# STA261 Lecture 8 — 2017-08-02

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Last edited: 2017-08-02 20:28

# interval estimation

# communicating uncertainty

Some population of interest is modeled by a random variable $X$, up to some parameter $\theta$, whose value is not known. So a sample $\mathbf{X} = X_1, \dots, X_n$ i.i.d. with the same distribution of $X$ will be gathered. We have extensively studied the problem of estimating $\theta$.

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Note: $\alpha$ is arbitrary and can technically be anything between 0 and 1, but it's usually small and almost always 0.05.

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**Example 8.3:** $X_{11}, \ldots, X_{1n}$ i.i.d $N(\mu_1, \sigma_1^2)$ and $X_{21}, \ldots, X_{2m}$ i.i.d $N(\mu_2, \sigma_2^2)$ and $X_{1i} \perp X_{2j}$ (with $\theta = \sigma_1^2/\sigma_2^2$))

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**Example 8.4:** Uniform$(0, \theta)$

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# (observed) confidence intervals

Suppose you actually encounter some data, say from a $N(\mu, 1)$ distribution. Sample size $n = 9$.

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To report your estimate for $\mu$ you might say "the 95% confidence interval is $[2.21, 3.52]$".

That's not strictly speaking a confidence interval, but we call them that anyway.

Just don't go thinking "there is a 95 chance that $\mu$ is inside $[2.21, 3.52]$"