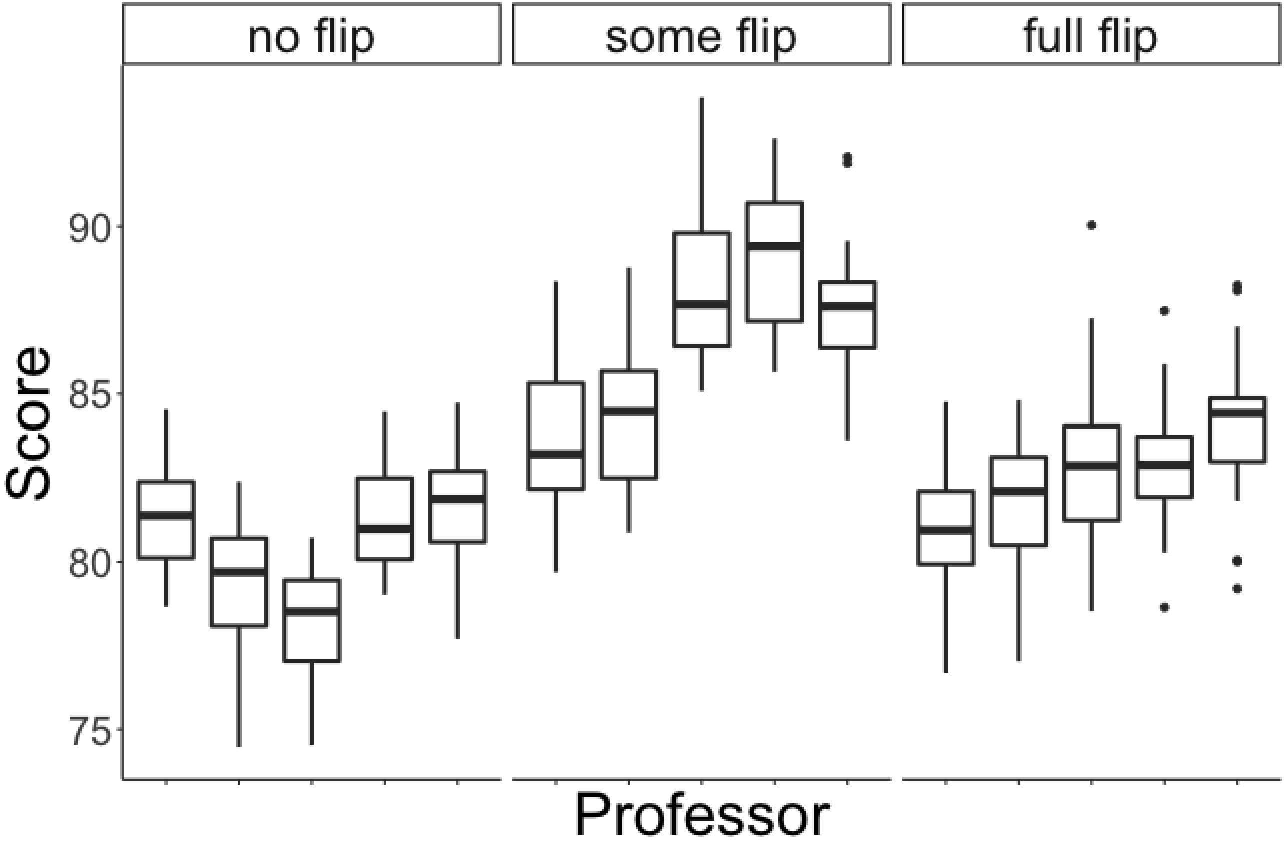
Beginning linear mixed effects models

Data: flipped classrooms?

Data set has 375 rows (one per student), and the following variables:

- `professor`: which professor taught the class (1 -- 15)
- `style`: which teaching style the professor used (no flip, some flip, fully flipped)
- `score`: the student's score on the final exam

Visualizing the data



Interpretation

$$Score_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

$$\varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma^2_\varepsilon) \qquad u_i \overset{iid}{\sim} N(0, \sigma^2_u)$$

- $\widehat{\beta}_0 = 77.66, \qquad \widehat{\beta}_1 = 11.07, \qquad \widehat{\beta}_2 = 2.81$
- $\widehat{\sigma}^2_\varepsilon = 4.25, \qquad \widehat{\sigma}^2_u = 21.37$

> How do I interpret $\widehat{\beta}_1 = 11.07$ and $\widehat{\beta}_2 = 2.81$?

