Poisson Regression

Data

2015 Family Income and Expenditure Survey (FIES) on households in the Phillipines. Variables include

- `age`: age of the head of household
- `numLT5`: number in the household under 5 years old
- `total`: total number of people other than head of household
- `roof`: type of roof (stronger material can sometimes be used as a proxy for greater wealth)
- `location`: where the house is located (Central Luzon, Davao Region, Ilocos Region, Metro Manila, or Visayas)

Data

Questions:

- How is the age of head of household related to the number of people in the household?
- Is the type of roof material related to the number of people in the household?

> To answer these questions, our response variable is `total` (total number of people other than head). What kind of variable is this?

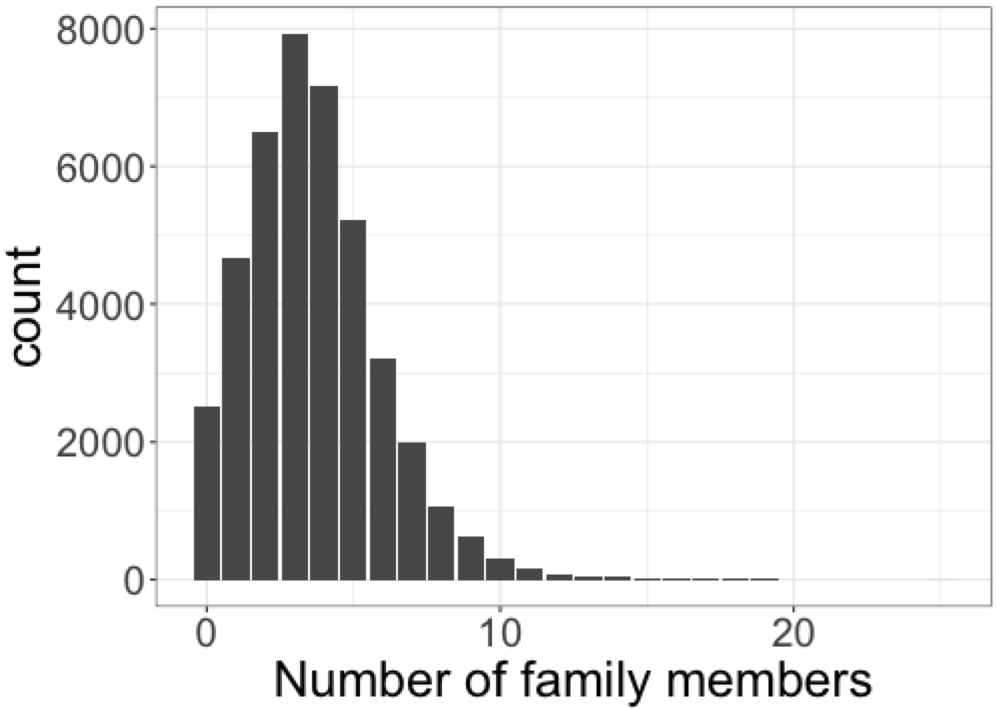
Building a parametric model

Step 1: Choose a reasonable distribution for Y

- $Y_i = total_i$ is a count variable!
- Unfortunately, we don't know any distributions for count data
- Bernoulli and Categorical distributions are for categorical data
- Normal distributions are for continuous data; count data is discrete

> We need a new distribution!

