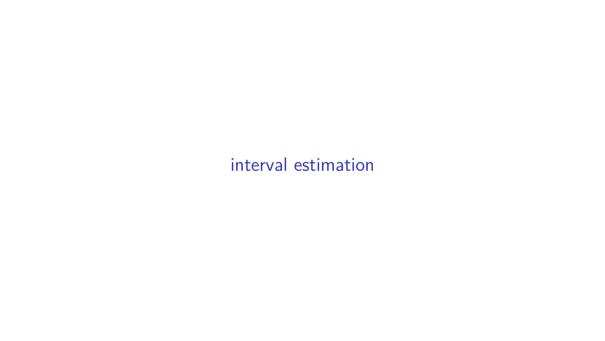
STA261 Lecture 8 — 2017-08-02

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Note: α is arbitrary and can technically be anything between 0 and 1, but it's usually small and almost always 0.05.

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Example 8.3: X_{11}, \ldots, X_{1n} i.i.d $N(\mu_1, \sigma_1^2)$ and X_{21}, \ldots, X_{2m} i.i.d $N(\mu_2, \sigma_2^2)$ and $X_{1i} \perp X_{2j}$ (with $\theta = \sigma_1^2/\sigma_2^2$))

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Example 8.7: Uniform $(0, \theta)$

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That's not strictly speaking a confidence interval, but we call them that anyway.

Just don't go thinking "there is a 95 chance that μ is inside [2.21, 3.52]"