Multilevel models

Estimation + Interpretation

🥧 day 2022

STA 310

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Announcements

- Quiz 03: Wed, Mar 16 - Fri, Mar 18
  - Feb 21 - Mar 14 lectures
- DataFest: April 1 - 3 in Penn Pavilion
  - Click here to sign up

Mini-project 02

Click here for project instructions

Updated timeline

- **Draft for peer review**: due Thu, Mar 24 at 12pm (noon)
- **Peer reviews**: due Thu, Mar 24 at 11:59pm
- **Presentation**: due Mon, Mar 28 at 3:30pm
- **Written report**: due Mon, Mar 28 at 11:59pm

Final project

Click here for project instructions

Timeline

- **Round 1 submission (optional)**: Wednesday, April 13 at 11:59pm
- **Final submission**: Wednesday, April 27 at 11:59pm

Mid-semester feedback

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Learning goals

- Fit and interpret multilevel models
- Compare maximum likelihood (ML) and restricted maximum likelihood (REML) estimation approaches
- Understand general process for fitting and comparing multilevel models

Data: Music performance anxiety

The data `musicdata.csv` come from the Sadler and Miller (2010) study of the emotional state of musicians before performances. The dataset contains information collected from 37 undergraduate music majors who completed the Positive Affect Negative Affect Schedule (PANAS), an instrument produces a measure of anxiety (negative affect) and a measure of happiness (positive affect). This analysis will focus on negative affect as a measure of performance anxiety.

The primary variables we'll use are

- **`na`**: negative affect score on PANAS (the response variable)
- **`LargeEnsemble`**: 1: performance is large ensemble; 0: performance is solo or small ensemble
- **`Orchestra`**: 1: orchestral instrument; 0: voice or piano
- **`mpqnem`**: negative emotionality (NEM) composite scale from MPQ

Look at data

id	diary	large_ensemble	mpqnem	orchestra
1	1	0	16	0
1	2	1	16	0
1	3	1	16	0
43	1	0	17	0
43	2	0	17	0
43	3	0	17	0

- **`diary`**, **`na`**, **`large_ensemble`** are Level One variables (performance-level variables)
- **`orchestra`**, **`mpqnem`** are Level Two variables (musician-level variables)

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The model

Model using `Orchestra` and `LargeEnsemble` to understand variability in performance anxiety (`na`)

Two-stage model

Level One Model

$$na_{ij} = a_i + b_i \; LargeEnsemble_{ij} + \epsilon_{ij}, \quad \epsilon_{ij} \sim N(0, \sigma^2)$$

Level Two Models

$$\begin{aligned} a_i &= \alpha_0 + \alpha_1 \; Orchestra_i + u_i \\ b_i &= \beta_0 + \beta_1 \; Orchestra_i + v_i \end{aligned}$$

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Composite model

Plug in the equations for a_i and b_i to get the **composite model**

$$
\begin{aligned}
Y_{ij} = &(\alpha_0 + \alpha_1\, Orchestra_i + \beta_0\, LargeEnsemble_{ij} \\
&+ \beta_1\, Orchestra_i : LargeEnsemble_{ij}) \\
&+ (u_i + v_i\, LargeEnsemble_{ij} + \epsilon_{ij})
\end{aligned}
$$

- The **fixed effects** to estimate are $\alpha_0, \alpha_1, \beta_0, \beta_1$
- The **error terms** are u_i, v_i, ϵ_{ij}
  - These describe the behavior of the random effects When we use the composite model, we no longer need to estimate a_i and b_i directly as we did with the two-stage approach.

Notation

- Greek letters denote the fixed effect model parameters that will be estimated
    - e.g., $\alpha_0, \alpha_1, \beta_0, \beta_1$
- English letters denote the preliminary effects at lower levels that will not be estimated directly
    - e.g. a_i, b_i
- σ and ρ denote variance components that will be estimated
- ϵ_{ij}, u_i, v_i denote error terms

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Error terms

- Assume random effects are normally distributed with mean 0 and variance that will be estimated

- Level One: Errors associated with each performance of a given musician are are normally distributed, i.e., $\epsilon_{ij} \sim N(0, \sigma^2)$

- Level Two: Errors are

  - u_i: deviation of musician i from the mean performance anxiety before solos and small ensembles after accounting for the instrument (musician-to-musician differences in intercept)

