STA221

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Last edited: 2017-01-11 14:57

t distributions

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We usually don't know σ, but we can estimate it from the data using s, but then:

$$\frac{\overline{X} - \mu}{s/\sqrt{n}} \sim t_{n-1}$$

$n - 1$ is called "degrees of freedom".

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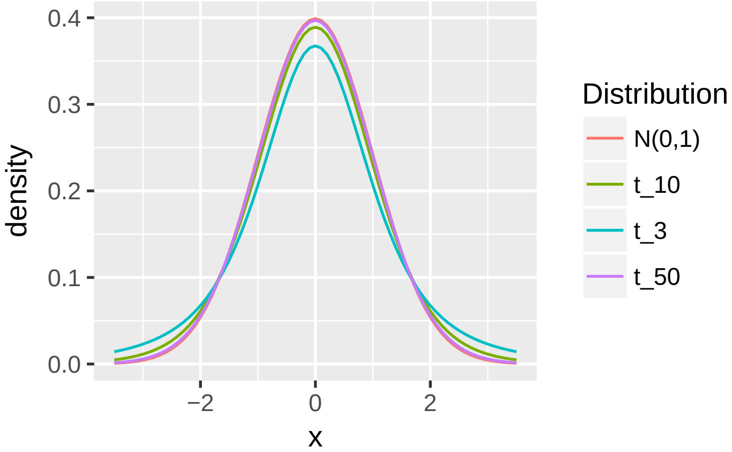
The phrase "degrees of freedom" comes from the realization that *given the value of* $\overline{x}$ the following list of number is redundant:

$$\{x_1, x_2, x_3, \ldots, x_n\}$$

From *any* $n-1$ of them, along with $\overline{x}$, you could calculate the missing value.

t distributions - II

The t distributions are (another) family of symmetric and bell-shaped distributions that look very much like $N(0, 1)$ distributions.



estimation - confidence intervals

From the following:

$$\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1) \qquad \text{and} \qquad \frac{\overline{X}-\mu}{s/\sqrt{n}} \sim t_{n-1}$$

which are approximately true for "large enough" n we get the usual 95% confidence intervals:

$$\overline{X} \pm 1.96\frac{\sigma}{\sqrt{n}} \qquad \text{and} \qquad \overline{X} \pm \text{``2''}\frac{s}{\sqrt{n}}$$

I put "2" because the value (for a 95% interval) is always close to 2.

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Modern inference is done using “p-values”, which are defined as *the probability of observing a summary of the data that is more extreme than what was observed.*

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Required reading: the ASA Statement on Statistical Significance and P-Values (pdf with lecture materials.)

example ("eye drops")

Which eye drop (A or B) for pupil dilation wears off faster?

40 people are each given both eye drops on different days. The wear-out times are recorded for each person.

```
## # A tibble: 40 × 3
##          A        B Difference
##      <dbl>    <dbl>      <dbl>
## 1 107.4709 115.8900  -8.419056
## 2 123.6729 128.5384  -4.865533
## 3 103.2874 146.1660 -42.878535
## 4 151.9056 189.1721 -37.266528
## 5 126.5902 114.0399  12.550228
## # ... with 35 more rows
```

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Mean and standard deviation of `Difference` are:

x-bar	sd
-19.81	37.03

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The "standard error" of $\overline{x}$ is $s/\sqrt{n} = 5.8554612$

the t test in R

```
##
##  One Sample t-test
##
## data:  eyedrops$Difference
## t = -3.3833, df = 39, p-value = 0.001642
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -31.654462  -7.966885
## sample estimates:
## mean of x
## -19.81067
```