STA286 Lecture 30

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hypothesis testing

context: a one-sample t interval example

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"Hypothesis testing" generally involves using data to make a principled statement about a parameter value.

hypotheses

Specific statements about parameter values can have practical meanings.

In the `Fe` example, the model used for the population was $X \sim N(\mu, \sigma)$.

Some statements include:

$$\mu = 4 \qquad \mu \neq 4 \qquad \sigma = 1 \qquad \sigma^2 > 5$$

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A statement about a parameter value is called a *hypothesis*.

some more natural hypotheses

Consider two populations $N(\mu_1, \sigma)$ and $N(\mu_2, \sigma)$. The most obviously interesting hypothesis is:

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Similarly, consider two population Bernoulli(p_1) and Bernoulli(p_2). We might also have:

$$p_1 = p_2$$

to mean "no difference"

null hypothesis and alternative hypothesis

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(Note: this is a scientific opinion, and not a mathematical opinion.)

“classical” hypothesis testing

Our first view of hypothesis testing has a clear goal, which is to use data to make a specific decision: to either *reject* H_0 or *not reject* H_0.

Sometimes *accept* is used as a synonym for *not reject*. The book uses the phrase *fail to reject*, which I’ve never seen anywhere alse.

The main thing is to avoid attaching positive or negative connotations to any of these phrases.

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Motivating example... a pre-fabricated furniture company needs its supplier to provide doors that are 700mm wide. Does the supplier meet this target?

The model for door width will be $N(\mu, \sigma)$, with $\sigma = 0.5$ magically known for now.

classical hypothesis testing—motivating example

The null and alternative hypotheses are:

$$H_0 : \mu = 700$$
$$H_1 : \mu \neq 700$$

We plan to gather a sample of size $n = 10$.

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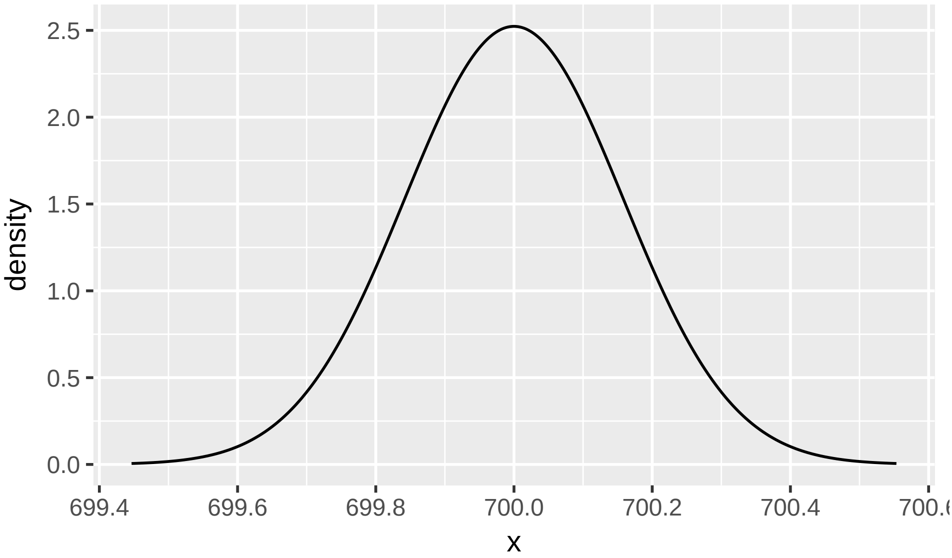
$$\overline{X} \sim N\left(700, \frac{0.5}{\sqrt{10}}\right) \quad \text{The } \textit{null distribution}$$

null distribution $N(700, 0.1581139)$



classical hypothesis testing - the details

"The values that would surprise us" are defined in advance according to a pre-set probabilty α.

This is called the "size" of the test, or the "level of significance". It is typically something small like: 0.05, 0.1, 0.05, 0.05, 0.01, 0.05, or 0.05.

