Multilevel Generalized Linear Models

04.11.22

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Announcements

- Quiz 04 open **Tue, Apr 12 - Thu, Apr 14**
- Final project - optional draft due
  - **Fri, Apr 15**
  - final report due **Wed, Apr 27**
- Please fill out course evaluations!
- Click here for answers to questions about multilevel models submitted on Quiz 03. Thanks to Jose for putting this document together!

Learning goals

- Exploratory data analysis for multilevel data with non-normal response variable
- Use a two-stage modeling approach to understand conclusions at each level
- Write Level One, Level Two and composite models for mutlilevel GLM
- Fit and interpret multilevel GLM

Data: College Basketball referees

The dataset **`basketball0910.csv`** contains data on 4972 fouls in 340 NCAA basketball games from the Big Ten, ACC, and Big East conferences during the 2009-2010 season. The goal is to determine whether the data from this season support a conclusion from Anderson and Pierce (2009) that referees tend to "even out" foul calls in a game. The variables we'll focus on are

- **`foul.home`**: foul was called on home team (1: yes, 0: no)
- **`foul.diff`**: difference in fouls before current foul was called (home - visitor)
- **`game`**: Unique game ID number
- **`visitor`**: visiting team abbreviation
- **`home`**: home team abbreviation

See BMLR: Section 11.3.1 for full codebook.

Data: College basketball referees

game	visitor	hometeam	foul.num	foul.home	foul.vis	foul.diff	foul.type	time
1	IA	MN	1	0	1	0	Personal	14.167
1	IA	MN	2	1	0	-1	Personal	11.433
1	IA	MN	3	1	0	0	Personal	10.233
1	IA	MN	4	0	1	1	Personal	9.733
1	IA	MN	5	0	1	0	Shooting	7.767
1	IA	MN	6	0	1	-1	Shooting	5.567
1	IA	MN	7	1	0	-2	Shooting	2.433
1	IA	MN	8	1	0	-1	Offensive	1.000
2	MI	MIST	1	0	1	0	Shooting	18.983
2	MI	MIST	2	1	0	-1	Personal	17.200

Modeling approach

- **`foul.home`** is a binary response variable (1: foul called on home team, 0: foul called on visiting team)

  ✅ Use a generalized linear model, specifically one with the logit link function

- Data has a multilevel structure
  - Covariates at individual foul level and game level

  ✅ Use a multilevel model

We will combine these and fit a **multilevel generalized linear model** with a logit link function

Exploratory data analysis

Exploratory data analysis

Univariate

- Visualizations and summary statistics for Level One and Level Two variables

Bivariate

- Segmented bar plots or mosaic plots for response vs. categorical predictors
- Conditional density plot for response vs. quantitative predictors
- Empirical logit plot for quantitative predictors
    - Is relationship reasonably linear?

Complete *Univariate EDA* in `lecture-25.Rmd`

05:00

Complete *Bivariate EDA* in `lecture-25.Rmd`

04:00

Logistic regression

Start with a logistic regression model treating all observations as independent

Quick analysis to get initial intuition about research question sand important variables, not be used for final conclusions

term	estimate	std.error	statistic	p.value
(Intercept)	-0.103	0.101	-1.023	0.306
foul.diff	-0.077	0.025	-3.078	0.002
score.diff	0.020	0.007	3.012	0.003
lead.home	-0.093	0.160	-0.582	0.560
time	-0.013	0.008	-1.653	0.098
foul.diff:time	-0.007	0.003	-2.485	0.013
lead.home:time	0.022	0.011	1.957	0.050

Two-stage modeling

Two-stage modeling

For now, let's fit a model focusing on the question: **Do the data provide evidence that referees tend to "even out" fouls?**

To explore this, we will fit a model with the Level One predictor **`foul.diff`** using a two-stage modeling approach.

