Modeling two-level longitudinal data

Inference

03.21.22

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Announcements

- Mini-project 02

  - draft due **Thu, Mar 24 at noon** (peer review in lab)

  - presentations **Mon, Mar 28** in class

  - report due **Mon, Mar 28** at 11:59pm

- Looking ahead

  - HW 04 due Fri, Apr 1 (assigned later this week)

  - Final project - optional draft due **Fri, Apr 15**, final report due **Wed, Apr 27**

- DataFest: April 1 - 3 in Penn Pavilion

  - [Click here](#) to register

Learning goals

- Compare multilevel models

- Conduct inference for random effects

- Conduct inference for fixed effects

Comparing multilevel models

Data: Charter schools in MN

The data set **charter-long.csv** contains standardized test scores and demographic information for schools in Minneapolis, MN from 2008 to 2010. The data were collected by the Minnesota Department of Education. Understanding the effectiveness of charter schools is of particular interest, since they often incorporate unique methods of instruction and learning that differ from public schools.

- **MathAvgScore**: Average MCA-II score for all 6th grade students in a school (response variable)

- **urban**: urban (1) or rural (0) location school location

- **charter**: charter school (1) or a non-charter public school (0)

- **schPctfree**: proportion of students who receive free or reduced lunches in a school (based on 2010 figures).

- **year08**: Years since 2008

Data

```
charter <- read_csv("data/charter-long.csv")
```

schoolName	year08	urban	charter	schPctfree	MathAvgScore
RIPPLESIDE ELEMENTARY	0	0	0	0.363	652.8
RIPPLESIDE ELEMENTARY	1	0	0	0.363	656.6
RIPPLESIDE ELEMENTARY	2	0	0	0.363	652.6
RICHARD ALLEN MATH&SCIENCE ACADEMY	0	1	1	0.545	NA
RICHARD ALLEN MATH&SCIENCE ACADEMY	1	1	1	0.545	NA
RICHARD ALLEN MATH&SCIENCE ACADEMY	2	1	1	0.545	631.2

Unconditional means model

Start with the **unconditional means model**, a model with no covariates at any level. This is also called the random intercepts model.

Level One : $Y_{ij} = a_i + \epsilon_{ij}, \quad \epsilon_{ij} \sim N(0, \sigma^2)$

Level Two: $a_i = \alpha_0 + u_i, \quad u_i \sim N(0, \sigma_u^2)$

Write the composite model.

Intraclass correlation

The **intraclass correlation** is the relative variability between groups

$$\hat{\rho} = \frac{\text{Between variability}}{\text{Total variability}} = \frac{\hat{\sigma}_u^2}{\hat{\sigma}_u^2 + \hat{\sigma}^2}$$

$\hat{\rho} = 0.798$. This means...

- About 79.8% of the variability in math scores can be attributed to differences between schools (school-to-school variability). About 20.2% of the variability can be attributed to changes over time.

- The average correlation between any two responses from the same school is about 0.798.

- The **effective sample size** (number of independent pieces of information available for modeling) is closer to the number of schools $(\rho \text{ close to } 1)$ than the number of

Unconditional growth model

A next step in the model building is the **unconditional growth model**, a model with Level One predictors but no Level Two predictors.

Level One: $Y_{ij} = a_i + b_i Year08_{ij} + \epsilon_{ij}$

Level Two:

- $a_i = \alpha_0 + u_i$
- $b_i = \beta_0 + v_i$

- Write the composite model.
- What can we learn from this model?

Pseudo R²

We can use **Pseudo R²** to explain changes in variance components between two models

$$\text{Pseudo } R^2 = \frac{\hat{\sigma}^2(\text{Model 1}) - \hat{\sigma}^2(\text{Model 2})}{\hat{\sigma}^2(\text{Model 1})}$$

Note: This should only be used when the definition of the variance component is the same between the two models

- The $\mathrm{Pseudo}\ R^2$ estimating the change in within school variance between the unconditional means and unconditional growth model is about 0.17. What does this value mean?

- Why can't we use $\mathrm{Pseudo}\ R^2$ to understand change in $\hat{\sigma}_u^2$ between the two models?

