Poisson Regression

- Project 1, Part 3 released later today
- Due Friday, March 17
- Exam 1 test corrections: due Friday, March 17
  - submit on Canvas
  - include both a picture of your original answer, and your corrected answer. Write the corrected answer on a new sheet of paper
  - can receive up to 20% of points back

    e.g. 80% → 84%

    65% → 72%

    etc.

Data

2015 Family Income and Expenditure Survey (FIES) on households in the Phillipines. Variables include

- `age`: age of the head of household
- `numLT5`: number in the household under 5 years old
- `total`: total number of people other than head of household
- `roof`: type of roof (stronger material can sometimes be used as a proxy for greater wealth)
- `location`: where the house is located (Central Luzon, Davao Region, Ilocos Region, Metro Manila, or Visayas)

Poisson regression model

$Y_i =$ number of people in household other than head

$$Y_i \sim Poisson(\lambda_i)$$ random component

$$\log(\lambda_i) = \beta_0 + \beta_1 Age_i$$ systematic component

Poisson: mean = variance = λ_i

λ_i = average # of people in household, given Age_i

Model assumptions

$Y_i =$ number of people in household other than head

$$Y_i \sim Poisson(\lambda_i)$$

$$\log(\lambda_i) = \beta_0 + \beta_1 Age_i$$

Assumptions:

- **Shape:** The shape of the regression model is correct
- **Independence:** The observations are independent
- **Poisson distribution:** A Poisson distribution is a good choice for Y_i

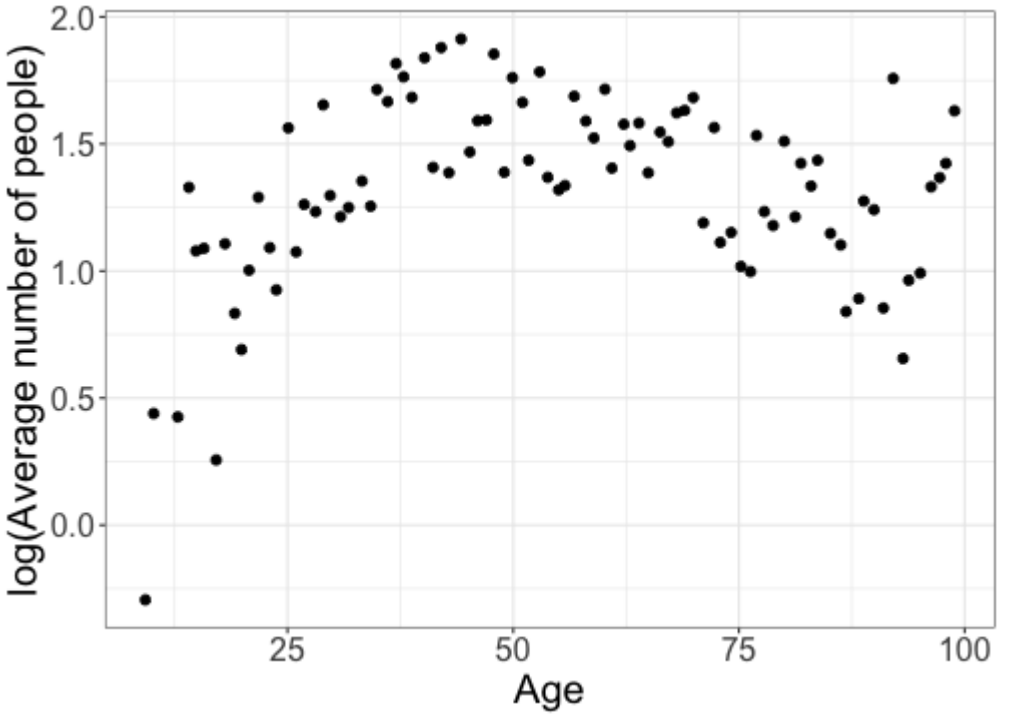
empirical log means plot, quantile residual plot

