STA286 Lecture 24

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interval estimation

estimation, with an assessment of the data collection plan

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It is possible to have $\hat{\theta}_L = -\infty$ or $\hat{\theta}_U = \infty$

When $\alpha = 0.05$ (as usual), we have a 95% confidence interval.

(artifical) example of a confidence interval

Suppose the underlying population is $N(\mu, \sigma_0)$ with σ_0 (magically) known.

We plan to gather a sample $X_1, \ldots, X_n$. There are *lots* of 95% confidence intervals for μ, obtained by isolating μ in the middle of:

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Define z_α as the solution of $P(Z \leqslant z_\alpha) = 1 - \alpha$, where $Z \sim N(0, 1)$. The *shortest possible* 95% confidence interval for μ comes from:

$$P\left(\overline{X} - z_{0.025}\frac{\sigma_0}{\sqrt{n}} < \mu < \overline{X} + z_{0.025}\frac{\sigma_0}{\sqrt{n}}\right) = 0.95$$

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"Two" is in quotation marks because the precise value will vary a little over and under 2, but it will always be close to 2 (for a 95% interval).

meaning, and some myths

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> *There is a 95% chance that μ is between 4.2 and 6.8.*

The statement is nonsense. Either μ is between 4.2 and 6.8, or it isn't.

things that affect the width of a typical C.I.

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The larger the α, the narrower the C.I. (But α is arbitrary.)

The larger the n, the narrower the C.I. (The sample size *is* under your control.)

a sample size calculation

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- use prior knowledge of the value of σ
- if the population is plausibly normal, use prior knowledge of the minimum m and maximum M plausible values you might ever see, and use $(M-m)/6$ as a rough guesstimate for σ.

the classic "one-sample" t interval - I

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Define $t_{n-1,\alpha}$ as the solution of $P(t_{n-1} \leqslant t_{n-1,\alpha}) = 1 - \alpha$. The new interval will be based on:

$$P\left(-t_{n-1,\alpha/2} < \frac{\overline{X} - \mu}{S/\sqrt{n}} < t_{n-1,\alpha/2}\right) = 1 - \alpha$$

the classic "one-sample" t interval - II

The interval is therefore:

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The value $\frac{S}{\sqrt{n}}$ is (also) called the (estimated) standard error for $\overline{X}$, or s.e.$(\overline{X})$, and in the usual 95% case we end up with another example of the universal formula:

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That's because t calculations aren't wildly different from $N(0,1)$ calculations as long as $n-1$ isn't tiny.

$t_{n-1,0.025}$ for some non-insane sample sizes

n	t
15	2.131449
30	2.042273
40	2.021075
50	2.008559
60	2.000298
150	1.975905

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So, for example, consider textbook question 9.14, which gives 15 values for the drying time, in hours, of a brand of latex paint.

x_bar	s	n
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The 95% confidence interval for the mean drying time is:

conf.low	conf.high
3.248994	4.324339