STA221

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Last edited: 2017-03-08 15:05

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Recall: a lower α gives a *wider* confidence interval. In the *t* case the full formula is:

$$(\overline{y}_i - \overline{y}_j) \pm t_{N-k,\alpha/2}\sqrt{MSE}\sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

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Definition: the *experimentwise error rate* is the probability of *any* Type I Errors among all tests done on the dataset from one experiment.

bounding the experimentwise error rate - I

Suppose we're going to do m hypothesis tests on a dataset.

Denote by $A_1, A_2, \ldots, A_m$ the events where A_i means "a Type I Error occurred when hypothesis test i took place", and $P(A_i) = \alpha$.

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This implies $P(A \cup B) \leq P(A) + P(B)$. You can extend this to any number of events, i.e.:

$$P(A_1 \cup A_2 \cup \cdots \cup A_m) \leq P(A_1) + P(A_2) + \cdots + P(A_m)$$

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How could the individual tests all be adjusted so that $\alpha^* = \alpha$?

An easy method uses:

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It's not a bad idea to apply a Bonferroni correct to any situation in which you are subjecting a dataset to lots of hypothesis tests.

full example, including some pairwise comparisons

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Suppose `List4` was some sort of "default list", and the other three lists were new word lists being evaluated. So it will be particularly interesting to investigate these three pairwise differences:

$$\mu_1 - \mu_4$$
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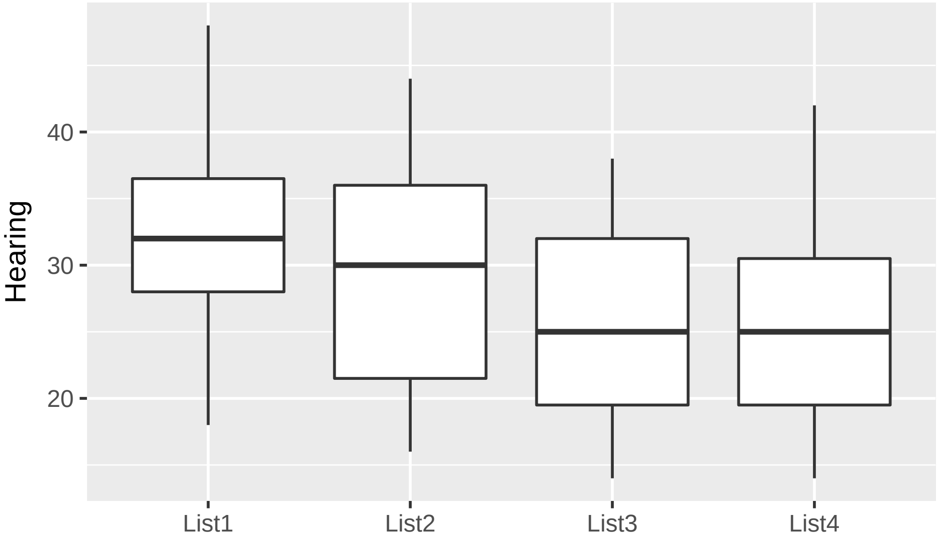
We will fix the experimentwise error rate at $\alpha = 0.05$ for the multiple comparisons.

hearing full example - I

First, look at a plot:



hearing full example - II

Next, verify the model assumptions starting with Levene's test:

```
## Levene's Test for Homogeneity of Variance (center = median)
##       Df F value Pr(>F)
## group  3  0.5701 0.6361
##       92
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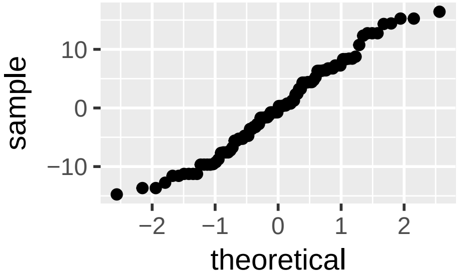
. . . followed by the normal quantile plot of the residuals:



hearing full example - III

Next we do the overall ANOVA F test:

```
##              Df Sum Sq Mean Sq F value  Pr(>F)
## ListID        3    920  306.82   4.919 0.00325
## Residuals    92   5738   62.37
```

hearing full example - III

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```

And since the p-value is low **we may proceed with the pairwise comparisons.**

hearing full example - IV

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To make the three confidence intervals we need the estimated mean differences and the group sample sizes:

```
## # A tibble: 4 × 3
##   ListID     n     mean
##   <fctr> <int>    <dbl>
## 1  List1    24 32.75000
## 2  List2    24 29.66667
## 3  List3    24 25.25000
## 4  List4    24 25.58333
```

We are doing three comparisons at an experimentwise error rate of 0.05, so we'll produce the $(1 - 0.05/3) \cdot 100\% = 98.33\%$ confidence intervals.

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The value of $t_{92,0.0167}$ is 2.1604869.

hearing full example - V

The three pairwise comparisons of interest can be made using these confidence intervals:

Comparison	Estimate	Margin of Error	Lower	Upper
$\mu_1 - \mu_4$	7.167	4.926	2.241	12.092
$\mu_2 - \mu_4$	4.083	4.926	-0.842	9.009
$\mu_3 - \mu_4$	-0.333	4.926	-5.259	4.592

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With k groups there will be $k(k-1)/2$ such comparisons.