think about data generating process

compare mean & variance, histogram, quantile residual plots

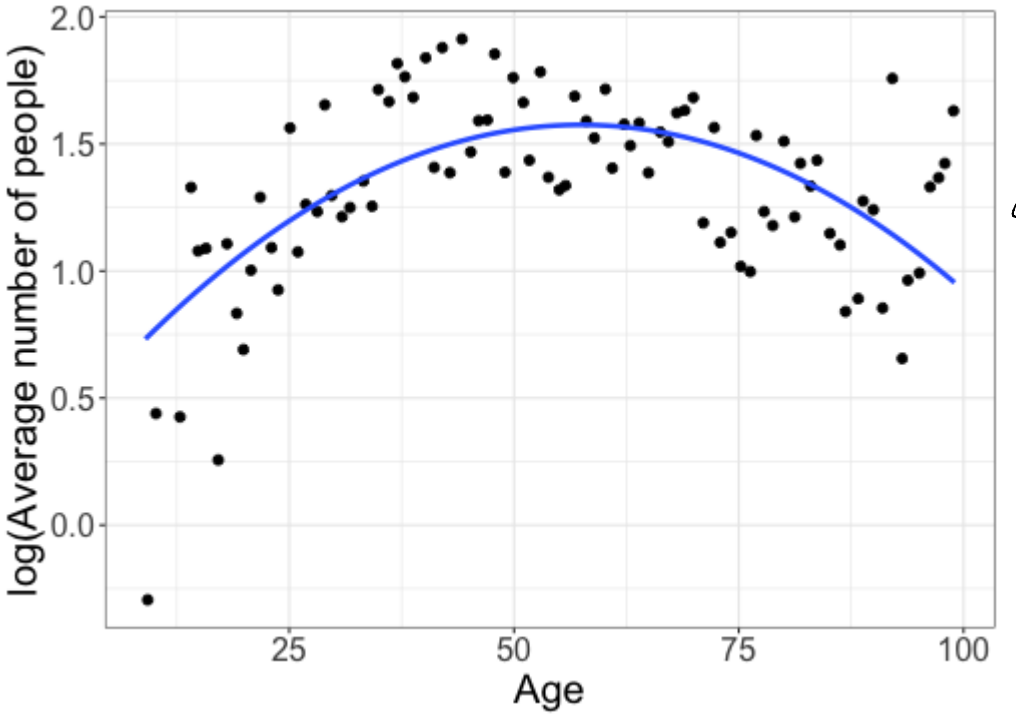
How could I assess these assumptions?

Shape: log empirical means plot



- Divide data into bins based on explanatory
- Calculate mean of Y in each bin
- Plot log(mean of Y) against X

