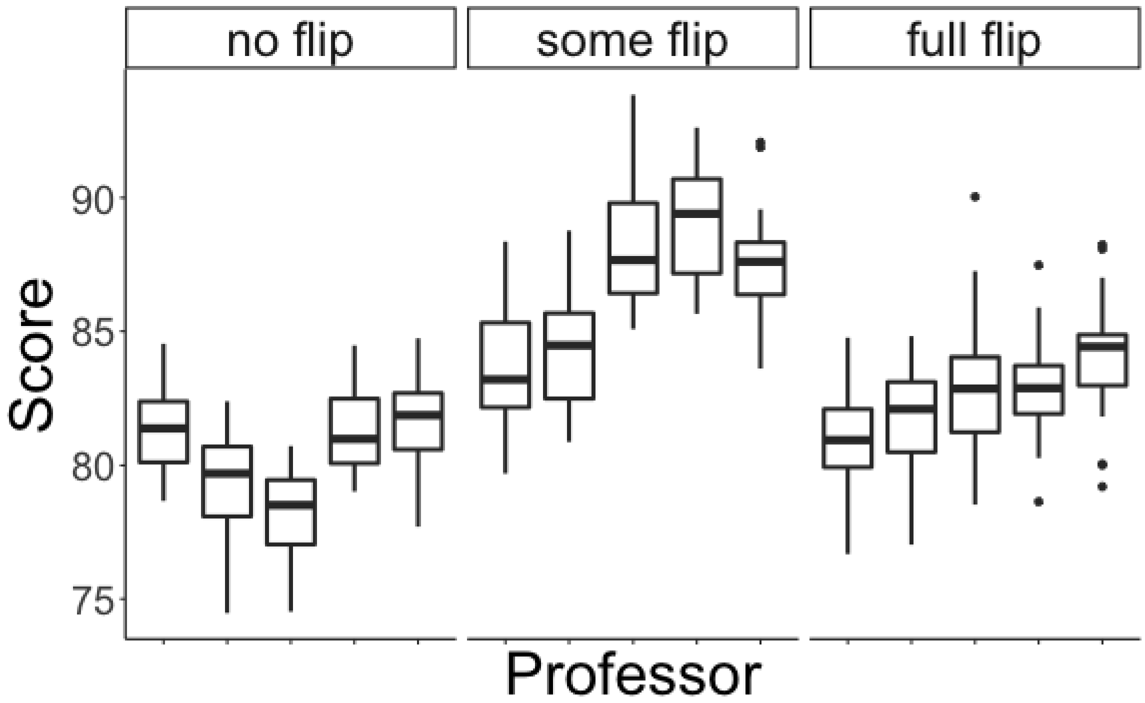
- `professor`: which professor taught the class (1 -- 15)
- `style`: which teaching style the professor used (no flip, some flip, fully flipped)
- `score`: the student's score on the final exam

Considering results



What do you notice about the scores?

Considering results



- There may be some differences between styles
- There may be some differences between professors

Considering results

> Suppose we notice that, on average, students in the “Some Flipped” classes have higher scores than students in the “Fully Flipped” classes. What might explain this difference?

Considering results

> Suppose we notice that, on average, students in the “Some Flipped” classes have higher scores than students in the “Fully Flipped” classes. What might explain this difference?

- The "Some Flipped" method may lead to higher test results.
- The professors assigned to teach "Some Flipped" may teach in such a way that their scores are higher than those in the "Fully Flipped" group (more experience, etc.).
- The students in the "Some Flipped" classes may have been stronger than those in the "Fully Flipped" group.

Different effects

- *Effect of interest* (*treatment effect*): The "Some Flipped" method may lead to higher test results; *the treatment imposed by the researchers has an effect on the outcome.*

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Different effects

- *Effect of interest* (*treatment effect*): The "Some Flipped" method may lead to higher test results; *the treatment imposed by the researchers has an effect on the outcome.*
- *Group effect*: The professors assigned to teach "Some Flipped" may have had an impact on the test scores; *the group the students are in has an effect on the outcome.*
- *Individual effect*: The students in the "Some Flipped" classes may have been stronger than those in the "Fully Flipped" group; *the individuals' characteristics or abilities have an effect on the outcome.*

Writing down a model

Score is a continuous response, so we can go back to linear models:

$$Score_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \varepsilon_i$$

$\varepsilon_i \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$

> Which effects does this model capture?

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> Which effects does this model capture?

- Treatment effect (β_0 is the average score in the no flip group, and β_1 and β_2 tell us how the score changes in the other groups)
- Individual effect (ε_i is the difference from the mean for student i)

Assumptions

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> What does this model assume about group effects (differences between professors)?

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> What does this model assume about group effects (differences between professors)?

That there are no systematic differences between professors (i.e., no group effects)

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> What does this model assume about correlation within a class?

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> What does this model assume about correlation within a class?

That there is no correlation between student scores within the same class

> Is this a good assumption?

Writing down a model

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> How can I incorporate systematic differences between classes?

Writing down a model

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> How can I incorporate systematic differences between classes?

Add a variable for the different professors:

$$Score_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \\ \beta_3 \text{Class2}_i + \cdots + \beta_{16} \text{Class15}_i + \varepsilon_i$$

$$\varepsilon_i \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

Writing down a model

$$Score_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i +$$
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> How many parameters did we add to the model to capture class differences?

Writing down a model

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> How many parameters did we add to the model to capture class differences?

14 (β_3,..., β_{16})

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> Do we want to do inference on β_3,..., β_{16} ?

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> Do we want to do inference on β_3,..., β_{16} ?

