Simulation and parametric bootstrap

- Project 2 due today
- HW 10 due wednesday
  - optional
  - will replace lowest HW 1-9 grade

- Today: wrap up parametric bootstrapping with mixed effects models
  + course evals
- Wednesday: Final exam review

Data and goal

Data on 497 performances by 37 undergraduate music majors (between 2 and 15 performances were measured for each musician). Each row in the data represents one performance:

- `id`: a unique identifier for the musician
- `na`: negative affect score (a measure of anxiety)
- `large`: whether the musician was performing as part of a large ensemble (large = 1), or as part of a small ensemble or solo (large = 0)
- `audience`: who attended (Instructor, Public, Students, or Juried)

Research question: Is there a difference in anxiety between large and small ensemble performances, after accounting for audience type, and accounting for systematic variation between musicians?

Models

Full model:

$$\begin{aligned} Anxiety_{ij} = \beta_0 &+ \beta_1\ JuriedPerformance_{ij} + \beta_2\ PublicPerformance_{ij} \\ &+ \beta_3\ StudentPerformance_{ij} + \beta_4\ LargeEnsemble_{ij} + u_i + \varepsilon_{ij} \end{aligned}$$

random effect for i^{th} musician

$u_i \overset{iid}{\sim} N(0, \sigma_u^2), \varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2).$

> We want to test whether there is a difference between large and small ensemble performances. What is the reduced model?

Models

Full model:

$$
\begin{aligned}
Anxiety_{ij} = \beta_0 &+ \beta_1\, JuriedPerformance_{ij} + \beta_2\, PublicPerformance_{ij} \\
&+ \beta_3\, StudentPerformance_{ij} + \beta_4\, LargeEnsemble_{ij} + u_i + \varepsilon_{ij}
\end{aligned}
$$

$u_i \overset{iid}{\sim} N(0, \sigma_u^2), \varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2).$

Reduced model:

$$
\begin{aligned}
Anxiety_{ij} = \beta_0 &+ \beta_1\, JuriedPerformance_{ij} + \beta_2\, PublicPerformance_{ij} \\
&+ \beta_3\, StudentPerformance_{ij} + u_i + \varepsilon_{ij}
\end{aligned}
$$

$u_i \overset{iid}{\sim} N(0, \sigma_u^2), \varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$

still have random effect u_i in the reduced model!

Fitting the models

```
m1 <- lmer(na ~ audience + large + (1|id),
           data = music)
m0 <- lmer(na ~ audience + (1|id), data = music)
```

> What test statistic should I calculate to compare the models?

Likelihood ratio test!

test stat $= 2(\log L_{full} - \log L_{reduced})$

LRT

```
m1 <- lmer(na ~ audience + large + (1|id),
           data = music)
m0 <- lmer(na ~ audience + (1|id), data = music)
```

Likelihood ratio test statistic:

```
as.numeric(2*(summary(m1)$logLik -
                summary(m0)$logLik))
```

```
## [1] 12.459
```

want to know whether 12.46 is unusual if H_0 is true

usually want to use a χ^2 distribution, but for mixed effects models a χ^2 distribution is only an approximation

Parametric bootstrapping

Observed test statistic: 12.46

> How would I use parametric bootstrapping to calculate a p-value for this test statistic?

1. Simulate data from reduced model
2. Fit full & reduced models on simulated data; calculate LRT statistic
3. Repeat 1-2 many times! (approximating null distribution of test stat.)
4. Compare observed test statistic to simulated test statistics

Simulating from the reduced model

```
summary(m0)
```

```
...
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  id       (Intercept)  5.599   2.366
##  Residual             20.852   4.566
## Number of obs: 497, groups:  id, 37
##
## Fixed effects:
##                                Estimate Std. Error t value
## (Intercept)                     14.9288     0.5560  26.849
## audienceJuried Recital           3.8268     0.8183   4.677
## audiencePublic Performance       0.9454     0.5452   1.734
## audienceStudent(s)               2.9242     0.6246   4.682
...
```