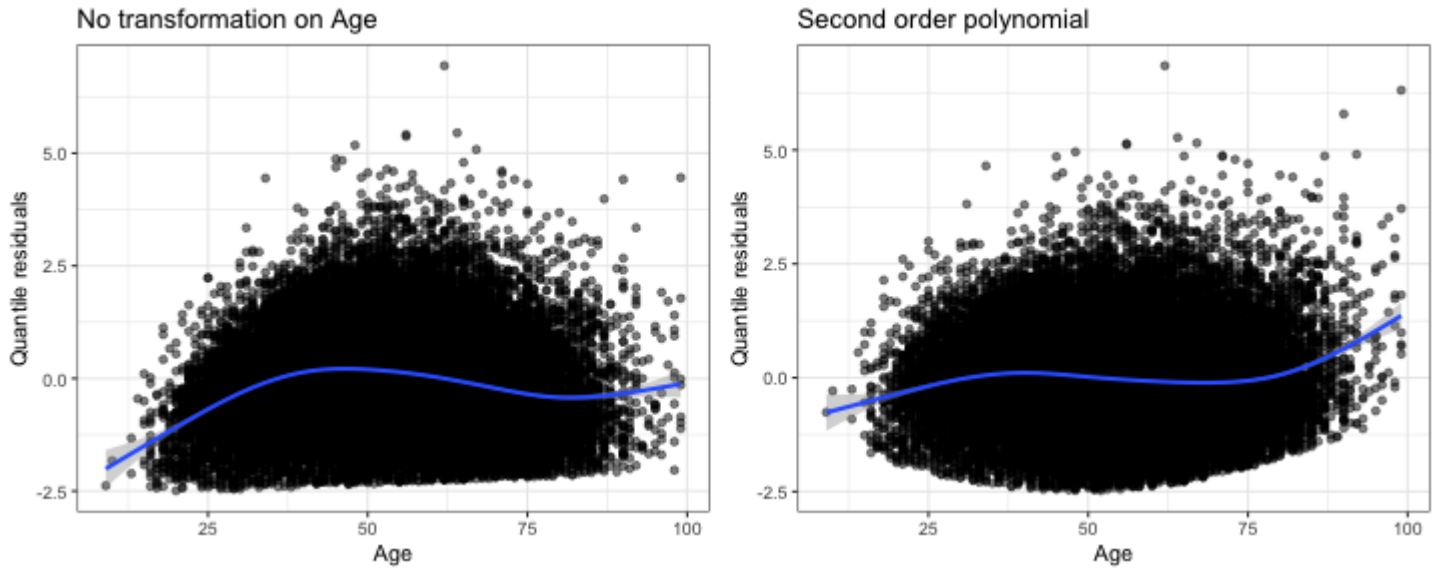
Shape: log empirical means plot



← second-order polynomial

Shape: quantile residual plot

```
m1 <- glm(total ~ age,
          data = fies, family = poisson)
m2 <- glm(total ~ poly(age, 2),
          data = fies, family = poisson)
```



- want residuals scattered around 0, no pattern

Class activity

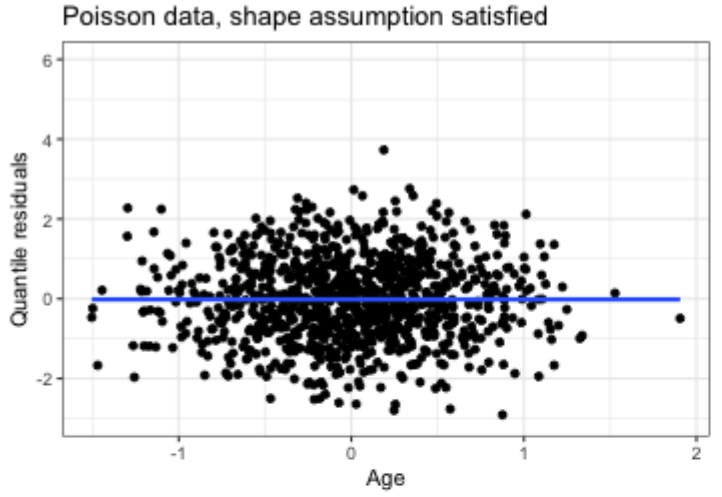
https://sta214-s23.github.io/class_activities/ca_lecture_19.html