Exploring the response



- **Right skewed, unimodal** distribution
- We can use a *Poisson* distribution

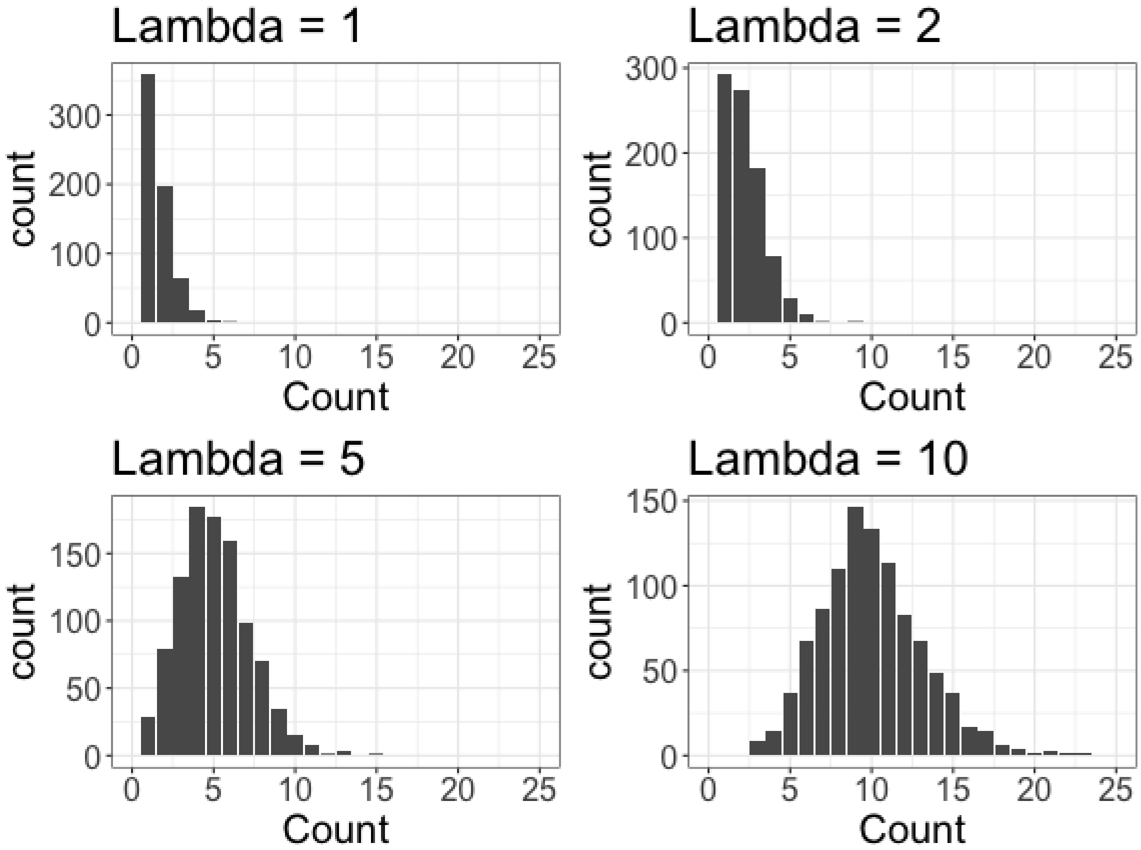
Poisson distribution

If $Y_i \sim Poisson(\lambda)$, then Y_i takes values $y = 0, 1, 2, \ldots$ with probabilities

$$P(Y_i = y) = \frac{e^{-\lambda}\lambda^y}{y!}$$

> Does this distribution look familiar?

Poisson distribution



How is λ related to the distribution?

Poisson distribution

If $Y_i \sim Poisson(\lambda)$, then Y_i takes values $y = 0, 1, 2, \ldots$ with probabilities

$$P(Y_i = y) = \frac{e^{-\lambda}\lambda^y}{y!}$$

- λ is the mean of the distribution
- λ is also the variance! (the mean and variance are the same)
- Our goal is to estimate λ, just like our goal was to estimate π in logistic regression

