

# STA221

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recap - pairwise comparisons

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If you plan to make  $m$  pairwise comparisons after “rejecting” the overall  $F$  test, you can report the following confidence intervals:

$$(\bar{y}_i - \bar{y}_j) \pm t_{N-k, \alpha/2m} \sqrt{MSE} \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

where the usual  $\alpha/2$  (which itself is usually 0.025 for a 95% interval) has been subjected to a Bonferroni correction to maintain the desired experimentwise error rate.

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where the usual  $\alpha/2$  (which itself is usually 0.025 for a 95% interval) has been subjected to a Bonferroni correction to maintain the desired experimentwise error rate.

The Bonferroni correction can also be used if you see an interesting pair or groups to compare only after the fact.

## post-hoc comparison trick

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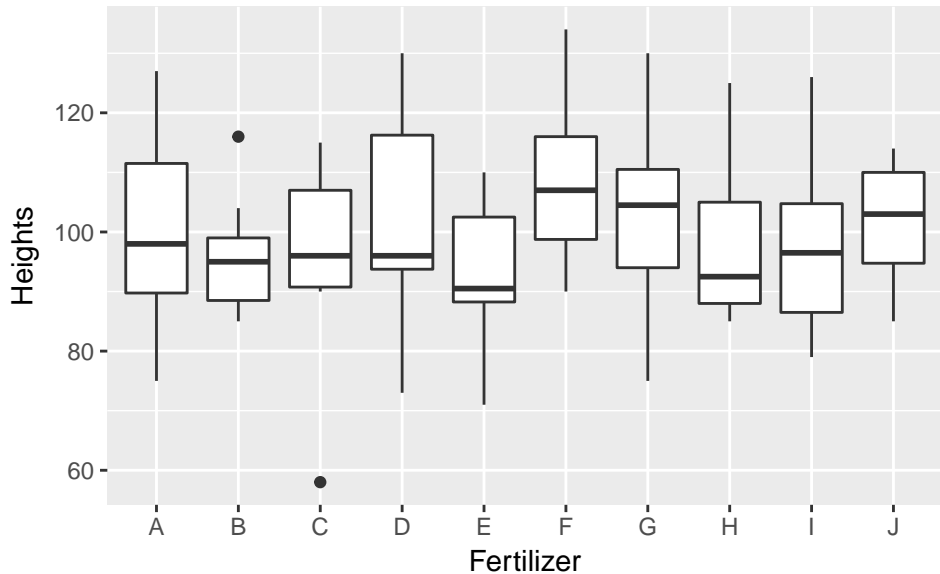
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Example using question 25.19 “Fertilizers”. There are  $k = 10$  fertilizers being compared with  $n = 10$  mung bean sprouts each. After a week, the bean heights are measured.



## “Fertilizers” example



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I need to run the ANOVA and verify the assumptions:

```
## Analysis of Variance Table
```

```
##
```

```
## Response: Heights
```

```
##           Df  Sum Sq Mean Sq F value Pr(>F)
```

```
## Fertilizer   9   2073.7   230.41   1.1882 0.3097
```

```
## Residuals  110  21331.1   193.92
```

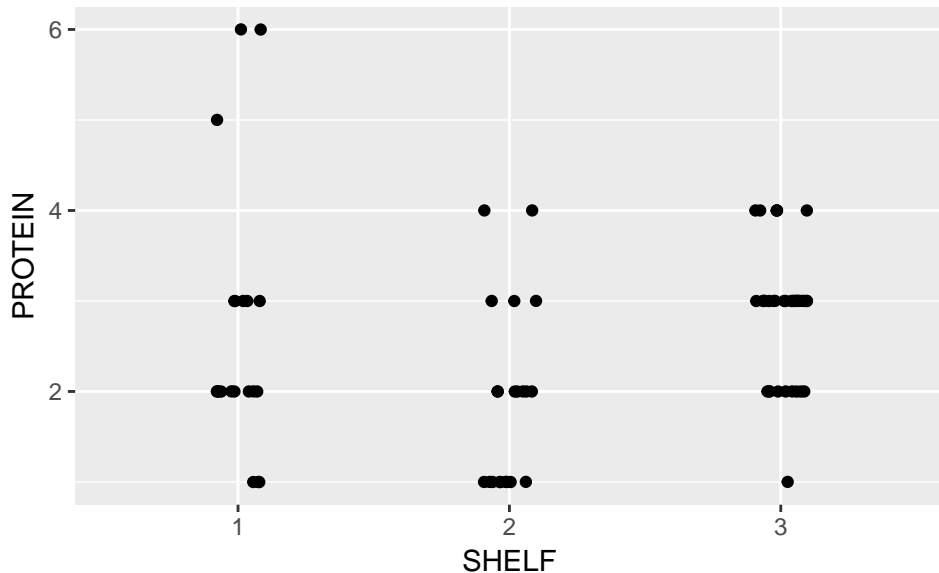
```
## Levene's Test for Homogeneity of Variance (center = median)
```

```
##           Df F value Pr(>F)
```

```
## group     9   0.7416 0.6701
```

```
##          110
```

OK, so then let's look at the "Cereals" data from Q25.21



## “Cereals redux”

I'd also like to see if there is a difference between shelves 2 and 3.

