2017-03-24 Tutorial

1. The failures within a fleet of pumps at a pulp mill occur according to a Poisson process at a rate of 3 per year.

a. Calculate the probability that the third failure will occur at some point in time after 8 months (2/3 of a year) have elapsed.

b. Compute the following integral:

$$\int_0^{2/3} \frac{9}{\Gamma(3)} u^2 e^{-3u}\, du$$

c. Suppose we know that after 6 months, only one failure has occurred. What is the probability that this failure occurred during the first 3 months of the year?

d. Suppose 4 months have elapsed, and no pump failures have occurred. The mill manager says "We should have had a failure by now, according to the average. We should start planning now for an imminent failure." Comment on the extent to which the mill manager is correct in this assessment.

2. Suppose $X \sim \text{Gamma}(\alpha_1, \lambda_1)$ and $Y \sim \text{Gamma}(\alpha_2, \lambda_2)$, with $X \perp Y$. Under what circumstances can we say anything about the distribution of $X + Y$?

3. What would the shape of a Gamma(131, 131) density have, and why?

4. Any time a patient arrives at Mount Sinai Hospital via ambulance and is not in a life-threatening situation, they will wait to see a doctor for more than 1 hour with probability 0.2.

a. 40 patients arrive at the hospital on one day. Approximate the probability that 10 or fewer will have to wait more than 1 hour to see a doctor.

b. What is the average number of patients to arrive at the hospital before the first has to wait more than 1 hour to see a doctor?

c. What is the probability that it takes more than the average number of patients before the first patient has to wait more than one hour to see a doctor. (expression is good enough - not necessary to compute the actual number.)