STA286 Lecture 22

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the t distributions - II-and-a-half

Rephrased from last time.

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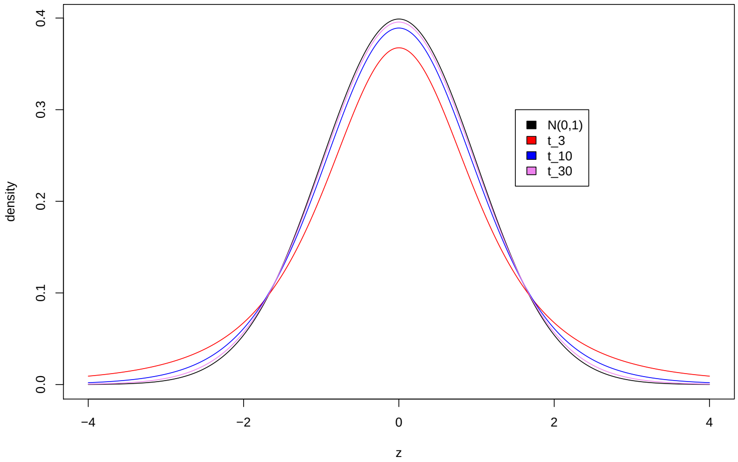
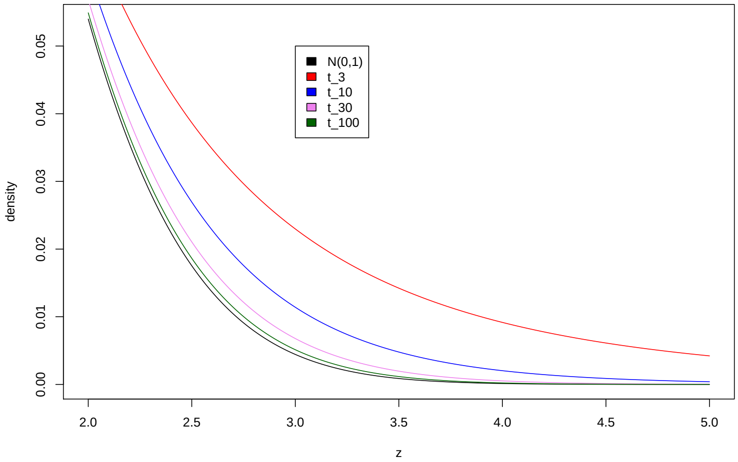
- As n gets big, T starts to look like $Z \sim N(0,1)\}$
- a table of t probabilities needed on tests.

overall pictures of t_ν



pictures of t_ν in the "tail"



the F distributions

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If $U \sim \chi^2_m$ and $V \sim \chi^2_n$ and $U \perp V$ then we say:

$$F = \frac{U/m}{V/n} \sim F_{m,n}$$

or "F has an F distribution with m and n degrees of freedom."

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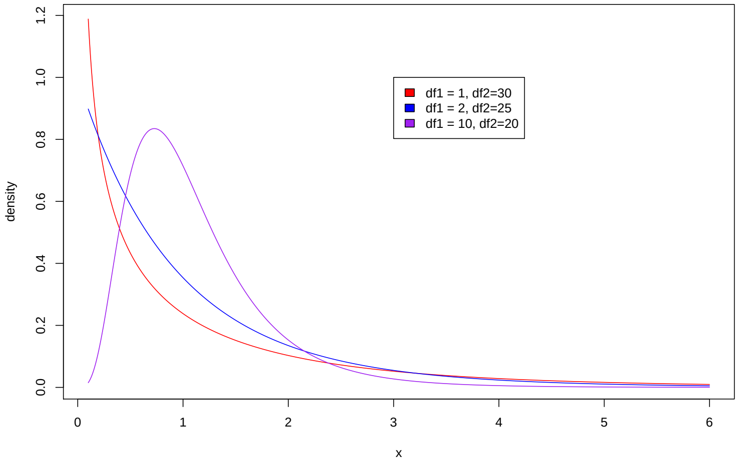
The density is nasty; tables must be used on tests, etc.