## “Cereals redux”

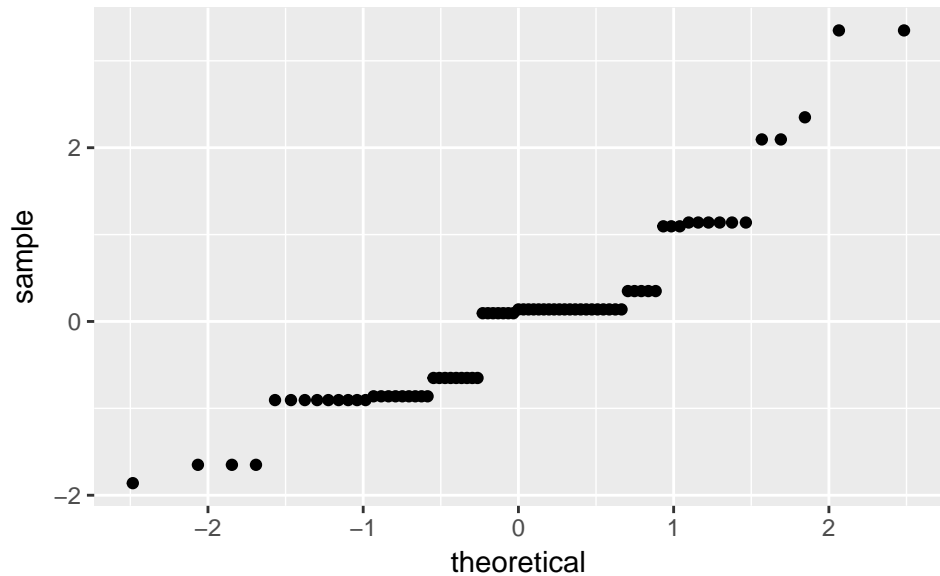
I'd also like to see if there is a difference between shelves 2 and 3.

Start with the analysis and assumption verification:

```
##              Df Sum Sq Mean Sq F value Pr(>F)
## SHELF         2  12.43   6.213   5.844 0.0044
## Residuals    74  78.67   1.063
```

```
## Levene's Test for Homogeneity of Variance (center = median)
##              Df F value Pr(>F)
## group      2  2.3446  0.103
##              74
```

## “Cereals redux”



## comparing shelves 2 and 3

There are 3 *possible* comparisons, so a Bonferroni correction with  $m = 3$  will be needed, even though I'm only actually doing one comparison.



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Computer says:  $t_{74,0.05/6} = 2.4496186$

```
##              Df Sum Sq Mean Sq F value Pr(>F)
## SHELF         2  12.43   6.213   5.844 0.0044
## Residuals    74  78.67   1.063
```

```
## # A tibble: 3 × 3
##   SHELF      n    mean
##   <fctr> <int>   <dbl>
## 1     1     20 2.650000
## 2     2     21 1.904762
## 3     3     36 2.861111
```

## “All pairwise comparisons”

Sometimes it is valuable to simply summarize all possible pairwise comparisons to determine which groups are the same and which are different.

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Sometimes it is valuable to simply summarize all possible pairwise comparisons to determine which groups are the same and which are different.

Here is an efficient algorithm for performing this task for cases when the group sample sizes are all the same (equal to some  $n$ ). Let's look at the Yeast example again.

```
##           Df Sum Sq Mean Sq F value    Pr(>F)
## Recipe      3 638968   212989    44.74 0.000000864
## Residuals   12  57128     4761
```

```
## # A tibble: 4 × 3
##   Recipe      n  mean
##   <fctr> <int>  <dbl>
## 1      D      4 183.75
## 2      B      4 196.25
## 3      A      4 486.25
## 4      C      4 656.00
```

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## “all pairwise” with Yeast

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##   Recipe      n   mean
##   <fctr> <int> <dbl>
## 1      D     4 183.75
## 2      B     4 196.25
## 3      A     4 486.25
## 4      C     4 656.00
```

The “margin of error” is 153.8204811.