$\hat{\sigma}_u^2$

$\hat{\sigma}_\varepsilon^2$

$\hat{\beta}$s

$u_i^* \sim N(0, \hat{\sigma}_u^2) \qquad \varepsilon_{ij}^* \sim N(0, \hat{\sigma}_\varepsilon^2)$

$\text{Anxiety}_{ij}^* = \hat{\beta}_0 + \hat{\beta}_1 \text{Juried}_{ij} + \hat{\beta}_2 \text{Public}_{ij} + \hat{\beta}_3 \text{Student}_{ij} + u_i^* + \varepsilon_{ij}^*$

Simulating from the reduced model

new random effects u_i^* 37 groups (musician)

```
re_new <- rnorm(n = 37, mean = 0,
                sd = sqrt(5.60))
```

$\hat{\sigma}_u^2 = 5.6$

ε_{ij}^*

```
noise_new <- rnorm(n = 497, mean = 0,
                   sd = sqrt(20.85))
```

497 rows $20.85 = \hat{\sigma}_\varepsilon^2$

```
fitted_values <- predict(m0, re.form=NA)
```

quick way to calculate $\hat{\beta}_0 + \hat{\beta}_1 \text{Juried}_{ij} + \dots + \hat{\beta}_3 \text{Student}_{ij}$

```
re_data <- data.frame(id = unique(music$id),
                      re = re_new) %>%
  right_join(dplyr::select(music, id), by = "id")

new_data <- data.frame(id = music$id,
                       audience = music$audience,
                       large = music$large,
                       na = fitted_values +
                         re_data$re +
                         noise_new)
```

u_i^*

Anxiety_{ij}^* ε_{ij}^*

Calculate a test statistic with simulated data

> How do I calculate a test statistic using the simulated data?

Calculate a test statistic with simulated data

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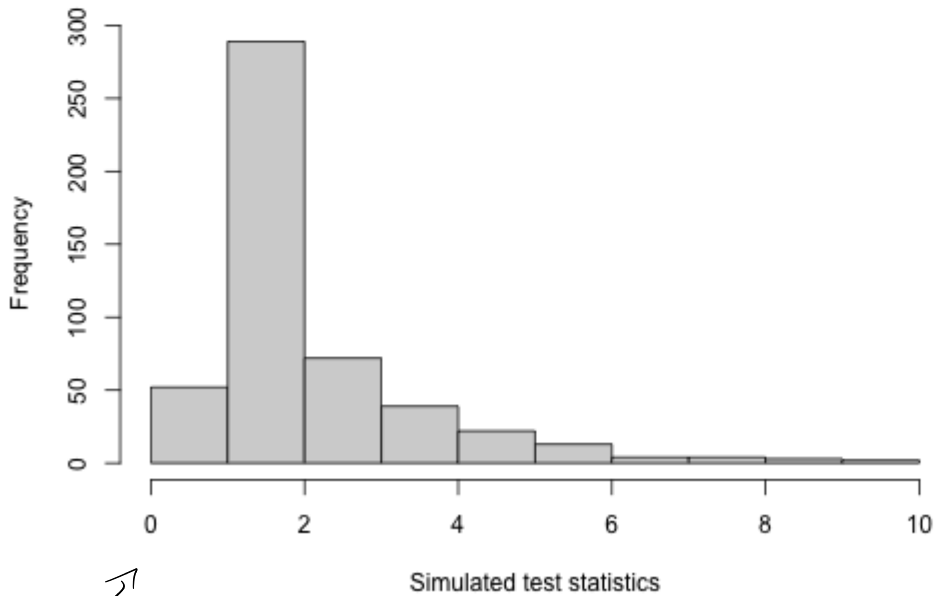
```
m1_sim <- lmer(na ~ audience + large + (1|id),
               data = new_data)
m0_sim <- lmer(na ~ audience + (1|id),
               data = new_data)
```

simulated data

```
as.numeric(2*(summary(m1_sim)$logLik -
                summary(m0_sim)$logLik))
```

```
## [1] 1.128246
```

Repeat many times!



12

12.46

Approximating null distribution of LRT statistic

p-value ≈ 0