Estimating λ with maximum likelihood

$$P(Y_i = y) = \frac{e^{-\lambda}\lambda^y}{y!}$$

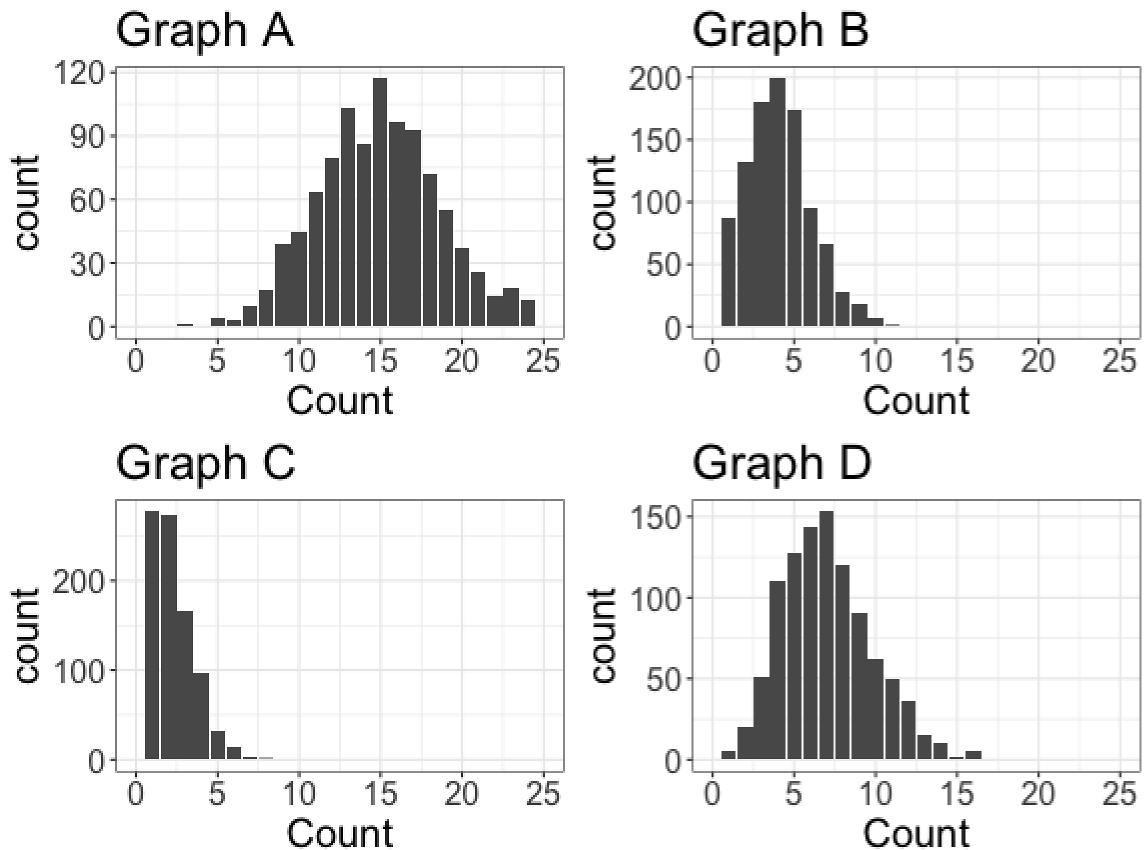
Suppose that $Y_1, \ldots, Y_n \overset{iid}{\sim} Poisson(\lambda)$.

What is the maximum likelihood estimate $\widehat{\lambda}$?

Class activity

https://sta214-s23.github.io/class_activities/ca_lecture_18.html

Class activity



What do you think λ is for each graph?

Class activity

$Y_i = \text{number of dogs adopted from animal shelter}$

$Y_i \sim Poisson(1.5)$

> What is the probability that at most two dogs are adopted?

Class activity

The Poisson distribution is for count data. Why is it ok for λ to not be a whole number?

Poisson regression

$Y_i =$ number of people in household other than head

How is Y_i related to the age of the head of the household?

Step 1: Choose a reasonable distribution for Y

$$Y_i \sim Poisson(\lambda_i)$$

Step 2: Choose a model for any parameters

$$\log(\lambda_i) = \beta_0 + \beta_1 Age_i$$

> Why do you think we use $\log(\lambda_i)$ instead of just λ_i?

Fitting the model

```
m1 <- glm(total ~ age, family = poisson,
          data = fies)
summary(m1)
```

```
...
##                Estimate Std. Error z value Pr(>|z|)
## (Intercept)  1.7135783  0.0085137  201.27   <2e-16 ***
## age         -0.0035255  0.0001619  -21.78   <2e-16 ***
...
```

$$\log(\widehat{\lambda}_i) = 1.714 - 0.0035 \; Age_i$$

> How can we interpret the slope?

Fitting the model

$$\log(\widehat{\lambda}_i) = 1.714 - 0.0035\ Age_i$$

For every additional year in age of the head of house, we expect the log of the *average* household size to decrease by 0.0035.

> Can I interpret on the un-logged scale?

Assumptions

$$Y_i \sim Poisson(\lambda_i)$$

$$\log(\lambda_i) = \beta_0 + \beta_1 Age_i$$

What assumptions does our Poisson regression model make?

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> What assumptions does our Poisson regression model make?