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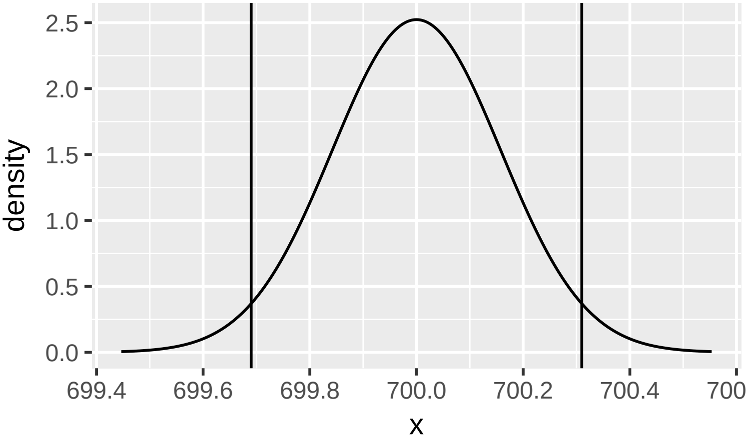
α is the *probability of rejecting* H_0 *when it is in fact true.*

classical hypothesis testing - the details

Suppose $\alpha = 0.05$. The "area of surprise" in our motivating example is defined as $\overline{X} \leqslant 699.6901$ or $\overline{X} \geqslant 700.3099$, as in:



classical hypothesis testing - the details

The "area of surprise" is really called the "rejection region" or "critical region".

We define two types of "error" in classical hypothesis testing:

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The probability $1 - \beta$ of rejecting H_0 when it is false is called the "power" of the test.

example of critical region

In our motivating example, the critical region comes from this expression that uses the null distribution:

$$P\left(-z_{\alpha/2} < \frac{\overline{X} - 700}{0.5/\sqrt{10}} < z_{\alpha/2}\right) = 1 - \alpha$$

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The region is:

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If we set $\alpha = 0.01$, say, this becomes:

$$\left\{\overline{X} < 699.593\right\} \cup \left\{\overline{X} > 700.407\right\}$$

example power calculation

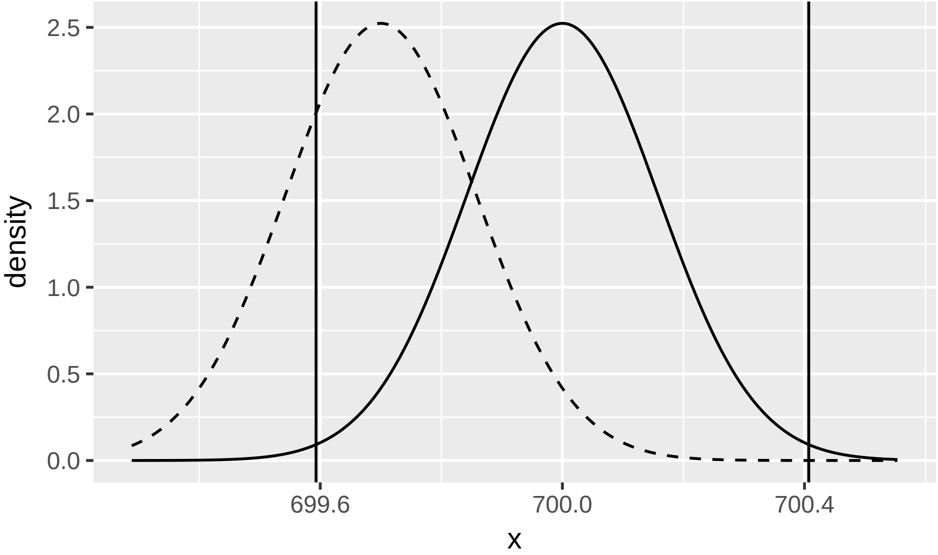
For an explicit power calculation, one needs a specific alternative.

So, suppose in fact the supplier makes doors that are $\mu_1 = 699.7$mm wide. So in fact $\overline{X} \sim N(699.7, 0.5/\sqrt(10))$

What is the probability of "rejecting H_0"?

$$\begin{aligned} P_{\mu_1}(\overline{X} < 699.593) + P_{\mu_1}(\overline{X} > 700.407) &= P(Z < -0.677) + P(Z > 4.471) \\ &= 0.249 + 0 \end{aligned}$$

power in pictures



size, power, and sample size

When the population is $N(\mu, \sigma)$ and the sample is $X_1, \ldots, X_n$ and the hypotheses are $H_0 : \mu = \mu_0$ versus $H_1 : \mu = \mu_1$, the generic rejection region is, for fixed α:

$$\left\{\overline{X} < \mu_0 - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right\} \cup \left\{\overline{X} > \mu_0 + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right\}$$