STA257

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joint cdf - continuous case

The joint cdf $F(x, y) = P(X \leq x, Y \leq y)$ can be calculated:

$$F(x, y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f(u, v)\, du\, dv$$

It is a mystery of multivariable calculus how to obtain f from F

"partial" derivatives crash course - I

Maybe you got this far in your co-requisite!

With a function $g(x, y)$ you can take the derivative with respect to one variable at a time, holding the other variable constant. Notation:

$$\frac{\partial}{\partial x} g(x, y) \quad \text{and} \quad \frac{\partial}{\partial y} g(x, y).$$

When g is "smooth" you get the nice result:

$$\frac{\partial}{\partial y}\left[\frac{\partial}{\partial x} g(x, y)\right] = \frac{\partial}{\partial x}\left[\frac{\partial}{\partial y} g(x, y)\right],$$

and we just call this:

$$\frac{\partial^2}{\partial x \partial y} g(x, y).$$

joint cdf to joint density

Just take all the "partial" derivatives in any order you like.

$$\frac{\partial^2}{\partial x \partial y} F(x, y) = f(x, y)$$

"Proof: …"

Examples can be challenging! Consider $f(x, y) = xy$ on $0 < x < 1$, $0 < y < 2$ (…to be revisited…)

marginal cdf and marginal density

Just like in the discrete case we can recover information about X and Y individually by "integrating out" the other variable. The marginal densities are:

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y)\, dx$$

What about the marginal cdfs?

Continue the example on the previous slide.

picture of F(x,y)

joint/marginal pdf example

Example D from the book.

$$f(x, y) = \begin{cases} \lambda^2 \exp(-\lambda y) & : 0 \leq x \leq y,\ \lambda > 0 \\ 0 & : \text{ otherwise.} \end{cases}$$

Exercise: review Example E from the book "Bivariate Normal". We will revisit this example.

independent random variables
(which the book gets a bit wrong)

recall some definitions and results

Events A and B are independent if $P(A \cap B) = P(A)P(B)$.

$A \perp B \iff A^c \perp B \iff A \perp B^c \iff A^c \perp B^c$

"Experiments" $\mathcal{E}_A = \{A_1, A_2, \dots\}$ and $\mathcal{E}_B = \{B_1, B_2, \dots\}$ are independent if $A_i \perp B_j$ for all $i,\ j$.

Definition: Random variables X and Y are *independent* if:

$$P(X \in A, Y \in B) = P(X \in A)P(Y \in B)$$

for any* $A, B \subset \mathbb{R}$.

the book's definition is actually a *theorem*

Theorem: $X \perp Y$ if and only if the joint cdf $F(x, y) = F_X(x)F_Y(y)$ is the product of the marginal cdfs.

Proof: $\Longleftarrow$ ("only if") too hard; $\Longrightarrow$ left as exercise.

Corollary: $X \perp Y$ if and only if the joint $f(x, y) = f_X(x)f_Y(y)$

To verify, in practice check two things:

1. The density factors.
2. The non-zero region is a rectangle (possibly infinite in either direction.)

examples

1. $f(x, y) = xy$ on $0 < x < 1$ and $0 < y < 2$.
2. $f(x, y) = \lambda^2 \exp(-\lambda y)$ on $0 < x < y < \infty$.
3. $f(x, y) = \frac{1}{2}\lambda^3 y \exp(-\lambda(x + y))$ on $x > 0$ and $y > 0$.