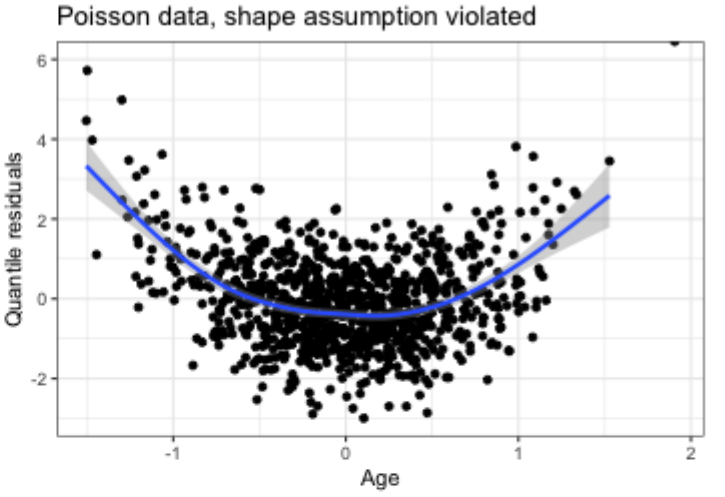
```
library(statmod)
```

Class activity

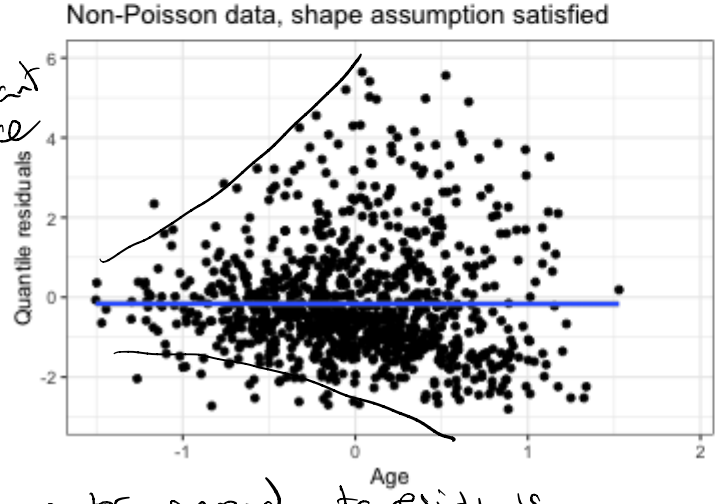
no pattern, scattered around 0, constant variance

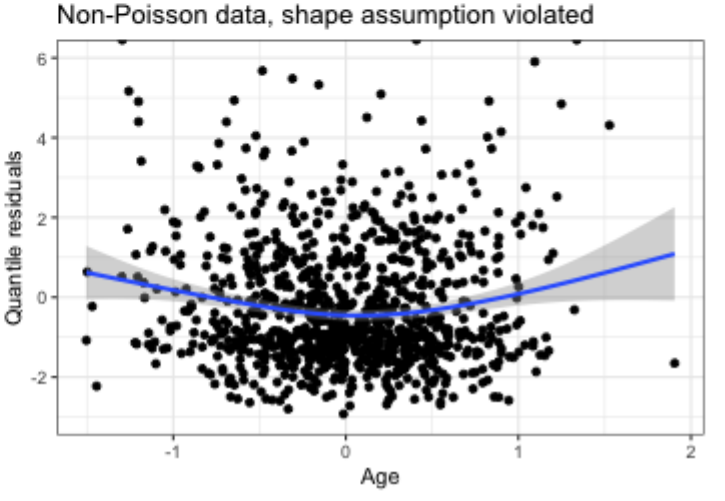
U-shaped pattern





mostly between -2 and 2





non-constant variance

U-shaped pattern

greater spread to residuals, residuals look a little skewed

Using quantile residual plots

We can use the quantile residual plot to assess the shape and distribution assumptions:

- Residuals mostly between -2 and 2, constant variance, no pattern => shape & Poisson assumptions are reasonable

- Residuals have greater spread, or nonconstant variance
  => violation of Poisson distribution assumption
  => consider adding more explanatory variables or use a different distribution

- Pattern in residuals
  => violation of shape assumption
  => consider transformations of explanatory variables

Another dataset

A concerned parent asks us to investigate crime rates on college campuses. We have access to data on 81 different colleges and universities in the US, including the following variables:

- `type`: college (C) or university (U)
- `nv`: the number of crimes for that institution in the given year
- `enroll1000`: the number of enrolled students, in thousands
- `region`: region of the US C = Central, MW = Midwest, NE = Northeast, SE = Southeast, SW = Southwest, and W = West)

Question

We want to know whether there are regional differences in the number of crimes on college campuses.

