4.9 $Y$ is the demand, with pmf $p(k)$

$n$ is the number stocked.

$X(n)$ is the income

$W(n) = \min(Y, n)$ is the number <u>sold</u>

(cannot exceed $n$)

$$X(n) = -cn + s\,W(n)$$

Want to select $n$ that maximizes:

$$E(X(n)) = -cn + s\,E(W(n))$$

The p.m.f. of $W(n)$ is: $P(W(n)=k) = \begin{cases} p(k) : k \in \{0,\dots,n-1\} \\ \sum_{j \ge n} p(j) : k = n \end{cases}$

To find the maximum of a "discrete" function like $E(X(n))$ one needs to look at differences or ratios of successive terms.

We'll look at differences


Let's find where $E(X(n)) - E(X(n-1)) < 0$

$$E(X(n)) - E(X(n-1)) = -cn + s\,E(W(n)) - \left(-c(n-1) + s\,E(W(n-1))\right)$$

$$= -cn + cn - c + s\left[E(W(n)) - E(W(n-1))\right]$$

$$= -c + s\left[\underbrace{\sum_{k=0}^{n-1} k p(k)}_{1} + n \underbrace{\sum_{j \ge n} p(j)}_{3} - \underbrace{\sum_{k=0}^{n-2} k p(k)}_{2} - \underbrace{(n-1)}_{4\ 5} \sum_{j \ge n-1} p(j)\right]$$

$$= -C + S\left[\underbrace{(n-1)p(n-1)}_{1-2} + n\left[\underbrace{\sum_{j \ge n} p(j)}_{3} - \underbrace{\sum_{j \ge n-1} p(j)}_{4}\right] + \underbrace{\sum_{j \ge n-1} p(j)}_{5}\right]$$

$$= -C + S\left[np(n-1) - p(n-1) + n\left[-p(n-1)\right] + \sum_{j \ge n-1} p(j)\right]$$

$$= -C + S\left[-p(n-1) + \sum_{j \ge n-1} p(j)\right] = -C + S\left[\sum_{j \ge n} p(j)\right]$$

Rule: Pick smallest $n$ so that

$$-c + s\left[\sum_{j \geq n} p(j)\right] < 0$$

or $$\sum_{j \geq n} p(j) < \frac{c}{s}$$