3.42a. $T \sim Exp(\lambda)$

$P(W = \pm 1) = 1/2 \qquad T \perp W$

$X = WT$. Note, X could take on any $-\infty < X < \infty$

$$F_X(x) = P(X \le x) = P(WT \le x)$$

$$= P(WT \le x \mid W = -1)P(W=-1) + P(WT \le x \mid W = 1)P(W=1)$$

$$= P(-T \le x)\cdot \tfrac{1}{2} + P(T \le x)\cdot\tfrac{1}{2}$$

$$= P(T \ge -x)\cdot\tfrac{1}{2} + P(T \le x)\cdot\tfrac{1}{2}$$

Case ① $x < 0$

$$F_X(x) = P(T \ge -x)\cdot\tfrac{1}{2} + 0\cdot\tfrac{1}{2}$$

$$= \frac{1}{2}\int_{-x}^{\infty} \lambda e^{-\lambda t}\,dt = \frac{1}{2}\left[-e^{-\lambda t}\right]_{-x}^{\infty} = \frac{1}{2}e^{-\lambda(-x)}$$

Case ② $x \ge 0$

$$F_X(x) = 1\cdot\tfrac{1}{2} + P(T \le x)\cdot\tfrac{1}{2}$$

$$= \frac{1}{2} + \frac{1}{2}\left(1 - e^{-\lambda x}\right)$$

So: $$F_X(x) = \begin{cases} \frac{1}{2}e^{-\lambda(-x)} & : x < 0 \\ \frac{1}{2} + \frac{1}{2}(1 - e^{-\lambda x}) & : x \ge 0 \end{cases}$$

and $$f_X(x) = \frac{d}{dx}F_X(x) = \begin{cases} \frac{1}{2}\lambda e^{-\lambda(-x)} & : x < 0 \\ \frac{1}{2}\lambda e^{-\lambda(x)} & : x \ge 0 \end{cases}$$

which can be succinctly written as:

$$f_X(x) = \frac{1}{2}\lambda e^{-\lambda|x|}$$