pictures of some F distributions



preview example of F theory

Populations

$N(\mu_1, \sigma_1)$

$N(\mu_2, \sigma_2)$

Question: are the two population standard deviations the same, or not?

preview example of F theory

Populations	Samples
$N(\mu_1, \sigma_1)$	$X_{11}, \ldots, X_{1n_1}$
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Answer might be based on:

$$\frac{S_{n_1}^2/(n_1 - 1)}{S_{n_2}^2/(n_2 - 1)} \sim F_{n_1-1, n_2-1}$$

a specialized plot for detecting deviations from normality

central limit theorem, and friends

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The answer is to use *statistics*, by which I mean gather a sample and estimate what you don't know.

limitations of “histogram”

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But a histogram requires a very large sample size (hundreds?) to give an accurate picture. By the time you have n in the hundreds, the normal approximation is going to be pretty good.

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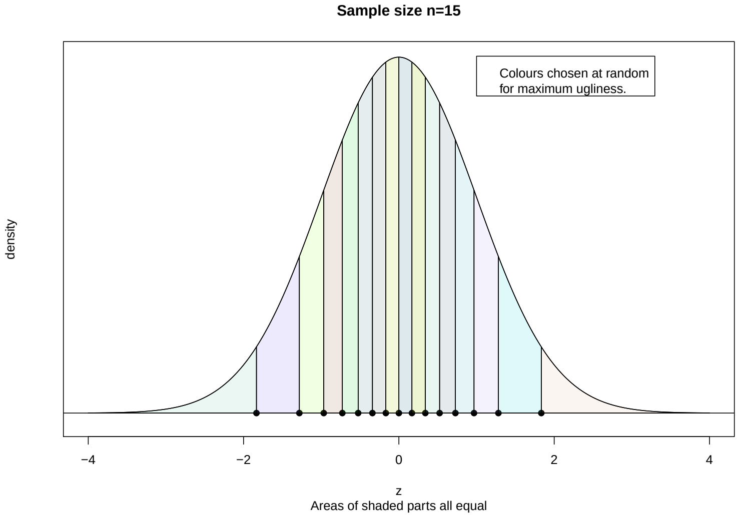
- (computer) makes scatterplot with perfect (horizontal) vs. ordered data (vertical)
- straight line means data are consistent with having come from a normal distribution. Other patterns also easy to interpret.

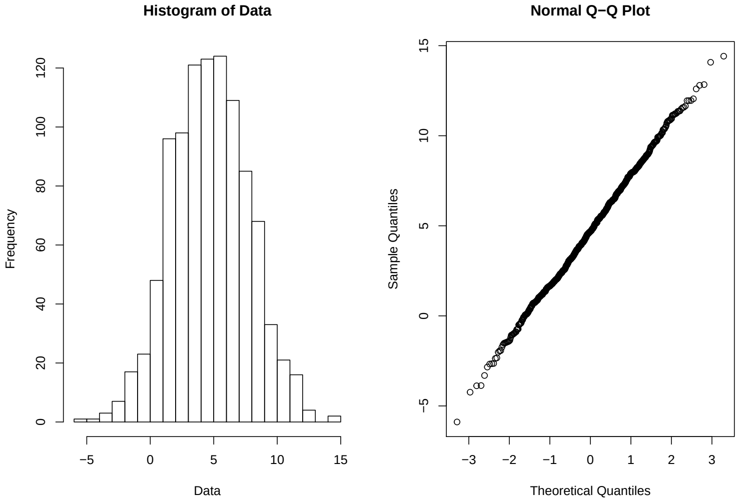
“perfect” standard normal data

Find the values that split the area under the curve into equal parts.



