4.13 "Direct" is also not so easy.

$$\int_0^\infty [1-F(x)]\,dx$$

Parts: $u = 1 - F(x) = P(X>x)$

$du = -f(x)$

$dv = dx$

$v = x$

$$= \left[x P(X>x)\right]_0^\infty + \int_0^\infty x f(x)\,dx$$

$$= \left[x P(X>x)\right]_0^\infty + E(X)$$

Clearly $\lim_{x\to 0} x P(X>x) = 0$.

But $\lim_{x\to\infty} x P(X>x) = 0$ is not so clear!

Since $E(X)=\mu$ exists we can write, for any $x>0$:

$$\mu = \int_0^\infty u f(u)\,du = \int_0^x u f(u)\,du + \int_x^\infty u f(u)\,du$$

so $\int_x^\infty u f(u)\,du$ exists and is finite.

Also $\int_x^\infty u f(u)\,du \geq \int_x^\infty x f(u)\,du$ (replace u with its lower limit x)

$$= x \int_x^\infty f(u)\,du = x P(X>x)$$

Finally $\lim_{x\to\infty} x P(X>x) \leq \lim_{x\to\infty} \int_x^\infty u f(u)\,du = 0$.