> What would be a reasonable model to investigate this question?

Model

$$Crimes_i \sim Poisson(\lambda_i)$$

$$\log(\lambda_i) = \beta_0 + \beta_1 MW_i + \beta_2 NE_i + \beta_3 SE_i + \beta_4 SW_i + \beta_5 W_i$$

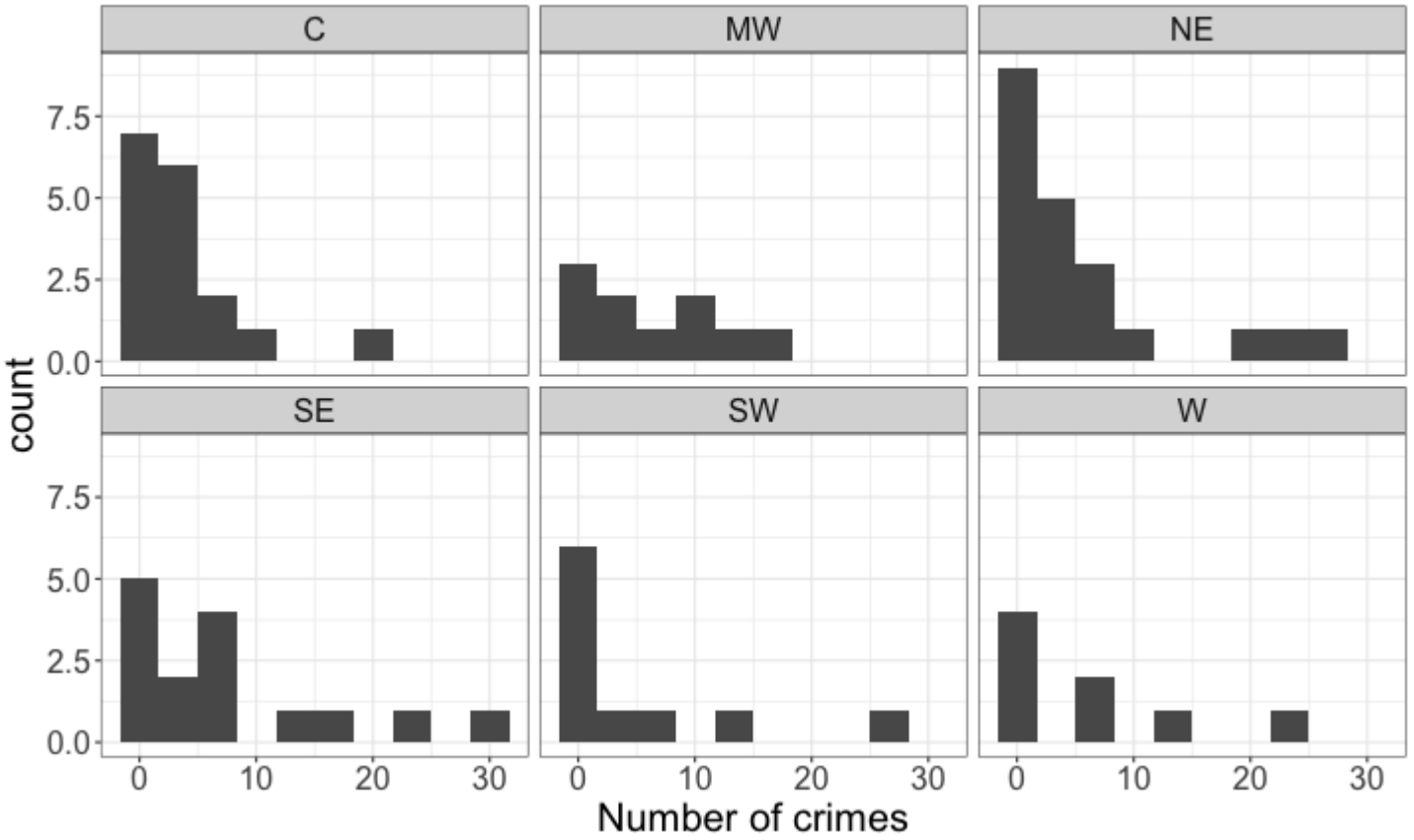
> What assumptions is this model making?

