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This is a classic that is hard until you know the "trick".

Note that $u = \int_0^u 1 \, dx$

Now; for X non-negative:

$$E(X) = \int_0^{\infty} u f_X(u) \, du$$

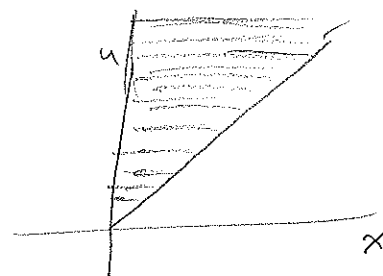
$$= \int_0^{\infty} \left[\int_0^u dx \right] f_X(u) \, du$$

$$= \int_0^{\infty} \int_0^u f_X(u) \, dx \, du$$

$$= \int_0^{\infty} \int_x^{\infty} f_X(u) \, du \, dx$$

$$= \int_0^{\infty} [F_X(u)]_x^{\infty} \, dx$$

$$= \int_0^{\infty} [1 - F_X(x)] \, dx$$



change order
of integration.

Note: 4.8 is similar using trick $k = \sum_{u=1}^k 1$