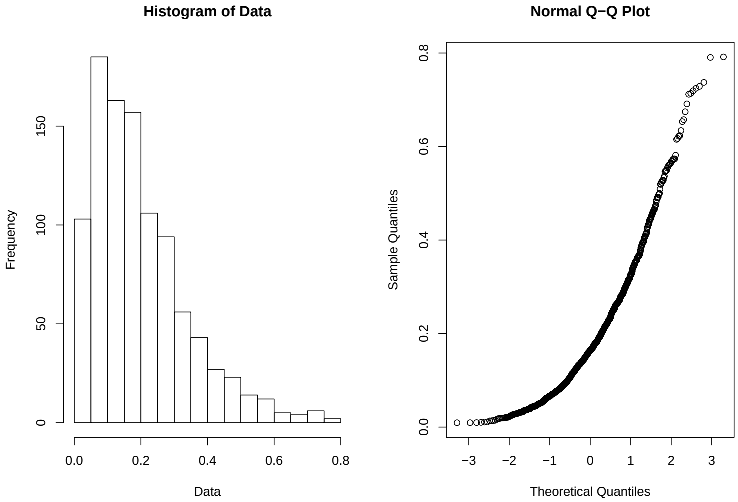
result: "normal quantile plot" of "normal q-q plot" (other names)

First set of examples will have $n = 1000$, in which case a histogram might have been OK anyway. First example: perfect normal $N(5, 3)$ data. Result: straight line.



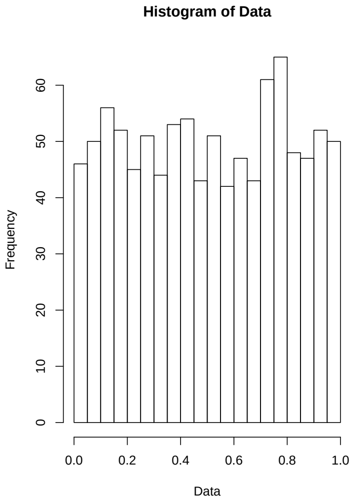
$n = 1000$ right skewed data

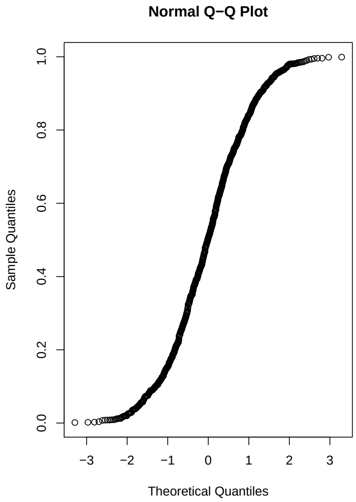
Gamma(2, 10). Result: curved ("concave 'up' ")



$n = 1000$ light tails

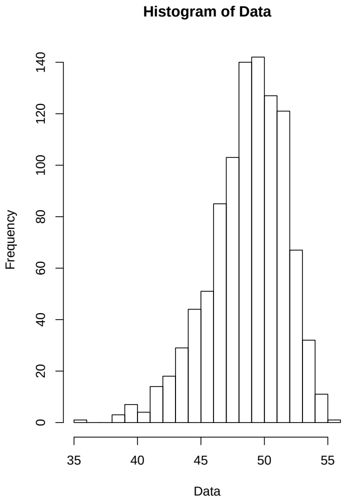
Uniform[0,1]. Result: S-shaped.

