STA286 Lecture 17

Neil Montgomery

Last edited: 2017-03-06 12:41

the normal distributions

We say Z has a "standard" normal distribution, or $Z \sim N(0,1)$, if its density is:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, \qquad -\infty < z < \infty$$

the normal distributions

We say Z has a "standard" normal distribution, or $Z \sim N(0,1)$, if its density is:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, \qquad -\infty < z < \infty$$

Is this a density?

the normal distributions

We say Z has a "standard" normal distribution, or $Z \sim N(0,1)$, if its density is:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, \qquad -\infty < z < \infty$$

Is this a density?

$E(Z) = 0$ (easy integral) and $\text{Var}(Z) = 1$ (easy integration by parts).

the normal distributions

We say Z has a "standard" normal distribution, or $Z \sim N(0,1)$, if its density is:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, \qquad -\infty < z < \infty$$

Is this a density?

$E(Z) = 0$ (easy integral) and $\text{Var}(Z) = 1$ (easy integration by parts).

Since $\int\limits_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\, dz = 1$, the change of variables $z = \frac{x-\mu}{\sigma}$ for any μ and any $\sigma > 0$ shows the following is also a valid density:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

the normal distributions

We say Z has a "standard" normal distribution, or $Z \sim N(0,1)$, if its density is:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, \qquad -\infty < z < \infty$$

Is this a density?

$E(Z) = 0$ (easy integral) and $\mathrm{Var}(Z) = 1$ (easy integration by parts).

Since $\int\limits_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\, dz = 1$, the change of variables $z = \frac{x-\mu}{\sigma}$ for any μ and any $\sigma > 0$ shows the following is also a valid density:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

We say X has a normal distribution with parameters μ and σ, or $X \sim N(\mu, \sigma)$, when it has this density.

normal mgf and some implications

Tedious algebra and calculus show (not interesting - see textbook or internet):

$$M_X(t) = e^{\mu t + \sigma^2 t^2/2}$$

normal mgf and some implications

Tedious algebra and calculus show (not interesting - see textbook or internet):

$$M_X(t) = e^{\mu t + \sigma^2 t^2/2}$$

Suppose $Z \sim N(0,1)$. What is the distribution of $X = \mu + \sigma Z$ for any μ and $\sigma > 0$?

normal mgf and some implications

Tedious algebra and calculus show (not interesting - see textbook or internet):

$$M_X(t) = e^{\mu t + \sigma^2 t^2/2}$$

Suppose $Z \sim N(0,1)$. What is the distribution of $X = \mu + \sigma Z$ for any μ and $\sigma > 0$?

Answer: $X \sim N(\mu, \sigma)$.

normal mgf and some implications

Tedious algebra and calculus show (not interesting - see textbook or internet):

$$M_X(t) = e^{\mu t + \sigma^2 t^2/2}$$

Suppose $Z \sim N(0,1)$. What is the distribution of $X = \mu + \sigma Z$ for any μ and $\sigma > 0$?

Answer: $X \sim N(\mu, \sigma)$.

Suppose $X \sim N(\mu, \sigma)$. What is the distribution of $Z = \frac{X-\mu}{\sigma}$?

normal mgf and some implications

Tedious algebra and calculus show (not interesting - see textbook or internet):

$$M_X(t) = e^{\mu t+\sigma^2 t^2/2}$$

Suppose $Z \sim N(0,1)$. What is the distribution of $X = \mu + \sigma Z$ for any μ and $\sigma > 0$?

Answer: $X \sim N(\mu, \sigma)$.

Suppose $X \sim N(\mu, \sigma)$. What is the distribution of $Z = \frac{X-\mu}{\sigma}$?

Answer: $Z \sim N(0,1)$.

normal mgf and some implications

Tedious algebra and calculus show (not interesting - see textbook or internet):

$$M_X(t) = e^{\mu t + \sigma^2 t^2/2}$$

Suppose $Z \sim N(0,1)$. What is the distribution of $X = \mu + \sigma Z$ for any μ and $\sigma > 0$?

Answer: $X \sim N(\mu, \sigma)$.

Suppose $X \sim N(\mu, \sigma)$. What is the distribution of $Z = \frac{X-\mu}{\sigma}$?

Answer: $Z \sim N(0,1)$.

A linear transformation of a normal is normal.

normal mgf and some implications

Tedious algebra and calculus show (not interesting - see textbook or internet):

$$M_X(t) = e^{\mu t + \sigma^2 t^2/2}$$

Suppose $Z \sim N(0,1)$. What is the distribution of $X = \mu + \sigma Z$ for any μ and $\sigma > 0$?

Answer: $X \sim N(\mu, \sigma)$.

Suppose $X \sim N(\mu, \sigma)$. What is the distribution of $Z = \frac{X-\mu}{\sigma}$?

Answer: $Z \sim N(0,1)$.

A linear transformation of a normal is normal.

If X_1 and X_2 are normal, what is $X_1 + X_2$?

normal probability calculations

Since the normal density has no anti-derivative (the usual case with functions, BTW), probability calculations are a problem for the computer, or a table of probabilities, which you will need to practice if necessary.

normal probability calculations

Since the normal density has no anti-derivative (the usual case with functions, BTW), probability calculations are a problem for the computer, or a table of probabilities, which you will need to practice if necessary.

Why only one table when there are many normal distributions?

normal probability calculations

Since the normal density has no anti-derivative (the usual case with functions, BTW), probability calculations are a problem for the computer, or a table of probabilities, which you will need to practice if necessary.

Why only one table when there are many normal distributions?

I'll do a few examples from the `normal_bootcamp.pdf` drill document contained with these notes.

some normal probabilities to remember

The classic plus-or-minus $k\sigma$ probabilities (with exact values):

some normal probabilities to remember

The classic plus-or-minus $k\sigma$ probabilities (with exact values):

$$P(\mu - \sigma < X < \mu + \sigma) \approx 0.68 \qquad (0.6826895)$$

some normal probabilities to remember

The classic plus-or-minus $k\sigma$ probabilities (with exact values):

$$P(\mu - \sigma < X < \mu + \sigma) \approx 0.68 \qquad (0.6826895)$$

$$P(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.95 \qquad (0.9544997)$$

some normal probabilities to remember

The classic plus-or-minus $k\sigma$ probabilities (with exact values):

$$P(\mu - \sigma < X < \mu + \sigma) \approx 0.68 \qquad (0.6826895)$$

$$P(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.95 \qquad (0.9544997)$$

$$P(\mu - 3\sigma < X < \mu + 3\sigma) \approx 0.997 \qquad (0.9973002)$$

some normal probabilities to remember

The classic plus-or-minus $k\sigma$ probabilities (with exact values):

$$P(\mu - \sigma < X < \mu + \sigma) \approx 0.68 \qquad (0.6826895)$$

$$P(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.95 \qquad (0.9544997)$$

$$P(\mu - 3\sigma < X < \mu + 3\sigma) \approx 0.997 \qquad (0.9973002)$$

Bonus probabilities:

$$P(\mu - 6\sigma < X < \mu + 6\sigma) = 0.999999998027$$

$$P(\mu - 8.5\sigma < X < \mu + 4.5\sigma) = 0.999996602327$$