

# Lecture 6

## HW 2, Question 2

- There are  $a$  boxes, and slips of paper with the numbers  $1, \dots, a$ . The slips of paper are randomly added to the boxes.
- Each player  $i = 1, \dots, a$  is going to try to find their slip of paper (the one with their number)
- Each player randomly selects  $a/2$  boxes to open
- What is the probability that *all* players find their slip of paper when opening the boxes?

# Tips on where to start

- There are  $a$  boxes, and slips of paper with the numbers  $1, \dots, a$ . The slips of paper are randomly added to the boxes.
- Each player  $i = 1, \dots, a$  is going to try to find their slip of paper (the one with their number)
- Each player randomly selects  $a/2$  boxes to open
- What is the probability that *all* players find their slip of paper when opening the boxes?

# Making a plan

Imagine we were doing this with real people. What would we do?

# Step 1: create the slips of paper

```
1 a <- 10  
2 slips <- 1:a
```

## Step 2: randomly assign the slips to boxes

```
1 a <- 10  
2 slips <- 1:a
```

**Question:** How do I randomly shuffle the entries in a vector?

## Step 2: randomly assign the slips to boxes

```
1 a <- 10
2 slips <- 1:a
3 boxes <- sample(slips, a, replace=F)
4 boxes
```

```
[1] 7 5 1 6 3 8 9 4 2 10
```

**Question:** What does `boxes[i]` represent?

## Step 3: a player randomly chooses boxes

```
1 a <- 10
2 slips <- 1:a
3 boxes <- sample(slips, a, replace=F)
```

**Question:** how should we randomly select which boxes to open?



## Step 3: a player randomly chooses boxes

```
1 a <- 10
2 slips <- 1:a
3 boxes <- sample(slips, a, replace=F)
```

```
1 opened_boxes <- sample(1:a, a/2, replace = F)
2 opened_boxes
```

```
[1] 9 6 4 1 5
```

**Question:** how do we see which slips of paper were in these boxes?

## Step 3: a player randomly chooses boxes

```
1 a <- 10
2 slips <- 1:a
3 boxes <- sample(slips, a, replace=F)
4 opened_boxes <- sample(1:a, a/2, replace = F)
```

```
1 boxes[opened_boxes]
```

```
[1] 2 8 6 7 3
```

# Step 4: check if players number is in the opened boxes

Suppose Player 1 has opened the boxes:

```
1 a <- 10
2 slips <- 1:a
3 boxes <- sample(slips, a, replace=F)
4 opened_boxes <- sample(1:a, a/2, replace = F)
```

```
1 boxes[opened_boxes]
```

```
[1] 2 8 6 7 3
```

```
1 1 %in% boxes[opened_boxes]
```

```
[1] FALSE
```

## Step 4: repeat for all the players

```
1 a <- 10
2 slips <- 1:a
3 boxes <- sample(slips, a, replace=F)
4 opened_boxes <- sample(1:a, a/2, replace = F)
5 1 %in% boxes[opened_boxes]
```

**Question:** How do I repeat this process for all a players?

## Step 4: repeat for all the players

```
1 a <- 10
2 slips <- 1:a
3 boxes <- sample(slips, a, replace=F)
4
5 for(player in 1:a){
6   opened_boxes <- sample(1:a, a/2, replace = F)
7   player %in% boxes[opened_boxes]
8 }
```

**Question:** How do we check whether all players saw their number?

## Step 4: repeat for all the players

```
1 a <- 10
2 slips <- 1:a
3 boxes <- sample(slips, a, replace=F)
4 player_results <- rep(NA, a)
5
6 for(player in 1:a){
7   opened_boxes <- sample(1:a, a/2, replace = F)
8   player_results[player] <- player %in% boxes[opened_boxes]
9 }
10 player_results
```

[1] TRUE TRUE FALSE TRUE TRUE TRUE TRUE FALSE FALSE TRUE

```
1 sum(player_results) == a
```