No -- we only care about inference for the treatment effect parameters (β_1 and β_2)

> Can we do something *different* to capture group effects?

Our first mixed effects model

Linear model:

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Linear mixed effects model: Let $Score_{ij}$ be the score of student j in class i

$$Score_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

$\varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2) \quad u_i \overset{iid}{\sim} N(0, \sigma_u^2)$

Anatomy of the mixed effects model

Linear mixed effects model: Let $Score_{ij}$ be the score of student j in class i

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- $\beta_0, \beta_1, \beta_2$: **fixed effect** terms (representing treatment effect)
- u_i : **random effect** terms (representing group effects)
- ε_{ij} : **noise** terms (representing individual effects)

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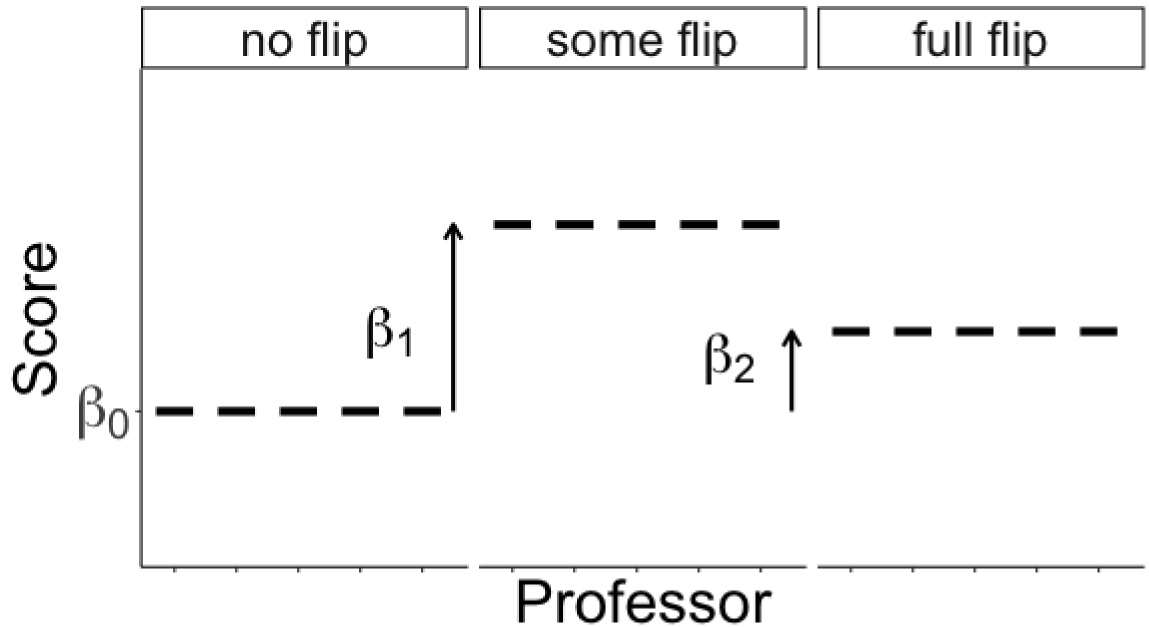
Part 1: Fixed effects (treatment effects)



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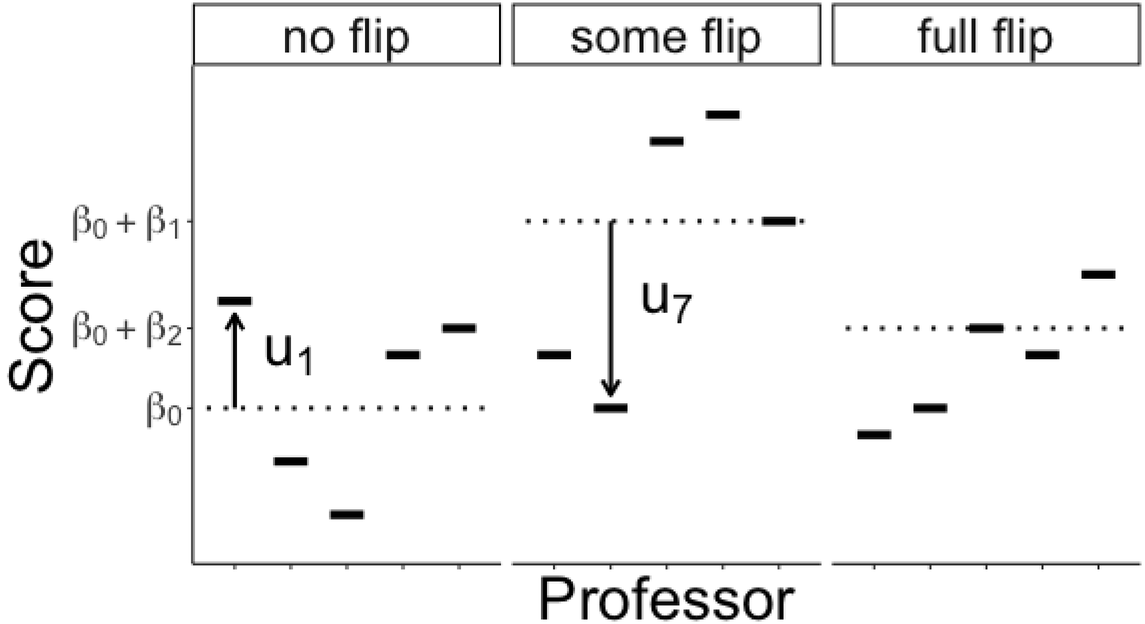
Part 2: Random effects (group effects)



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Part 3: Noise (individual effects)