STA 310

Model with school-level covariates

Fit a model with school-level covariates that takes the following form:

Level One

$$Y_{ij} = a_i + b_i Year08_{ij} + \epsilon_{ij}$$

Level Two

$$a_i = \alpha_0 + \alpha_1 Charter_i + \alpha_2 urban_i + \alpha_3 schpctfree_i + u_i$$
$$b_i = \beta_0 + \beta_1 Charter_i + \beta_2 urban_i + v_i$$

- Write out the composite model.

- Fit the model in R.

Consider a simpler model

Level One

$$Y_{ij} = a_i + b_i Year08_{ij} + \epsilon_{ij}$$

Level Two

$$a_i = \alpha_0 + \alpha_1 Charter_i + \alpha_2 urban_i + \alpha_3 schpctfree_i + u_i$$
$$b_i = \beta_0 + \beta_1 Charter_i + \beta_2 urban_i$$

In this model, the effect of year is the same for all schools with a given combination of **Charter** and **urban**

STA 310

14

Full and reduced models

```
full_model <- lmer(MathAvgScore ~ charter + urban + schPctfree +
                   charter:year08  + urban:year08 + year08 +
                   (year08|schoolid), REML = T, data = charter)
```

```
reduced_model <-  lmer(MathAvgScore ~ charter + urban + schPctfree +
                   charter:year08  + urban:year08 + year08 +
                       (1 | schoolid), REML = T, data = charter)
```

Issues with the drop-in-deviance test

```
anova(full_model, reduced_model, test = "Chisq") %>%
  kable(digits = 3)
```

	npar	AIC	BIC	logLik	deviance	Chisq	Df	Pr(>Chisq)
reduced_model	9	9952.992	10002.11	-4967.496	9934.992	NA	NA	NA
full_model	11	9953.793	10013.83	-4965.897	9931.793	3.199	2	0.202

- χ^2 test conservative, i.e., the p-values are larger than they should be, when testing random effects at the boundary (e.g., $\sigma_v^2 = 0$) or those with bounded ranges (e.g., ρ_{uv})

- If you observe small p-values, you can feel relatively certain the tested effects are statistically significant

- Use bootstrap methods to obtain more accurate p-values

Parametric bootstrapping

- **Bootstrapping** is from the phrase "pulling oneself up by one's bootstraps"

- Accomplishing a difficult task without any outside help

- **Task**: conduct inference for model parameters (fixed and random effects) using only the sample data

Parametric bootstrapping for likelihood ratio test

1 Fit the null (reduced) model to obtain the fixed effects and variance components (*parametric* part).

2 Use the estimated fixed effects and variance components to generate a new set of response values with the same sample size and associated covariates for each observation as the original data (*bootstrap* part).

3 Fit the full and null models to the newly generated data.

4 Compute the likelihood test statistic comparing the models from the previous step.

5 Repeat steps 2 - 4 many times (~ 1000).

Parametric bootstrapping for likelihood ratio test

6️⃣ Create a histogram of the likelihood ratio statistics to get the distribution of likelihood ratio statistic under the null hypothesis.

7️⃣ Get the p-value by calculating the proportion of bootstrapped test statistics greater than the observed statistic.

Go to `lecture-19.Rmd` to calculate the bootstrapped p-value for the likelihood ratio test statistic testing whether v_i and ρ_{uv} can be removed from the model.

Acknowledgements

The content in the slides is from

- BMLR: Chapter 9 - Two-level Longitudinal Data
  - Sections 9.1 - 9.6

- Hierarchical Linear Modeling with Maximum Likelihood, Restricted Maximum Likelihood, and Fully Bayesian Estimation by Peter Boedeker

- *Applied longitudinal data analysis: Modeling change and event occurrence.* by J.D. Singer and J.B. Willett

  - Online copy available through Duke library

Acknowledgements

- *Extending the linear model with R.* by Julian Faraway

  - Online copy available through Duke library

- More mathematical details in *Computational methods for mixed models* by Douglas Bates (a vingette for the **lme4** R package)