STA286 Lecture 30

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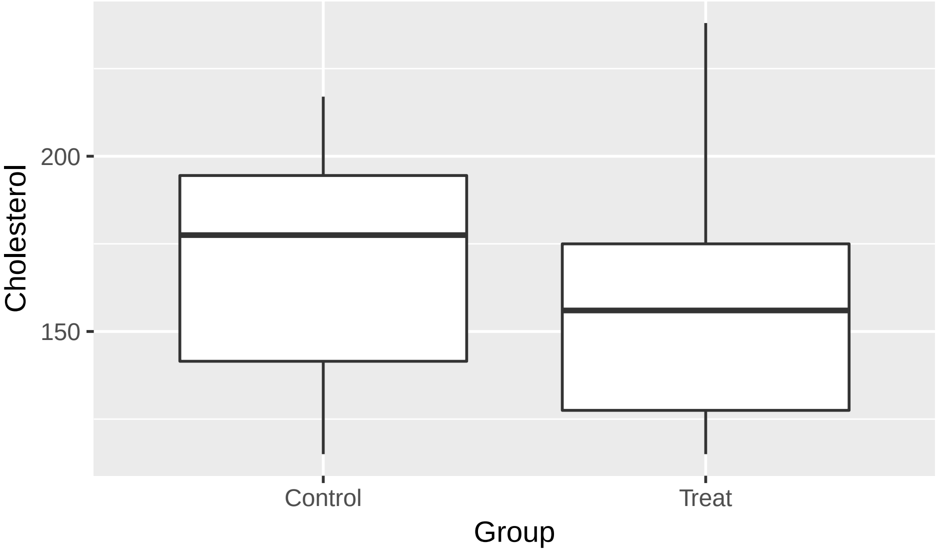
two-sample t-test example - plot



two-sample t-test example - equal variance version

```
## # A tibble: 2 × 4
##     Group     n    X_bar        S
##    <fctr> <int>    <dbl>    <dbl>
## 1 Control    18 170.0000 30.78770
## 2   Treat    15 156.3333 33.09006
```

```
##
##  Two Sample t-test
##
## data:  Cholesterol by Group
## t = 1.2275, df = 31, p-value = 0.2289
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -9.041658 36.374992
## sample estimates:
## mean in group Control   mean in group Treat
##              170.0000              156.3333
```

two-sample t-test example - no variance assumption version

```
##
##  Welch Two Sample t-test
##
## data:  Cholesterol by Group
## t = 1.2192, df = 29.039, p-value = 0.2326
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -9.258355 36.591688
## sample estimates:
## mean in group Control   mean in group Treat
##              170.0000              156.3333
```

two-sample t-test example - or is it?

Question 10.54. Nine people had breathing rates measured with and without elevated CO levels.

Subject	WithCO	WithoutCO
1	30	30
2	45	40
3	26	25
4	25	23
5	34	30
6	51	49
7	46	41
8	32	35
9	30	28

Does CO impact breathing frequency?

two-sample t-test example - or is it?

Two populations are $N(\mu_1, \sigma_1)$ and $N(\mu_2, \sigma_2)$.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

The two samples are $X_{11}, \ldots, X_{19}$ and $X_{21}, \ldots, X_{29}$.

two-sample t-test example - or is it?

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The two samples are $X_{11}, \ldots, X_{19}$ and $X_{21}, \ldots, X_{29}$.

But they are surely not independent. We should examine the differences $D_1, \ldots, D_9$, which will be $N(\mu_D, \sigma_D)$ where $\mu_D = \mu_1 - \mu_2$.

two-sample t-test example - or is it?

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But they are surely not independent. We should examine the differences $D_1, \ldots, D_9$, which will be $N(\mu_D, \sigma_D)$ where $\mu_D = \mu_1 - \mu_2$.

Here's a case where the null and alternatives are actually self-evident:

$$H_0 : \mu_D = 0$$
$$H_1 : \mu_D \neq 0$$

two-sample t-test example - or is it?

The analysis:

n	X_Bar_1	X_Bar_2	S_1	S_2	X_bar_D	S_D
9	35.444	33.444	9.462	8.502	2	2.55

```
##
##  One Sample t-test
##
## data:  co$WithCO - co$WithoutCO
## t = 2.3534, df = 8, p-value = 0.04643
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.04027332 3.95972668
## sample estimates:
## mean of x
##         2
```

single proportion example (something funny happens)

From the second test, that gas company "knew" the proportion of defective meters was 0.01. Let's change that to "assumes" (perhaps based on some industry knowledge). As usual, the single sample scenarios tend to be a bit contrived.

Work Team Beta inspects 2000 meters and finds 24 defective ones. Is there evidence that the company's assumption is inaccurate?

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$$H_0 : p = 0.01$$
$$H_1 : p \neq 0.01$$

We’ll use the MLE $\hat{p}$, for which we know:

$$\hat{p} \sim^{approx} N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$$

To calculate the p-value (or to get a critical region) we plug the H_0 value to obtain the null distribution. This happens to eliminate the unknown variance problem!

single proportion example

single proportion example

We observe $\hat{p}_{obs} = 0.012$. What is the p-value?

$$
\begin{aligned}
P(\hat{p} < 0.088) + P(\hat{p} > 0.012) &= P\left(Z < \frac{0.088 - 0.01}{\sqrt{\frac{0.01(1-0.01)}{2000}}}\right) + P\left(Z > \frac{0.012 - 0.01}{\sqrt{\frac{0.01(1-0.01)}{2000}}}\right) \\
&= P(Z < -0.899) + P(Z > 0.899) \\
&= 0.369
\end{aligned}
$$

two proportion example - a little trick

Much more natural.

Let's say Work Team Beta found $x_1 = 24$ defective meters in $n_1 = 2000$ inspections, and Work Team Delta found $x_2 = 14$ in $n_2 = 1500$ inspections. Do the teams find defectives at the same rate?

We are comparing a Bernoulli(p_1) with a Bernoulli(p_2). The null and alternative are self-evident:

$$H_0 : p_1 = p_2 \qquad H_1 : p_1 \neq p_2$$

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We will use $\hat{p}_1 - \hat{p}_2$, which we know satisfies:

$$\hat{p}_1 - \hat{p}_2 \sim^{approx} N\left(p_1 - p_2, \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}\right)$$

two proportion example - null distribution little trick

When computing the p-value, we plug in the H_0 fact that $p_1 = p_2$, which we will denote by just p. The variance of the "null distribution" reduces to:

$$p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$$

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We don't know p. Use the data, which, under the null hypothesis, are just 0's and 1's from the same Bernoulli(p) distribution. We pool them together to get:

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

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So the null distribtion is:

$$\hat{p}_1 - \hat{p}_2 \sim^{approx} N\left(0, \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right)$$

two proportion example

In our example we had $x_1 = 24$, $n_1 = 2000$, $x_2 = 14$, $n_2 = 1500$. So:

$$\hat{p} = 0.010857$$

and the standard deviation of the null distribution is 0.00354.

Also, $\hat{p}_1 - \hat{p}_2 = 0.002667$

The p-value is 0.451228 based on $2P(Z < -0.75337)$