Two-stage modeling approach

- Fit a separate model of the association between **`foul.diff`** and **`foul.home`** for each game (Level One models)
- Fit models to explain game-to-game variability in the estimated slopes and intercepts (Level Two models)

Model set up

- Y_{ij}: 1 if j^{th} foul in Game i was called on home team; 0 otherwise.
- p_{ij}: True probability the j^{th} foul in Game i was called on home team

$$Y_{ij} \sim \text{Bernoulli}(p_{ij})$$

Model Set Up

Level One models

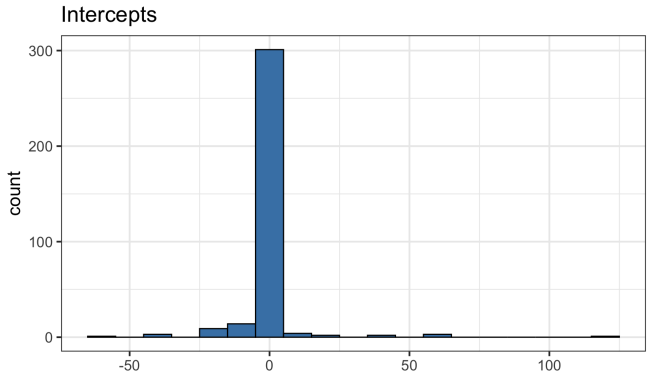
$$\log\left(\frac{p_{ij}}{1-p_{ij}}\right) = a_i + b_i \,\text{foul.diff}_{ij}$$

Level Two models

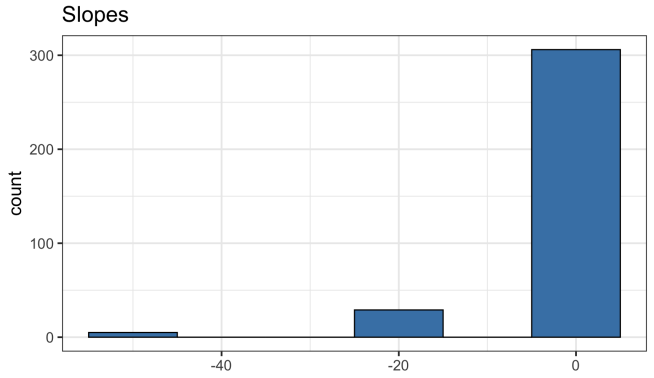
$$a_i = \alpha_0 + u_i \qquad b_i = \beta_0 + v_i$$

