STA261 Lecture 8 — 2017-08-02

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interval estimation

communicating uncertainty

Some population of interest is modeled by a random variable X, up to some parameter θ, whose value is not known. So a sample $\mathbf{X} = X_1, \ldots, X_n$ i.i.d. with the same distribution of X will be gathered. We have extensively studied the problem of estimating θ.

It would be nice to communicate an amount of uncertainty along with the estimate itself, which will depend on:

- the population variance itself
- the estimator being used
- the sample size

confidence interval

A $(1-\alpha)\cdot 100$ % *confidence interval* for θ is a pair of statistics $L(\mathbf{X})$ and $U(\mathbf{X})$ with these properties:

1. $L(\mathbf{X}) < U(\mathbf{X})$
2. $P(\{L(\mathbf{X}) > \theta)\} \cup \{U(\mathbf{X}) < \theta)\}) = 1 - \alpha$

It's not clear where such a pair of statistics might come from. But they are often easy to find.

Note: We can allow one of L or U to be a constant, including $\pm\infty$, resulting in a so-called "one-sided" confidence interval.

Note: α is arbitrary and can technically be anything between 0 and 1, but it's usually small and almost always 0.05.

pivots

A *pivot*, or *pivotal quantity*, for θ is a function $g(\mathbf{X}, \theta)$ whose distribution is the same for all θ.

Less formally, a pivot contains θ in its formula, but not in its distrbution.

Example 8.0: $N(\mu, 1)$.

Examples 8.1: $N(\mu, \sigma^2)$ (with $\theta = \mu$, and then with $\theta = \sigma^2$)

Example 8.2: Gamma(α, λ) (with $\theta = \lambda$)

Example 8.3: $X_{11}, \ldots, X_{1n}$ i.i.d $N(\mu_1, \sigma_1^2)$ and $X_{21}, \ldots, X_{2m}$ i.i.d $N(\mu_2, \sigma_2^2)$ and $X_{1i} \perp X_{2j}$ (with $\theta = \sigma_1^2/\sigma_2^2$))

Example 8.4: Uniform$(0, \theta)$

from pivot to confidence interval

Since a pivot has θ in the formula, but not in the distribution, confidence intervals can be produces in great abundance.

Example 8.5: $N(\mu, 1)$.

Definition: z_τ is the solution of $P(Z > z_\tau) = \tau$ and $t_{\nu,\tau}$ is the solution of $P(t_\nu > t_{\nu,\tau}) = \tau$.

Examples 8.6: $N(\mu, \sigma^2)$ (with $\theta = \mu$, and then with $\theta = \sigma^2$)

Example 8.7: Uniform$(0, \theta)$

(observed) confidence intervals

Suppose you actually encounter some data, say from a $N(\mu, 1)$ distribution. Sample size $n = 9$.

$$4.03, 1.92, 2.06, 3.45, 3.69, 3.14, 2.79, 2.93, 1.81$$

The observed value of $\overline{X}$ is what I'd call $\overline{x} = 2.87$.

To report your estimate for μ you might say "the 95% confidence interval is $[2.22, 3.52]$".

That's not strictly speaking a confidence interval, but we call them that anyway.

Just don't go thinking "there is a 95 chance that μ is inside $[2.22, 3.52]$"