STA221

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Last edited: 2017-01-31 13:48

regression

linear models

Basic model: Output $=$ Input $+$ Noise

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The "one sample t-test" can be thought of a way to analyze data that can be modeled as:

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where ε_i are independent $N(0, \sigma)$ and n is the sample size.

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The "two sample t-test" could be modeled as:

$$Y_{ij} = \mu_i + \varepsilon_{ij}$$

where $i = 1, 2$ and the μ_i are the two population means. (There are a few ways to treat the ε_{ij}.)

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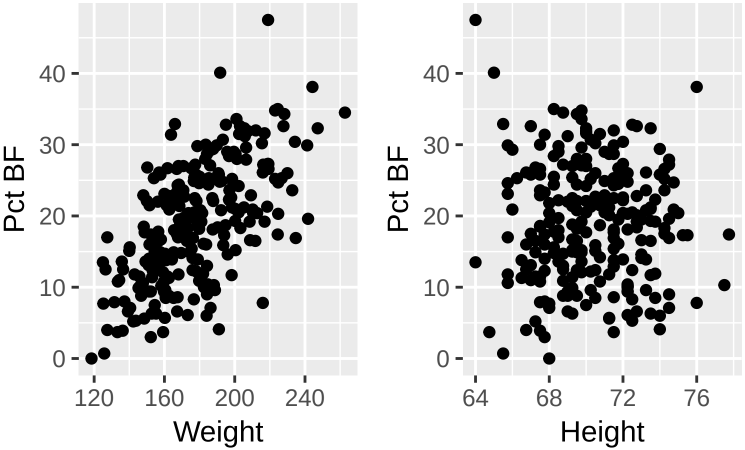
Let's consider the male body fat dataset that is discussed in the textbook (Chapter 24).

```
## # A tibble: 250 × 15
##   `Pct BF`   Age Weight Height  Neck Chest Abdomen    waist   Hip
##      <dbl> <int>  <dbl>  <dbl> <dbl> <dbl>   <dbl>    <dbl> <dbl>
## 1     12.3    23 154.25  67.75  36.2  93.1    85.2 33.54331  94.5
## 2      6.1    22 173.25  72.25  38.5  93.6    83.0 32.67717  98.7
## 3     25.3    22 154.00  66.25  34.0  95.8    87.9 34.60630  99.2
## 4     10.4    26 184.75  72.25  37.4 101.8    86.4 34.01575 101.2
## 5     28.7    24 184.25  71.25  34.4  97.3   100.0 39.37008 101.9
## # ... with 245 more rows, and 6 more variables: Thigh <dbl>,
## #   Knee <dbl>, Ankle <dbl>, Bicep <dbl>, Forearm <dbl>, Wrist <dbl>
```

body fat EDA


Pct BF
40
30
20
10
0
120
160
200
240
Weight
Pct BF
40
30
20
10
0
64
68
72
76
Height

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In general:

$$y = \beta_0 + \beta_1 x + \varepsilon$$

where y and x are the variables and ε is the random noise.

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When there are only two variables this is called a *simple regression model.*

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The *parameter* β_1 is the slope of the line and is of primary interest. (The parameter β_0 is the y-intercept and not normally of any interest.)

model details; terminology

y and x are *not* interchangeable; i.e. these are completely different:

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- It could be a pre-specified grid of values.
- The "grid" could consist of as few as two values!

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We'll add another requirement when the time comes.

estimating the slope (and intercept)

Now we proceed through the typical steps of a data analysis (the χ^2 procedures were an exception!)

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The classic method of regression parameter estimation given data is called *least squares regression*.

- The data come in pairs $(y_1, x_1), (y_2, x_2), \ldots (y_n, x_n)$.

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- The data come in pairs $(y_1, x_1), (y_2, x_2), \ldots (y_n, x_n)$.
- For any "candidate" slope b_0^* and intercept b_1^* we could construct the set of "predictions" $\hat{y}_i = b_0^* + b_1^* x_i$ and their "residuals" $\varepsilon_i = y_i - \hat{y}_i$

more least squares

Here's the actual "least squares" part...

It is possible to find the unique slope and intercept that makes this sum of squared residuals:

$$\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - (b_0^* + b_1^* y_i))^2$$

as small as possible.

We'll call the unique intercept and slope b_0 and b_1, respectively.

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The formula for the slope estimator b_1 turns out to be (corrected from class!):

$$b_1 = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{\sum (x_i - \overline{x})^2} = \frac{S_{xy}}{S_{xx}}$$

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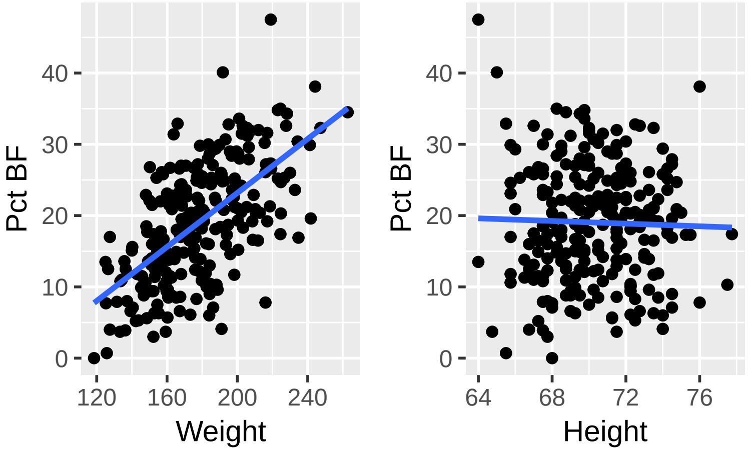
The formula for the intercept is $b_0 = \overline{y} - b_1 \overline{x}$

body fat examples

Here are the plots with the least squares regression lines added:



bodyfat calculation examples

Obviously don't do these by hand! Here is basic R regression output:

```
##
## Call:
## lm(formula = `Pct BF` ~ Weight, data = bodyfat)
##
## Coefficients:
## (Intercept)       Weight
##    -14.6931       0.1894
```

```
##
## Call:
## lm(formula = `Pct BF` ~ Height, data = bodyfat)
##
## Coefficients:
## (Intercept)       Height
##    25.58078     -0.09316
```