We expect that, on average, scores in some-flipped classes are 11.07 points higher than for no-flipped classes.

We expect that, on average, scores in fully-flipped classes are 2.81 points higher than for no-flipped classes.

Interpretation

$$Score_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

$$\varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2) \qquad u_i \overset{iid}{\sim} N(0, \sigma_u^2)$$

- $\widehat{\beta}_0 = 77.66, \quad \widehat{\beta}_1 = 11.07, \quad \widehat{\beta}_2 = 2.81$
- $\widehat{\sigma}_\varepsilon^2 = 4.25, \quad \widehat{\sigma}_u^2 = 21.37$

> How do I interpret $\widehat{\sigma}_u^2 = 21.37$?

Interpretation

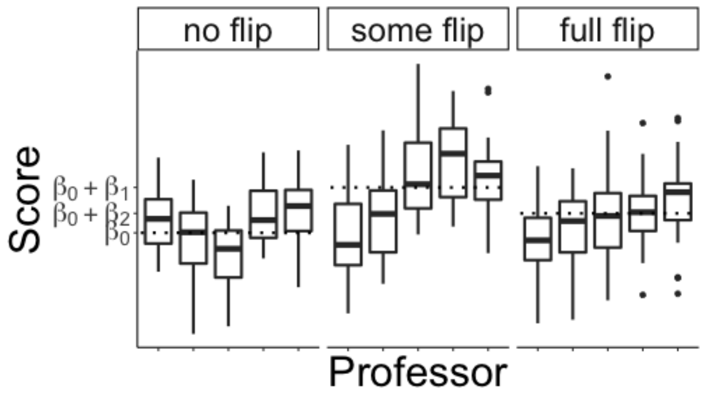
$$Score_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

$$\varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma^2_\varepsilon) \qquad u_i \overset{iid}{\sim} N(0, \sigma^2_u)$$

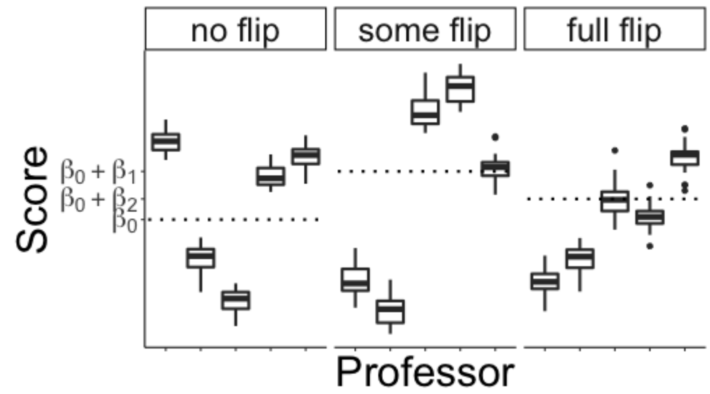
- $\widehat{\beta}_0 = 77.66, \quad \widehat{\beta}_1 = 11.07, \quad \widehat{\beta}_2 = 2.81$
- $\widehat{\sigma}^2_\varepsilon = 4.25, \quad \widehat{\sigma}^2_u = 21.37$

> How do I interpret $\widehat{\sigma}^2_\varepsilon = 4.25$?

Intra-class correlation



σ_ε^2 is large relative to σ_u^2



σ_ε^2 is small relative to σ_u^2

- Observations within a group are *more correlated* when σ_ε^2 is small relative to σ_u^2
- **Intra-class correlation:**

$$\rho_{group} = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\varepsilon^2} = \frac{\text{between group variance}}{\text{total variance}}$$

Intra-class correlation

$$Score_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

$$\varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma^2_\varepsilon) \qquad u_i \overset{iid}{\sim} N(0, \sigma^2_u)$$

- $\widehat{\beta}_0 = 77.66, \quad \widehat{\beta}_1 = 11.07, \quad \widehat{\beta}_2 = 2.81$
- $\widehat{\sigma}^2_\varepsilon = 4.25, \quad \widehat{\sigma}^2_u = 21.37$

$$\hat{\rho}_{group} = \frac{21.37}{21.37 + 4.25} = 0.83$$

So 83% of the variation in student's scores can be explained by differences in average scores from class to class (after accounting for teaching style). That's huge!

Class activity

https://sta214-s23.github.io/class_activities/ca_lecture_30.html

Class activity

Interpret the estimate fixed effect coefficients $\widehat{\beta}_0$ and $\widehat{\beta}_1$.

Class activity

Calculate and interpret the estimated intra-class correlation.