Wald Test: use $N(0,1)$

$$\frac{\hat{\beta} - b}{SE_{\hat{\beta}}} \approx N(0,1) \quad \text{(under } H_0\text{)}$$

$H_0: \beta = b$

$H_a: \beta \neq b$

$\beta > b$

etc.

(In R:)

	Estimate	Std Error
(Intercept)		
variable	$\hat{\beta}$	$SE_{\hat{\beta}}$

on question

1) write H_0 & H_A
2) calculate $z = \dfrac{\hat{\beta} - b}{SE_{\hat{\beta}}}$
3) p-value

Likelihood ratio test (LRT)
(aka Drop in deviance test)

H_0: $\beta_1 = \beta_2 = \beta_3 = 0$

H_A: at least one of $\beta_1, \beta_2, \beta_3 \neq 0$

G = deviance for reduced model
— deviance for full model

Under H_0:
$$G \sim \chi^2_{\text{\#parameters tested}}$$

R output: compare residual deviances
for full & reduced models

To know about nested F test for LMMs:

1) Test fixed effects (βs) for full & reduced models

2) upper & lower bounds on denominater df

Poisson regression:

$$Y_i \sim \text{Poisson}(\lambda_i)$$

$$\log(\lambda_i) = \beta_0 + \beta_1 X_i$$

$$\text{Mean}(Y_i) = \text{Variance}(Y_i) = \lambda_i$$

$$\Rightarrow \quad \phi = \frac{\text{Variance}(Y_i)}{\text{Mean}(Y_i)} = 1$$

$\Rightarrow$ we assume $\phi = 1$

But what if $\phi > 1$?

Quasi-Poisson:

$$\text{Mean}(Y_i) = \lambda_i$$

$$\text{variance}(Y_i) = \phi \lambda_i \quad \text{(linear function of } \lambda_i\text{)}$$

$$\text{St. Dev.}(Y_i) = \sqrt{\hat{\phi} \lambda_i}$$

we can estimate ϕ, & use estimate to adjust SEs:

$$SE_{\text{quasiPoisson}} = \underbrace{\sqrt{\hat{\phi}}}_{>1} \cdot SE_{\text{Poisson}}$$

$\Rightarrow$ CIs wider,
test statistics smaller,
p-values are larger,

pro: easy to interpret, same $\hat{\beta}$s as Poisson
con: uses quasi-likelihood

Negative Binomial

$$Y_i \sim NB(\theta, p_i)$$

Mean of Y_i = $\dfrac{p_i \theta}{1-p_i} = \mu_i$

Variance of Y_i = $\mu_i + \dfrac{\mu_i^2}{\theta}$

(quadratic function of mean)

$$\log(\mu_i) = \beta_0 + \beta_1 X_i$$

interpreted the same as in (quasi-) Poisson regression

Lab 9: $Y_i \sim NB(\theta, p)$

$$P(Y_i = y) = \frac{(y+\theta-1)!}{y!\,(\theta-1)!}(1-p)^{\theta} p^{y}$$

Observe $Y_1, \ldots, Y_n$. Want MLE $\hat{p}$

$$L(\hat{p}) = \prod_{i=1}^{n} \frac{(Y_i+\theta-1)!}{Y_i!\,(\theta-1)!}(1-\hat{p})^{\theta}\hat{p}^{Y_i}$$

$$\log L(\hat{p}) = \sum_{i=1}^{n}\left(\log\left(\frac{(Y_i+\theta-1)!}{Y_i!\,(\theta-1)!}\right) + \theta\log(1-\hat{p}) + Y_i\log(\hat{p})\right)$$

$$\frac{d}{d\hat{p}}\log L(\hat{p}) = \sum_{i=1}^{n}\left(0 - \frac{\theta}{1-\hat{p}} + \frac{Y_i}{\hat{p}}\right) \overset{set}{=} 0$$

$$\sum_{i=1}^{n}\frac{Y_i}{\hat{p}} = \sum_{i=1}^{n}\frac{\theta}{1-\hat{p}} = \frac{n\theta}{1-\hat{p}}$$

$$\sum_{i=1}^{n} Y_i = \frac{n\theta\hat{p}}{1-\hat{p}} \Rightarrow \frac{\hat{p}}{1-\hat{p}} = \frac{n\theta}{\sum_{i=1}^{n} Y_i}$$

$$\frac{\hat{p}}{1-\hat{p}} = \frac{n\theta}{\sum_{i=1}^{n} Y_i}$$

$$\Rightarrow \frac{1-\hat{p}}{\hat{p}} = \frac{\sum_{i=1}^{n} Y_i}{n\theta}$$

$$\Rightarrow \frac{1}{\hat{p}} = \frac{\sum_{i=1}^{n} Y_i}{n\theta} + 1 = \frac{\sum_{i=1}^{n} Y_i + n\theta}{n\theta}$$

$$\Rightarrow \hat{p} = \frac{n\theta}{n\theta + \sum_{i=1}^{n} Y_i}$$