[1] FALSE

**Question:** How do we repeat this code many times to estimate a probability?

# Step 5: repeat the whole game many times

```
1 set.seed(27)
2 a <- 10
3 slips <- 1:a
4 ngames <- 1000
5 game_results <- rep(NA, ngames)
```

```
1 for(i in 1:ngames){
2   boxes <- sample(slips, a, replace=F)
3   player_results <- rep(NA, a)
4   for(player in 1:a){
5     opened_boxes <- sample(1:a, a/2, replace = F)
6     player_results[player] <- player %in% boxes[opened_boxes]
7   }
8   game_results[i] <- sum(player_results) == a
9 }
10
11 mean(game_results)
```

```
[1] 0.002
```

## HW 2, Question 3: modifying the game

- Each slip is labeled  $1, \dots, a$  and randomly colored red or blue
- Each player  $i = 1, \dots, a$  is going to try to find their slip of paper (the one with their number)
- Each player randomly selects  $a/2$  boxes to open
- If the player does not see their slip, they randomly guess a color
- What is the probability that *all* players correctly announce their color?



# Activity

Work with a neighbor to discuss how we could modify the code from Question 2 for this new scenario.

## HW 2, Question 3

```
1 set.seed(27)
2 a <- 10
3 slips <- 1:a
4 ngames <- 1000
5 game_results <- rep(NA, ngames)

1 for(i in 1:ngames){
2   boxes <- sample(slips, a, replace=F)
3   player_results <- rep(NA, a)
4   for(player in 1:a){
5     opened_boxes <- sample(1:a, a/2, replace = F)
6     player_results[player] <- player %in% boxes[opened_boxes]
7   }
8   game_results[i] <- sum(player_results) == a
9 }
10
11 mean(game_results)
```

**Question:** What needs to change?

# Modifying the logic

- Randomly assign a color to each slip
- Store whether each player correctly identifies their color
- If a player sees their slip, do they also see their color?

# Modifying the logic

- Randomly assign a color to each slip
- Store whether each player correctly identifies their color
- If a player sees their slip, do they also see their color?  
Yes!
- If a player does not see their slip, what happens?

# Modifying the logic

```
1 boxes <- sample(slips, a, replace=F)
2 slip_colors <- sample(c("red", "blue"), a, replace=T)
3 player_results <- rep(NA, a)
4
5 for(player in 1:a){
6   opened_boxes <- sample(1:a, a/2, replace = F)
7   if(player %in% boxes[opened_boxes]){
8     ...
9   } else {
10    ...
11  }
12 }
```

**Question:** How do we fill in the `if...else...` here?

# Modifying the logic

```
1 boxes <- sample(slips, a, replace=F)
2 slip_colors <- sample(c("red", "blue"), a, replace=T)
3 player_results <- rep(NA, a)
4
5 for(player in 1:a){
6   opened_boxes <- sample(1:a, a/2, replace = F)
7   if(player %in% boxes[opened_boxes]){
8     player_results[player] <- TRUE
9   } else {
10     random_guess <- sample(c("red", "blue"), 1)
11     player_results[player] <- random_guess == slip_colors[player]
12   }
13 }
```

# Putting it all together

```
1 set.seed(27)
2 a <- 10
3 slips <- 1:a
4 ngames <- 1000
5 game_results <- rep(NA, ngames)

1 for(i in 1:ngames){
2   boxes <- sample(slips, a, replace=F)
3   slip_colors <- sample(c("red", "blue"), a, replace=T)
4   player_results <- rep(NA, a)
5
6   for(player in 1:a){
7     opened_boxes <- sample(1:a, a/2, replace = F)
8     if(player %in% boxes[opened_boxes]){
9       player_results[player] <- TRUE
10    } else {
11      random_guess <- sample(c("red", "blue"), 1)
12      player_results[player] <- random_guess == slip_colors[player]
13    }
14  }
15 }
```