- Independence
- Poisson distribution

(Not really a shape assumption, b/c we only have a single categorical explanatory variable)

Exploratory data analysis

· unimodal, right-skewed count data
⇒ Poisson could be reasonable



Does it look reasonable to assume a Poisson distribution for the response?

Exploratory data analysis

```
crimes %>%
  group_by(region) %>%
  summarize(mean_crimes = mean(nv),
            var_crimes = var(nv))
```

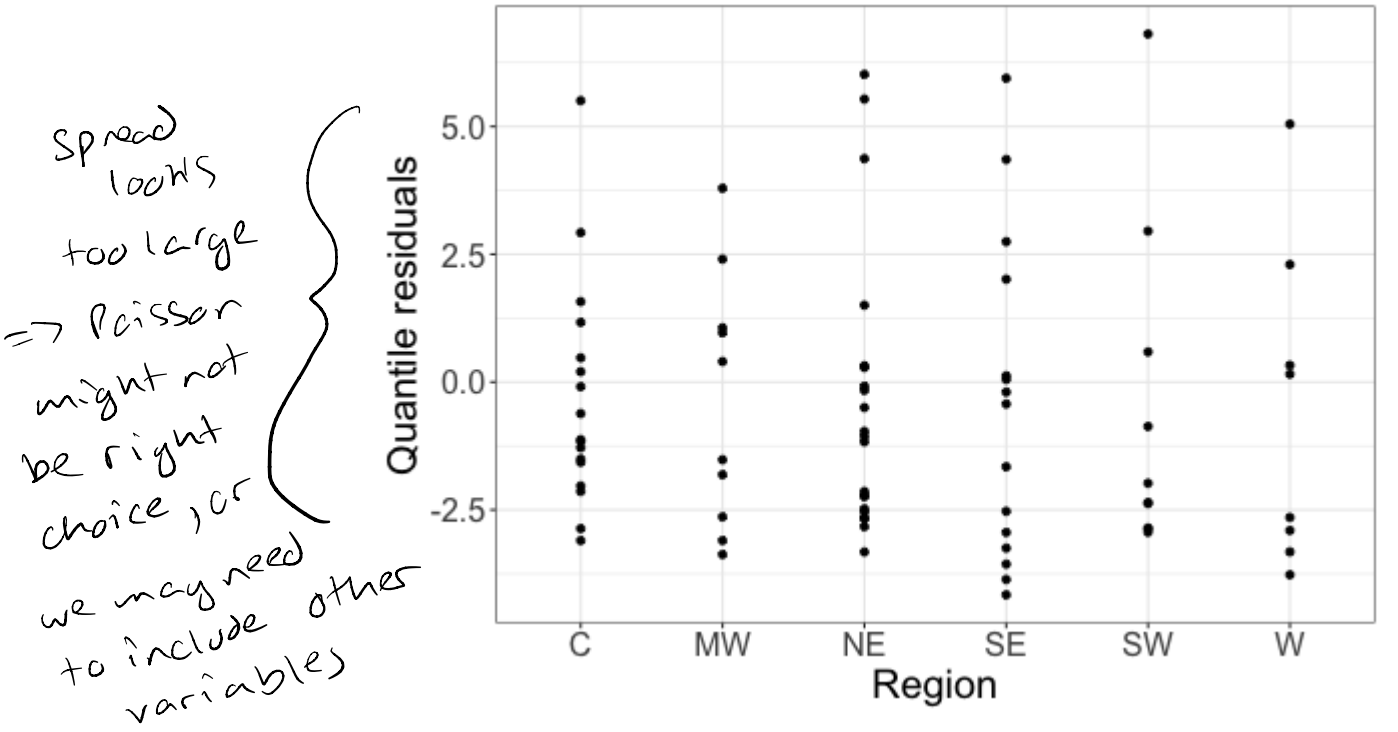
```
## # A tibble: 6 × 3
##   region mean_crimes var_crimes
##   <chr>        <dbl>      <dbl>
## 1 C             3.82       24.3
## 2 MW            6.2        37.1
## 3 NE            5.95       59.0
## 4 SE            8.27       84.4
## 5 SW            5.3        75.3
## 6 W             6.5        65.7
```