  - v_i: deviance of musician i from the mean difference in performance anxiety between large ensembles and other performance types after accounting for instrument (musician-to-musician differences in slope)

- Need to account for correlation between u_i and v_i for the i^{th} musician

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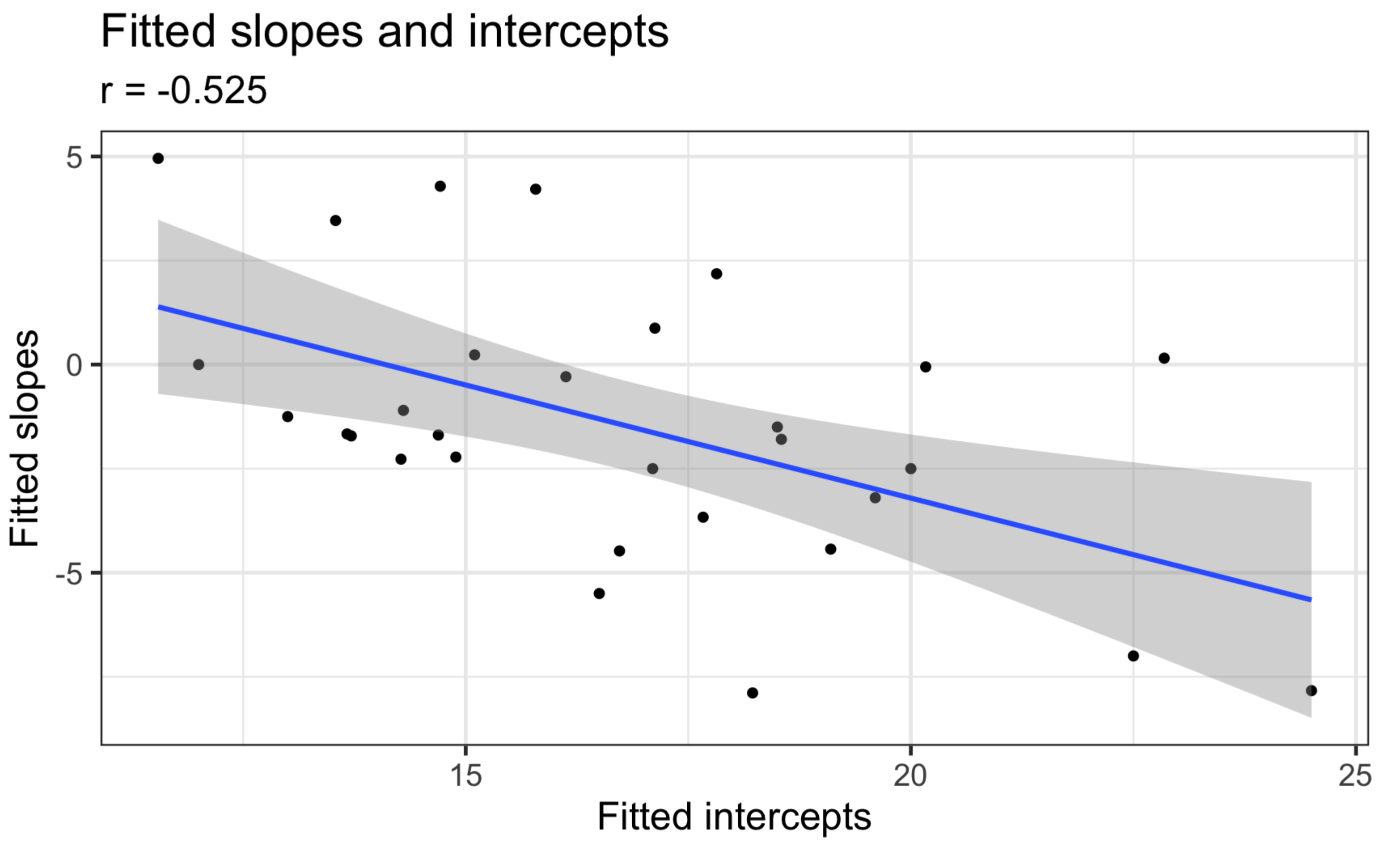

Recreated from Figure 8.11

Describe the association between the slopes and intercepts.