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix}\right)$$

Level One Models

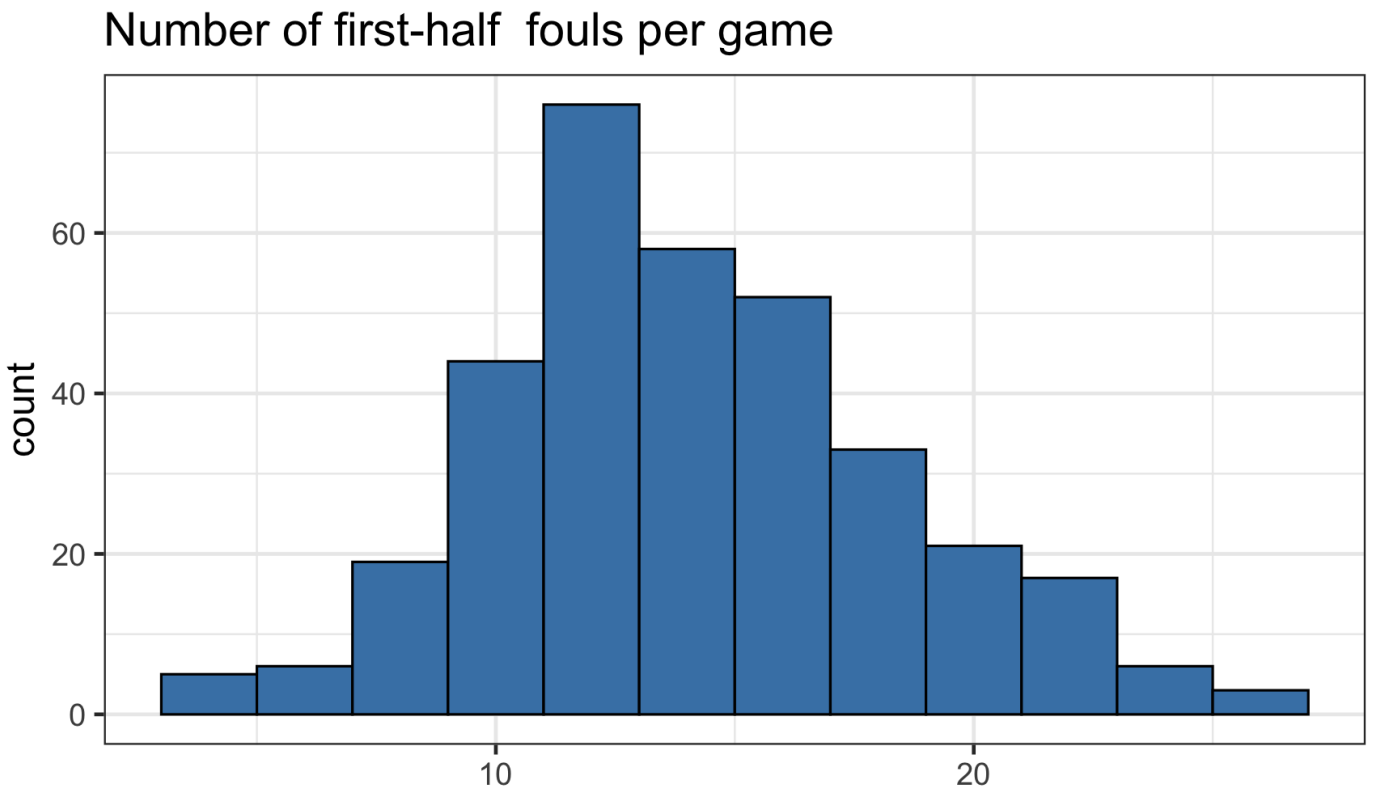


alpha0	sigma2_u
-0.301	128.858



beta0	sigma2_v
-3.328	60.049

Number of fouls per game



⚠️ In the two-stage approach, games with 3 fouls are treated with equal weight as games with 20 + fouls

Unified multilevel model

Composite model

Composite model

$$\log\left(\frac{p_{ij}}{1-p_{ij}}\right) = \alpha_0 + \beta_0 \, \text{foul.diff}_{ij} + [u_i + v_i \, \text{foul.diff}_{ij}]$$

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix}\right)$$

Fit the model in R

Use the **`glmer`** function in the **lme4** package to fit multilevel GLMs.

```
model1 <- glmer(foul.home ~ foul.diff + (foul.diff|game),
                data = basketball, family = binomial)
```

```
## boundary (singular) fit: see help('isSingular')
```

effect	group	term	estimate	std.error	statistic	p.value
fixed	NA	(Intercept)	-0.157	0.046	-3.382	0.001
fixed	NA	foul.diff	-0.285	0.038	-7.440	0.000
ran_pars	game	sd__(Intercept)	0.542	NA	NA	NA
ran_pars	game	cor__(Intercept).foul.diff	-1.000	NA	NA	NA
ran_pars	game	sd__foul.diff	0.035	NA	NA	NA

Boundary constraints

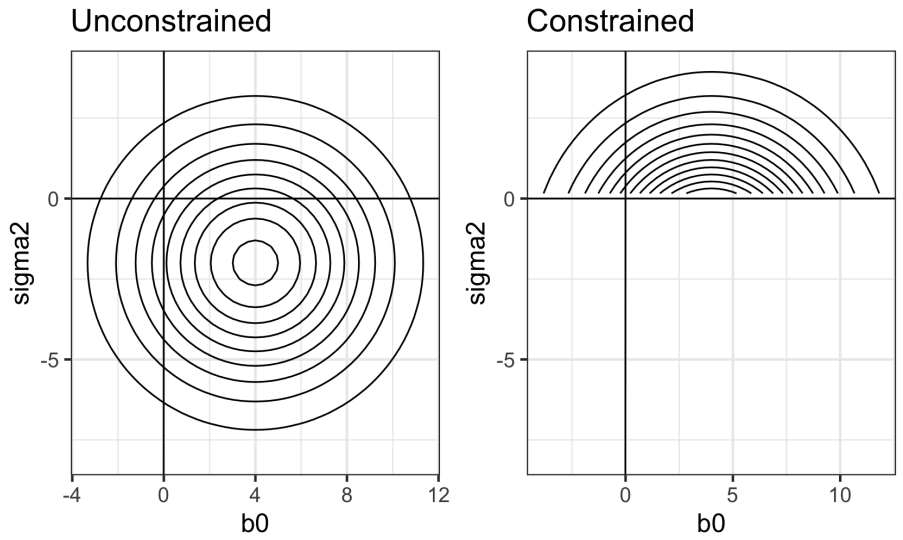
- The estimates of the parameters $\alpha_0, \beta_0, \sigma_u, \sigma_v, \rho_{uv}$ are those that maximize the likelihood of observing the data
- The fixed effects, e.g., α_0 and β_0, can take any values, but the terms associated with the error terms are constrained to a set of "allowable" values

