3.46   X , Y   continuous. $Z = X - Y$

$$F_z(z) = P(Z \le z)$$

$$\boxed{dy \, dx \text{ version}}$$

$$= P(X - Y \le z) \quad \text{which is the integral of}$$

$$f(x, y) \quad \text{over} \quad x - y \le z :$$

$$= \int_{-\infty}^{\infty} \int_{x-z}^{\infty} f(x, y) \, dy \, dx$$
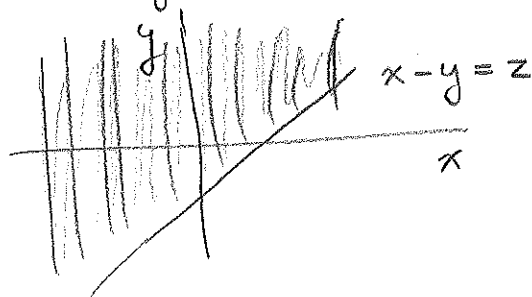
Change of variables:

$$y = x - u \qquad u = x - y$$
$$dy = - du$$

$$= \int_{-\infty}^{\infty} \int_{z}^{-\infty} f(x, x-u)(-1) \, du \, dx \qquad \left(\begin{array}{l} \text{as } y \text{ goes from} \\ x-z \text{ to } \infty, \quad u = x-y \\ \text{goes from } "z" \text{ to } -\infty \end{array}\right)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{z} f(x, x-u) \, du \, dx \qquad (\text{flip limits})$$

$$= \int_{-\infty}^{z} \int_{-\infty}^{\infty} f(x, x-u) \, du \, dx$$

So $\quad f_z(z) = \int_{-\infty}^{\infty} f(x, x-z) \, dx$