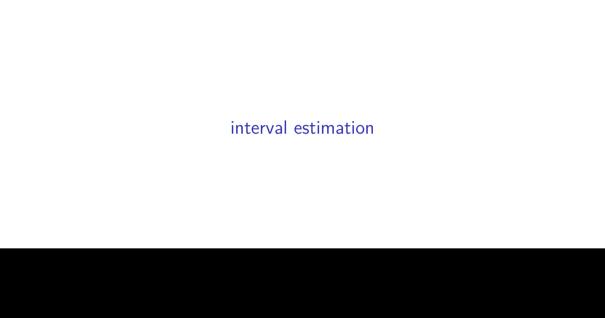
## STA261 Lecture 8 — 2017-08-02

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Note:  $\alpha$  is arbitrary and can technically be anything between 0 and 1, but it's usually small and almost always 0.05.

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Suppose you actually encounter some data, say from a  $N(\mu,1)$  distribution. Sample size n=9.

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That's not strictly speaking a confidence interval, but we call them that anyway.

Just don't go thinking "there is a 95 chance that  $\mu$  is inside [1.9, 3.21]"