

3.46

 X, Y

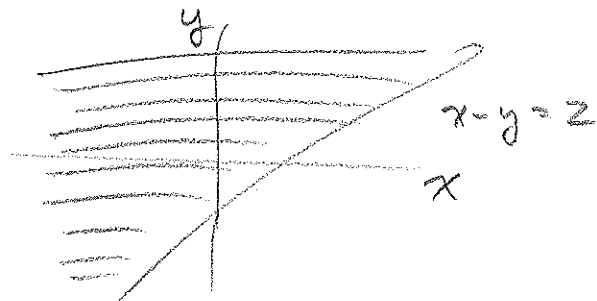
continuous

 $Z = X - Y$

$dx dy$ version

$$F_Z(z) = P(Z \leq z) \\ = P(X - Y \leq z)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{z+y} f(x, y) dx dy$$



$$x = u + y \\ dx = du$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^z f(u+y, y) du dy$$

$$= \int_{-\infty}^z \int_{-\infty}^{\infty} f(u+y, y) dy du$$

$$\text{So } f_Z(z) = \frac{d}{dz} F_Z(z) = \int_{-\infty}^{\infty} f(z+y, y) dy$$

Both versions correct.