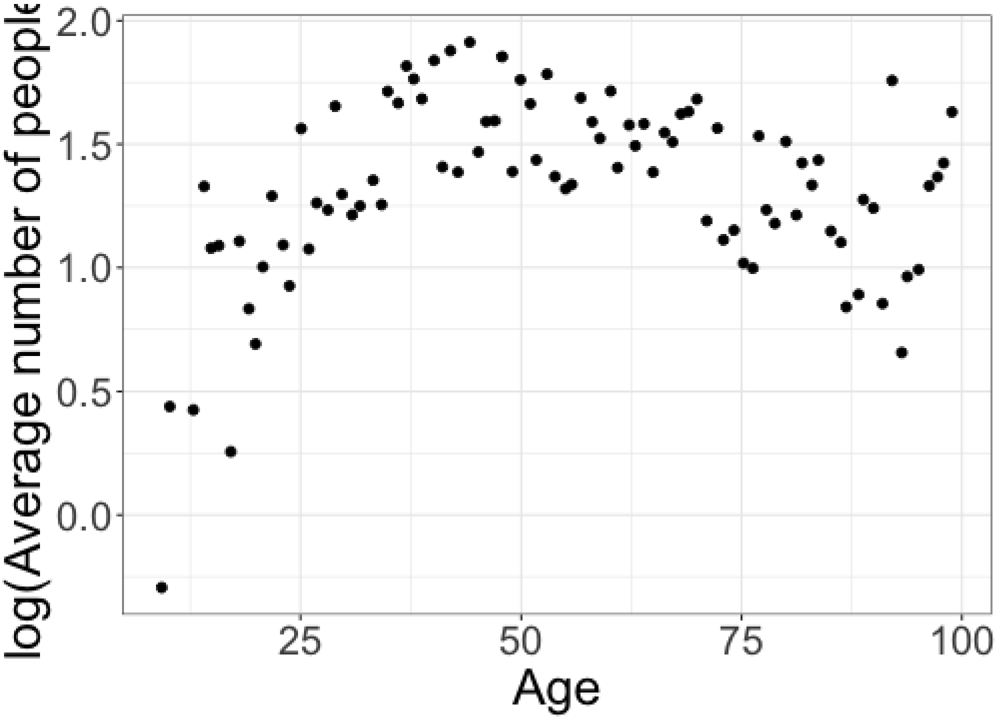
- **Shape:** The shape of the regression model is correct
- **Independence:** The observations are independent
- **Poisson distribution:** A Poisson distribution is a good choice for Y_i

The shape assumption

Shape assumption: The shape of the regression model is correct

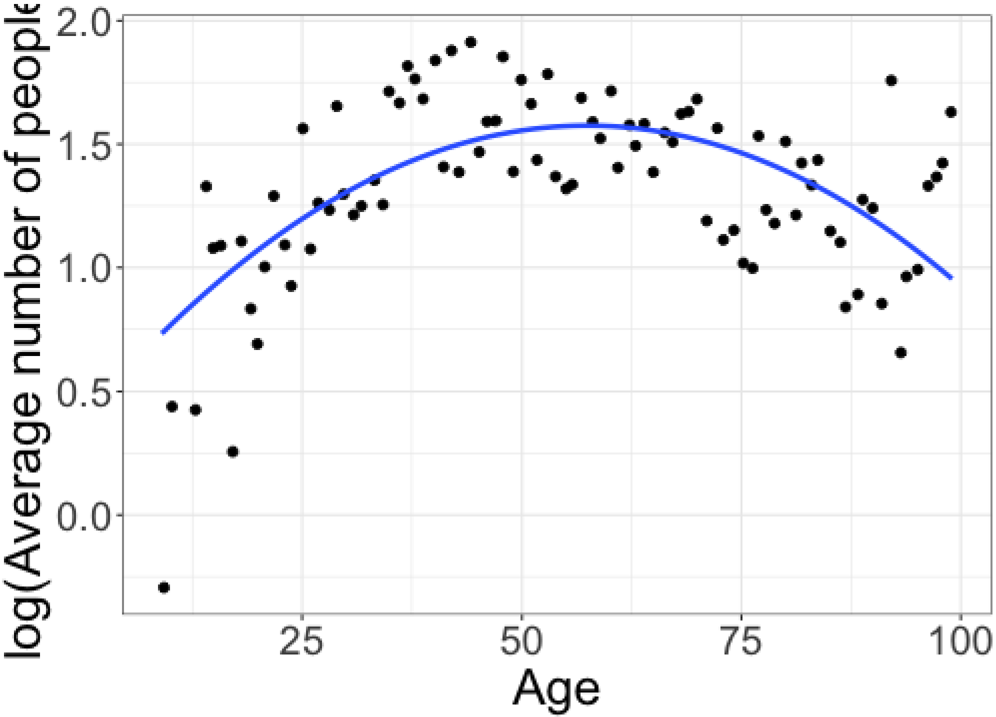
> How can I assess this assumption?

Checking the shape assumption



What shape seems appropriate?

Second order polynomial



Poisson distribution assumption

$$Y_i \sim Poisson(\lambda_i)$$

$$\log(\lambda_i) = \beta_0 + \beta_1 Age_i + \beta_2 Age_i^2$$

Poisson distribution assumption: The Poisson distribution is a good choice for Y_i.

> What are some characteristics of the Poisson distribution we could check?

Checking distribution shape

Look at the distribution of Y_i for different ages: