STA221

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the analysis

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We'll call the factor variables `A` and `B`, with I and J levels respectively.

In the additive case we get, assuming a common sample size of n for each combination of factor levels (strongly encouraged in practice!):

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When the experiment is *balanced* (strongly encouraged!) with n in each combination of levels, the error degrees of freedom simplifies to $IJ(n - 1)$.

fire example - no interaction (?!)

```
##             Df Sum Sq Mean Sq F value    Pr(>F)
## Bath         1  715.3   715.3  23.954 0.0000131
## Launderings  1  202.1   202.1   6.769    0.0125
## Residuals   45 1343.8    29.9
```

sums of squares - a few details

The total sum of squares SS_{Total} is (as always) $(N-1)$ times the sample varianace of the response variable:

$$\sum_{i,j,k} \left(y_{ijk} - \overline{\overline{y}}\right)^2$$

The treatment sums of squares will be:

$$SS_A = nI \sum_i \left(\overline{y}_{i\cdot\cdot} - \overline{\overline{y}}\right)^2$$

$$SS_B = nJ \sum_j \left(\overline{y}_{\cdot j\cdot} - \overline{\overline{y}}\right)^2$$

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...which only makes sense when there is no interaction.

error sum of squares

For the sake of completeness:

$$SS_{Error} = \sum_{i,j,k} \left(y_{ijk} - \overline{y}_{ij\cdot} \right)^2$$

Note that $y_{ijk} - \overline{y}_{ij\cdot}$ is also called a “residual”.

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Mostly the same as with one treatment factor, with the same verification techniques.

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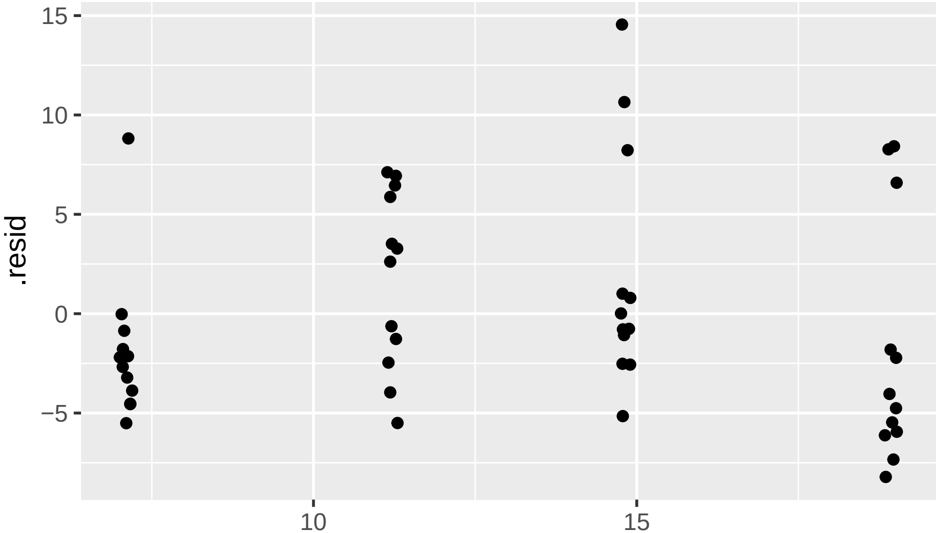
When $n = 1$, the lack of interaction is also an *assumption*.

fire retardant model assumptions - equal variance

Plot of residuals versus “fitted values” (in this case, just the group averages):

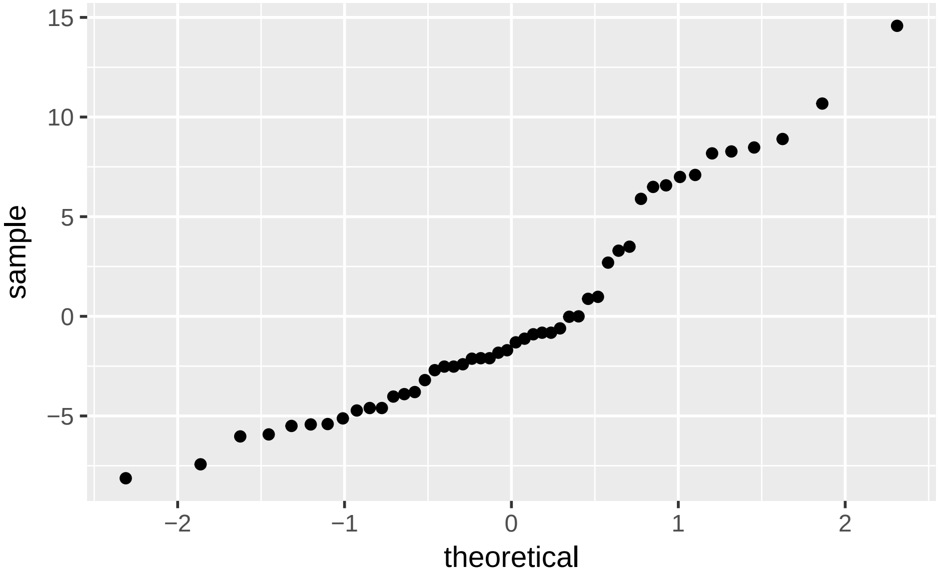


fire retardant model assumptions - equal variance

Since $n = 12$ Levene's test also works:

```
## Levene's Test for Homogeneity of Variance (center = median)
##       Df F value Pr(>F)
## group  3  0.7138  0.549
##       44
```

fire retardant model assumptions - normality



the general model and analysis (with interaction)

This model has the $(\tau\gamma)$ interaction term:

$$y_{ijk} = \mu + \tau_i + \gamma_j + (\tau\gamma)_{ij} + \varepsilon_{ijk} \qquad \text{general model}$$

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The interaction sum of squares is:

$$n \sum_{i,j} \left(y_{ijk} - \overline{y}_{i..} - \overline{y}_{.j.} + \overline{\overline{y}} \right)^2$$

Small when additive; large when not.

degrees of freedom - balanced case

$$SS_{Total} = SS_A + SS_B + SS_{AB} + SS_{Error}$$
$$(N-1) = (I-1) + (J-1) + (I-1)(J-1) + IJ(n-1)$$

Note: $IJ(n-1) = N - IJ$

We get (in addition):

$$\frac{SS_{AB}/(I-1)(J-1)}{SS_{Error}/IJ(n-1)} \sim F_{(I-1)(J-1),IJ(n-1)}$$

flame retardant with interaction

Flame retardant example, with interaction (and without):

```
##                    Df Sum Sq Mean Sq F value     Pr(>F)
## Bath                1  715.3   715.3  26.726 0.00000549
## Launderings         1  202.1   202.1   7.552    0.00866
## Bath:Launderings    1  166.1   166.1   6.207    0.01657
```