STA261 Lecture 8 — 2017-08-02

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Last edited: 2017-08-02 20:27

interval estimation

communicating uncertainty

Some population of interest is modeled by a random variable X, up to some parameter θ, whose value is not known. So a sample $\mathbf{X} = X_1, \ldots, X_n$ i.i.d. with the same distribution of X will be gathered. We have extensively studied the problem of estimating θ.

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- the sample size

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Note: α is arbitrary and can technically be anything between 0 and 1, but it's usually small and almost always 0.05.

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Example 8.3: $X_{11}, \ldots, X_{1n}$ i.i.d $N(\mu_1, \sigma_1^2)$ and $X_{21}, \ldots, X_{2m}$ i.i.d $N(\mu_2, \sigma_2^2)$ and $X_{1i} \perp X_{2j}$ (with $\theta = \sigma_1^2/\sigma_2^2$)

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(observed) confidence intervals

Suppose you actually encounter some data, say from a $N(\mu, 1)$ distribution. Sample size $n = 9$.

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To report your estimate for μ you might say "the 95% confidence interval is $[1.9, 3.21]$".

That's not strictly speaking a confidence interval, but we call them that anyway.

Just don't go thinking "there is a 95 chance that μ is inside $[1.9, 3.21]$"