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Distribution of Level Two errors

The Level Two error terms follow a **multivariate normal** distribution

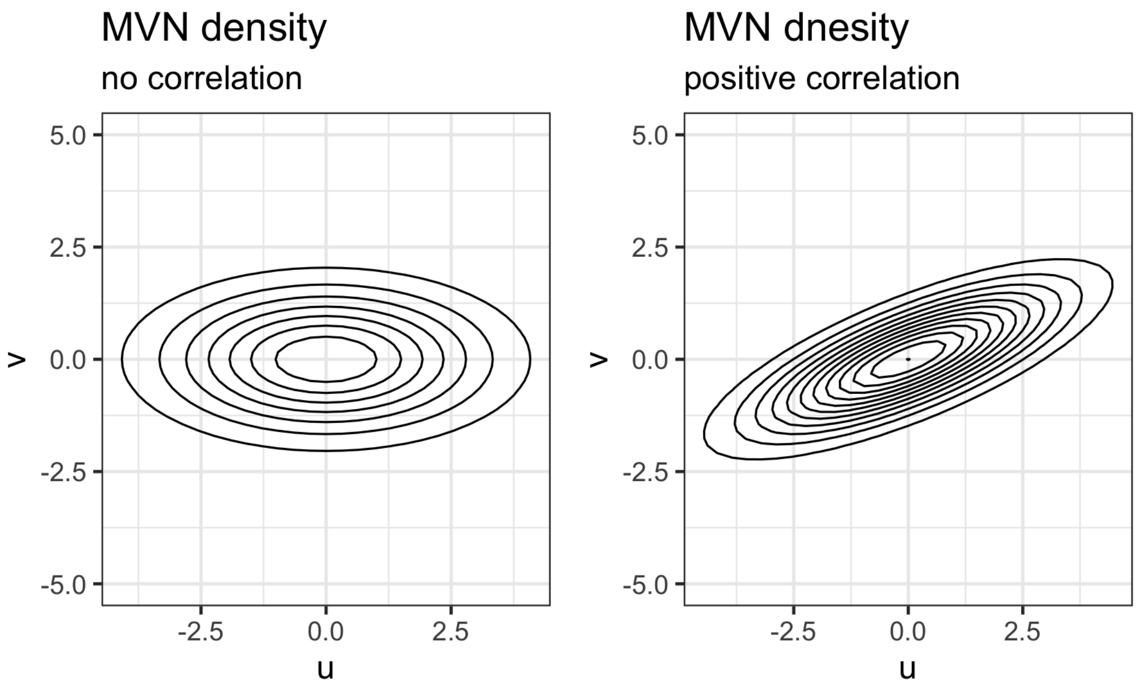
$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v \\ \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 \end{bmatrix}\right)$$

where σ_u^2 and σ_v^2 are the variances of u_i's and v_i's respectively, ρ_{uv} is the correlation between u_i and v_i

Covariance: $\sigma_{uv} = \rho_{uv}\sigma_u\sigma_v$

- Indicates how those terms vary together

Visualizing multivariate normal distribution



Recreated from Figure 8.12

Fit the model in R

Fit multilevel model using the **`lmer`** ("linear mixed effects in R") function from the lme4 package.

```
library(lme4)
music_model <- lmer(na ~ orchestra + large_ensemble +
                      orchestra:large_ensemble + (large_ensemble|id),
                    data = music)
```

`na ~ orchestra + large_ensemble + orchestra:large_ensemble`: Represents the fixed effects + intercept

`(large_ensemble|id)`: Represents the error terms and associated variance components

- Specifies two error terms: u_i corresponding to the intercepts, v_i corresponding to effect of large ensemble

Tidy output

Display results using the **`tidy`** function from the broom.mixed package.

```
library(broom.mixed)
tidy(music_model)
```

- Get fixed effects only

```
tidy(music_model) %>% filter(effect == "fixed")
```

- Get errors and variance components only

```
tidy(music_model) %>% filter(effect == "ran_pars")
```

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Model output: Fixed effects

effect	group	term	estimate	std.error	statistic
fixed	NA	(Intercept)	15.930	0.641	24.833
fixed	NA	orchestra1	1.693	0.945	1.791
fixed	NA	large_ensemble1	-0.911	0.845	-1.077
fixed	NA	orchestra1:large_ensemble1	-1.424	1.099	-1.295