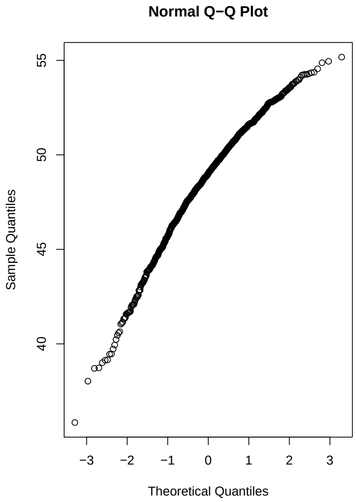




$n = 1000$ rarer case: left skewed

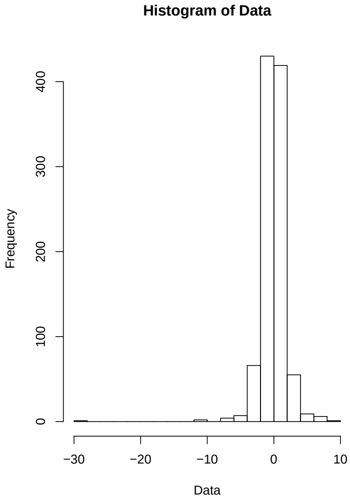
Result: curved (“concave ‘down’ ”)

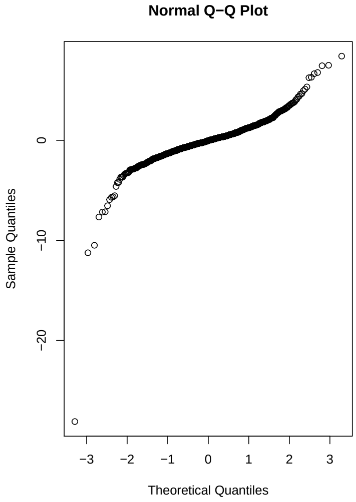




$n = 1000$ very rare: "heavy tails"

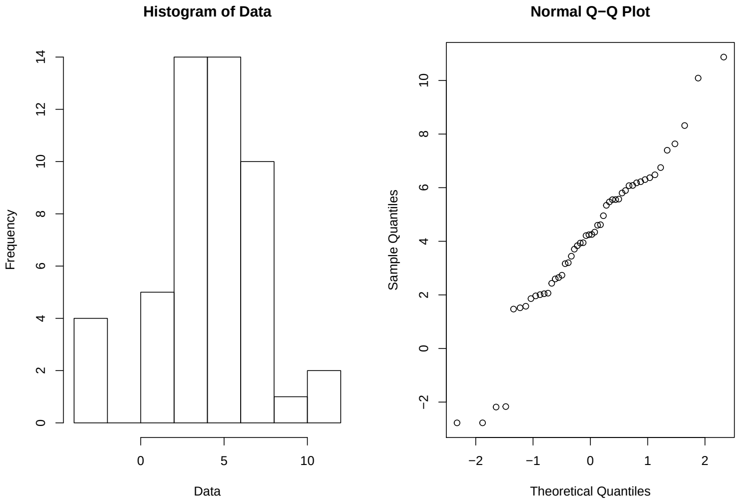
Result: "reverse-S" shaped



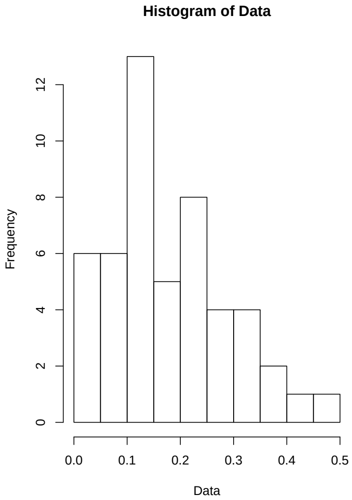


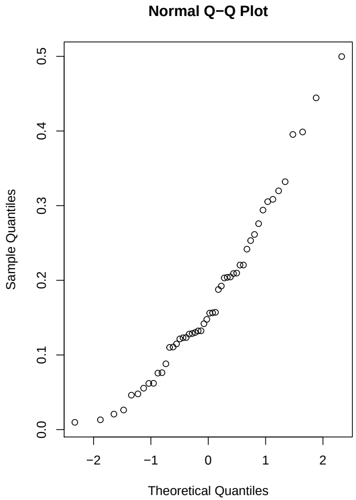
value of normal quantile plots is with small samples

Bring the sample size down to $n = 50$. Histogram not useful. Here's the $N(5, 3)$ example again.



$n = 50$ right skewed





$n = 50$ light tails