$$\sigma_u \geq 0 \quad \sigma_v \geq 0 \quad -1 \leq \rho_{uv} \leq 1$$

- Because of these boundaries, a "constrained" search is used to find the MLEs.
- The error message **`"## boundary (singular) fit"`**, means the estimate of one or more terms was set at the maximum (or minimum) allowable value, not the value it would've been if an unconstrained search were allowable

Illustrating boundary constraints

Contour plots from a hypothetical likelihood $L(\beta_0, \sigma^2)$



- In the unconstrained search, the likelihood $L(\beta_0, \sigma^2)$ is maximized at $\hat{\beta}_0 = 4, \hat{\sigma}^2 = -2$
- In reality σ^2 must be non-negative, so the search for the MLE is restricted to the region such that $\sigma^2 \geq 0$.
- The constrained likelihood is maximized at $\hat{\beta}_0 = 4, \hat{\sigma}^2 = 0$

From BMLR Figure 10.14

Addressing boundary constraints

Address boundary constraints by reparameterizing the model

- Remove variance and/or correlation terms estimated at the boundary
- Reduce the number of parameters to be estimated by fixing the values of some parameters
- Apply transformation to covariates, such as centering variables, standardizing variables, or changing units. Extreme values, outliers, or highly correlated covariates can sometimes cause issues with MLEs.

Is it OK to use model that faces boundary constraints?

Best to try to deal with boundary constraints, but you can sometimes leave the model as is if...

- You're not interested in estimating parameters with boundary issues
- Removing the parameters does not impact conclusions about the variables of interest

Original model

effect	group	term	estimate	std.error	statistic	p.value
fixed	NA	(Intercept)	-0.157	0.046	-3.382	0.001
fixed	NA	foul.diff	-0.285	0.038	-7.440	0.000
ran_pars	game	sd__(Intercept)	0.542	NA	NA	NA
ran_pars	game	cor__(Intercept).foul.diff	-1.000	NA	NA	NA
ran_pars	game	sd__foul.diff	0.035	NA	NA	NA

1. Which term(s) should we remove to try to address boundary constraint?
2. Refit the model with these terms removed.
3. Which model do you choose? Explain.

04:00

Interpret model coefficients

Interpret the following coefficients in the selected model (if applicable):

- $\hat{\alpha}_0$
- $\hat{\beta}_0$
- $\hat{\sigma}_u$
- $\hat{\sigma}_v$
- $\hat{\rho}_{uv}$

Looking ahead

So far we've only considered random effects within a nested structure, but sometimes we may want to consider random effects that aren't nested.

- For example, we want to consider a random effect for the home team, but home team is not nested within game (since teams can be the home team for multiple games)
- We will deal with this using **crossed random effects**

Acknowledgements

- BMLR: Section 10.5 - Encountering boundary constraints
- Chapter 11: Multilevel generalized linear models structure
    - Sections 11.1 - 11.4
- Anderson, Kyle J., and David A. Pierce. 2009. “Officiating Bias: The Effect of Foul Differential on Foul Calls in NCAA Basketball.” Journal of Sports Sciences 27 (7): 687–94. https://doi.org/10.1080/02640410902729733.