Label the fixed effects

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Model output: Random effects

effect	group	term	estimate	std.error	statistic
ran_pars	id	sd__(Intercept)	2.378	NA	NA
ran_pars	id	cor__(Intercept).large_ensemble1	-0.635	NA	NA
ran_pars	id	sd__large_ensemble1	0.672	NA	NA
ran_pars	Residual	sd__Observation	4.670	NA	NA

Label the error terms and variance components

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Interpret the effects

- Split into 4 groups.
- Each group will write the interpretation for one main effect and one variance component. The terms on each slide do not necessarily have a direct correspondence to each other.
- One person write the group's interpretations on the slide.
- Click here for the slides.

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06:00

Fitting the model

Comparing fixed effects estimates

Below are the estimated coefficients and standard errors for four modeling approaches.

from BMLR Table 8.3

Variable	Independence	Two.Stage	LVCF	Multilevel
Intercept	15.72(0.36)	16.28(0.67)	15.20(1.25)	15.93(0.64)
Orch	1.79(0.55)	1.41(0.99)	1.45(1.84)	1.69(0.95)
Large	-0.28(0.79)	-0.77(0.85)	-	-0.91(0.85)
Orch*Large	-1.71(1.06)	-1.41(1.20)	-	-1.42(1.10)

- How do the coefficient estimates compare?
- How do the standard errors compare?

ML and REML

Maximum Likelihood (ML) and Restricted (Residual) Maximum Likelihood (REML) are the two most common methods for estimating the fixed effects and variance components

Maximum Likelihood (ML)

- Jointly estimate the fixed effects and variance components using all the <u>sample data</u>
- Can be used to draw conclusions about fixed and random effects
- *Issue*: Fixed effects are treated as known values when estimating variance components
  - Results in biased estimates of variance components (especially when sample size is small)

ML and REML

Restricted Maximum Likelihood (REML)

- Estimate the variance components using the sample residuals not the sample data
- It is conditional on the fixed effects, so it accounts for uncertainty in fixed effects estimates. This results in unbiased estimates of variance components.

Example using OLS

See the post Maximum Likelihood (ML) vs. REML for details and illustration of ML vs. REML.

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ML or REML?

- Research has not determined one method absolutely superior to the other
- REML (`REML = TRUE`; default in `lmer`) is preferable when
  - the number of parameters is large or primary, or
  - primary objective is to obtain estimates of the model parameters
- ML (`REML = FALSE`) must be used if you want to compare nested fixed effects models using a likelihood ratio test (e.g., a drop-in-deviance test).
  - For REML, the goodness-of-fit and likelihood ratio tests can only be used to draw conclusions about variance components

Source: Singer, J. D. & Willett, J. B. (2003). *Applied longitudinal data analysis: Modeling change and event occurrence*. Oxford university press.

Comparing ML and REML

	Maximum Likelihood		Restricted Maximum Likelihood	
Term	**Estimate**	**Std. Error**	**Estimate**	**Std. Error**
(Intercept)	15.924	0.623	15.930	0.641
orchestra1	1.696	0.919	1.693	0.945
large_ensemble1	-0.895	0.827	-0.911	0.845
orchestra1:large_ensemble1	-1.438	1.074	-1.424	1.099
sd__(Intercept)	2.286	NA	2.378	NA
cor__(Intercept).large_ensemble1	-1.00	NA	-0.635	NA
sd__large_ensemble1	0.385	NA	0.672	NA
sd__Observation	4.665	NA	4.670	NA

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Strategy for building multilevel models

- Conduct exploratory data analysis for Level One and Level Two variables
- Fit model with no covariates to assess variability at each level
- Create Level One models. Start with a single term, then add terms as needed.
- Create Level Two models. Start with a single term, then add terms as needed. Start with equation for intercept term.
- Begin with the full set of variance components, then remove variance terms as needed.

Alternate model building strategy in BMLR Section 8.6

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Open **`lecture-17.Rmd`**.

See BMLR Sections 8.6 - 8.11 for full step-by-step analysis.

Acknowledgements

The content in the slides is from

- BMLR: Chapter 8 - Introduction to Multilevel Models
  - Sections 8.5 - 8.12
  - Singer, J. D. & Willett, J. B. (2003). *Applied longitudinal data analysis: Modeling change and event occurrence*. Oxford university press.