variances are too big, compared to the mean

> Does the Poisson distribution still seem reasonable?

Quantile residual plot

```
m1 <- glm(nv ~ region, data = crimes, family = poisson)
```



Goodness of fit

H_0: model is a good fit
H_A: model is not a good fit
Under H_0, residual deviance $\sim \chi^2_{n-p}$

Another way to assess whether our model is reasonable is with a *goodness of fit* test.

Goodness of fit test: If the model is a good fit for the data, then the residual deviance follows a χ^2 distribution with the same degrees of freedom as the residual deviance

```
...
##     Null deviance: 649.34  on 80  degrees of freedom
## Residual deviance: 621.24  on 75  degrees of freedom
...
```

$n-p$

n = # observations
p = # parameters

Residual deviance = 621.24, df = 75

> How likely is a residual deviance of 621.24 if our model is correct?

Goodness of fit

Goodness of fit test: If the model is a good fit for the data, then the residual deviance follows a χ^2 distribution with the same degrees of freedom as the residual deviance

Residual deviance = 621.24, df = 75

```
pchisq(621.24, df=75, lower.tail=F)
```

```
## [1] 5.844298e-87
```

So our model might not be a very good fit to the data.

> Why might our model not be a good fit?

Potential issues with our model

- The Poisson distribution might not be a good choice
- There may be additional factors related to the number of crimes which we are not including in the model

> Which other factors might be related to the number of crimes?

Offsets

We will account for school size by including an **offset** in the model:

$$\log(\lambda_i) = \beta_0 + \beta_1 MW_i + \beta_2 NE_i + \beta_3 SE_i + \beta_4 SW_i + \beta_5 W_i + \log(Enrollment_i)$$

Motivation for offsets

We can rewrite our regression model with the offset:

$$\log(\lambda_i) = \beta_0 + \beta_1 MW_i + \beta_2 NE_i + \beta_3 SE_i + \beta_4 SW_i + \beta_5 W_i + \log(Enrollment_i)$$

Fitting a model with an offset

```
m2 <- glm(nv ~ region, offset = log(enroll1000),
          data = crimes, family = poisson)
summary(m2)
```

```
...
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.30445    0.12403 -10.517  < 2e-16 ***
## regionMW     0.09754    0.17752   0.549  0.58270
## regionNE     0.76268    0.15292   4.987 6.12e-07 ***
## regionSE     0.87237    0.15313   5.697 1.22e-08 ***
## regionSW     0.50708    0.18507   2.740  0.00615 **
## regionW      0.20934    0.18605   1.125  0.26053
...
```

- The offset doesn't show up in the output (because we're not estimating a coefficient for it)

Fitting a model with an offset

$$\log(\widehat{\lambda}_i) = -1.30 + 0.10MW_i + 0.76NE_i + 0.87SE_i + 0.51SW_i + 0.21W_i + \log(Enrollment_i)$$

How would I interpret the intercept -1.30?

When to use offsets

Offsets are useful in Poisson regression when our counts come from groups of very different sizes (e.g., different numbers of students on a college campus). The offset lets us interpret model coefficients in terms of rates instead of raw counts.

> With your neighbor, brainstorm some other data scenarios where our response is a count variable, and an offset would be useful. What would our offset be?