

# STA286 Lecture 05

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real-valued functions with arguments that live inside sample spaces that eventually we will pretty much forget even exist

## sample spaces: too general?

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- ▶ all possible combinations of the six horse distances run as functions of time.

But really all you care about is whether you suffer a \$2 loss or a \$4.75 profit, and it turns out to be better just to focus on that.



## a new style of function to add to the stable

We will now consider functions that have a sample space  $S$  as a domain and range  $\mathbb{R}$ , e.g.

Element of $S$	Value in $\mathbb{R}$
CBFADE	-2.00
DFACBE	4.75
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So if all the usual function things like derivatives, local maxima, integrals etc. are not of interest, what do we do with these things?

All we care about are the *distributions* of such functions—roughly speaking the possible values in  $\mathbb{R}$  and their probabilities.

## “random variables”

Sadly these functions have a terrible name.

Definition: A *random variable* is a real-valued function of a sample space.

More examples:

**Toss to first head:** Count the number of tosses of a coin until the first H appears.

Element of $S$	Value in $\mathbb{R}$
H	1
TH	2
TTH	3
TTTH	4
TTTTH	5
$\vdots$	$\vdots$

## examples of random variables

**See if a product is defective:** Select an item at random from a factory. See if it is defective. Define the following function:

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Defective	1
Not Defective	0

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Seems like a stupid example, but this is merely an instance of one of the most important random variables of all. To generalize:

**“Bernoulli trial”** (book: *Bernoulli random variable*) Observe a random process to see if an event  $A$  occurred. The *indicator function*  $I_A$  which takes on the value 1 if  $A$  occurred and 0 otherwise is called a Bernoulli trial.

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**Failure time(s)** Observe a pump until its bearing fails. Observe a pump until its seal fails. Observe a pump until either component fails.

These random variables could take on any positive real number.

## notation and naming conventions

Random variables are given names that tend to be capital Roman letters near the end of the alphabet.

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Usual names:  $X$ ,  $Y$ ,  $Z$ ,  $X_1$ ,  $X_2$ ,  $X_3$ , etc.

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Is notation important? If you like to watch the world burn put this on a calculus exam:

$$\int_0^{2\pi} \sin(f) df$$

## values of random variables imply events

For any random variable  $X$ , any subset of the real line you could think of implies an event.

Examples:

**Toss to first head:** Let  $X$  be the number of tosses.

$X$ taking on any of these values	Implies this event
$X \in \{1\}$	

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The distribution of a random variable is the mapping between values of  $X$  and their probabilities of the implied events.

(Over-)simplified: the values, and their probabilities.

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## distribution in the bus stop example

$X$  is the amount of time you wait for the bus, with the idea that the bus could come at any random time in the next 10 minutes “uniformly”

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## more notation

Here's how we will actually write the probability statements:

Inconvenient	Usual Notation
$X \in [2, 4]$	$P(2 \leq X \leq 4) = \frac{2}{10}$
$X \in [3, 5]$	$P(3 \leq X \leq 5) = \frac{2}{10}$
$X \in (-9, -3.2]$	$P(-9 < X \leq -3.2) = 0$
$X \in \{2\}$	$P(X = 2) = 0$

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It's clear from the examples that “distribution” is a complex object, so we'll need convenient representations for them. Here is the first one.

## distributions and their representations

**When you know the distribution of a random variable, you know everything** (*as far as probability theory goes.*)

It's clear from the examples that “distribution” is a complex object, so we'll need convenient representations for them. Here is the first one.

(“Theorem”:)the distribution of any random variable  $X$  is completely determined by the following function  $F : \mathbb{R} \rightarrow \mathbb{R}$ , defined at every  $x \in \mathbb{R}$  :

$$F(x) = P(X \leq x).$$

$F$  is called the *cumulative distribution function* of  $X$ , or cdf for short.