# BIL 133 HW1

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## 1

### 1.1

p : Alice is a friend of Bob
q : Bob is a friend of Carol
r : Alice is a friend of Carol
$p \wedge q \rightarrow r$

### 1.2

p : You have read the lecture notes
q : You have done the first three homework assignments
r : You should be in good shape for the first exam
s : You will have a problem
$(p \wedge q \rightarrow r) \wedge (\neg(p \wedge q) \rightarrow s)$

### 1.3

p : Cancer will not be cured
q : Cancer's cause is determined
r : New drug for cancer is found
$\neg(q \wedge r) \rightarrow p$

### 1.4

p : Smith has installed central heating
q : Smith has sold his car
r : Smith has not paid his mortgage.
$p \rightarrow q \vee r$

# 2

**(A)**

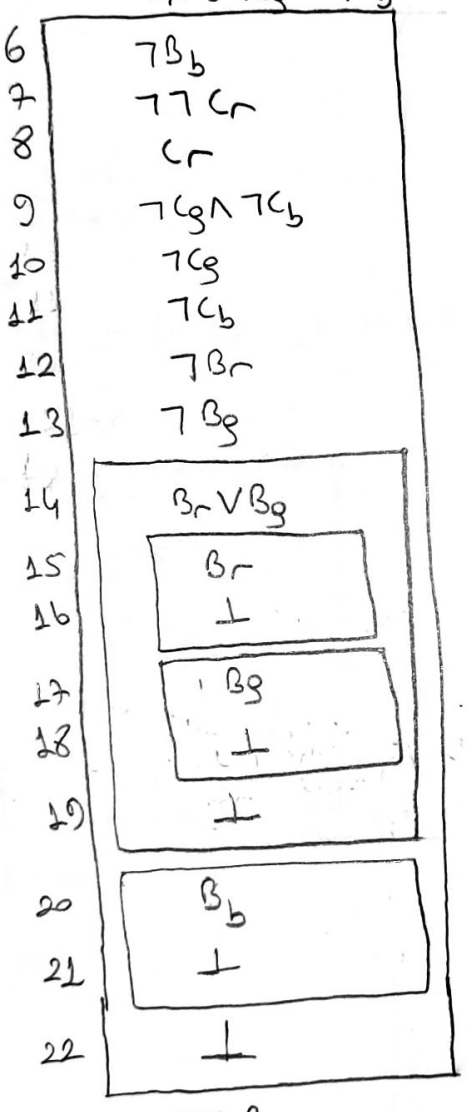
According to the last statement, if Cansu does not wear red (which means if Cansu wear green or blue), Banu wears blue.
According to the first 2 statements, if Banu wears red or green(which means if Banu does not wear blue), Cansu wears either green or blue.
Which means Cansu does not wear red.
According to the last statement, Cansu must have wear red. This is a contradiction. So, Banu can not wear red or green.
Therefore, Banu's outfit is blue.

**(B)**

$$\mathrm{B}_r \rightarrow C_g$$
$$\mathrm{B}_g \rightarrow C_b$$
$$\neg C_r \rightarrow B_b$$
$$\mathrm{B}_r \vee B_g \vee B_b$$
$$\mathrm{C}_r \rightarrow \neg C_g \wedge \neg C_b$$

(c)

$B_r \to C_g, B_g \to C_b, \neg C_r \to B_b, (B_r \vee B_g) \vee B_b, C_r \to \neg C_g \wedge \neg C_b \vdash B_b$



| | | |
|---|---|---|
| 1 | $B_r \to C_g$ | Premise |
| 2 | $B_g \to C_b$ | Premise |
| 3 | $\neg C_r \to B_b$ | Premise |
| 4 | $(B_r \vee B_g) \vee B_b$ | Premise |
| 5 | $C_r \to \neg C_g \wedge \neg C_b$ | Premise |
| 6 | $\neg B_b$ | Assumption |
| 7 | $\neg\neg C_r$ | MT 3, 6 |
| 8 | $C_r$ | $\neg\neg_e$ 7 |
| 9 | $\neg C_g \wedge \neg C_b$ | $\to_e$ 8, 5 |
| 10 | $\neg C_g$ | $\wedge e_1$ 9 |
| 11 | $\neg C_b$ | $\wedge e_2$ 9 |
| 12 | $\neg B_r$ | MT 1, 10 |
| 13 | $\neg B_g$ | MT 2, 11 |
| 14 | $B_r \vee B_g$ | Assumption |
| 15 | $B_r$ | Assumption |
| 16 | $\bot$ | $\neg_e$ 15, 12 |
| 17 | $B_g$ | Assumption |
| 18 | $\bot$ | $\neg_e$ 17, 13 |
| 19 | $\bot$ | $\vee_e$ 14, 15-16, 17-18 |
| 20 | $B_b$ | Assumption |
| 21 | $\bot$ | $\neg_e$ 20, 6 |
| 22 | $\bot$ | $\vee_e$ 4, 14-19, 20-21 |
| 23 | $\neg\neg B_b$ | $\neg_i$ 6-22 |
| 24 | $B_b$ | $\neg\neg_e$ 23 |

qed

3

3.a

$(p \to r) \land (q \to r) \vdash p \land q \to r$

| | | |
|---|---|---|
| 1. | $(p \to r) \land (q \to r)$ | premise |
| 2. | $p \to r$ | $\land e_1$ 1 |
| 3. | $p \land q$ | assumption |
| 4. | $p$ | $\land e_1$ 3 |
| 5. | $r$ | $\to e$ 4,2 |
| 6. | $p \land q \to r$ | $\to i$ 3-5 |

qed

3.b

$q \to r \vdash (p \to q) \to (p \to r)$

| | | |
|---|---|---|
| 1. | $q \to r$ | premise |
| 2. | $p \to q$ | assumption |
| 3. | $p$ | assumption |
| 4. | $q$ | $\to e$ 3,2 |
| 5. | $r$ | $\to e$ 4,1 |
| 6. | $p \to r$ | $\to i$ 3-5 |
| 7. | $(p \to q) \to (p \to r)$ | $\to i$ 2-6 |

qed

3.c

$p \to q \vdash ((p \wedge q) \to p) \wedge (p \to (p \wedge q))$

| | | |
|---|---|---|
| 1. | $p \to q$ | premise |
| 2. | $p \wedge q$ | assumption |
| 3. | $p$ | $\wedge e_1$ 2 |
| 4. | $(p \wedge q) \to p$ | $\to i$ 2-3 |
| 5. | $p$ | assumption |
| 6. | $q$ | $\to e$ 5,1 |
| 7. | $p \wedge q$ | $\wedge i$ 5,6 |
| 8. | $p \to (p \wedge q)$ | $\to i$ 5-7 |
| 9. | $((p \wedge q) \to p) \wedge (p \to (p \wedge q))$ | $\wedge i$ 4,8 |

qed

3.d

$\vdash (p \to q) \to ((r \to s) \to (p \wedge r \to q \wedge s))$

| | | |
|---|---|---|
| 1. | $p \to q$ | assumption |
| 2. | $r \to s$ | assumption |
| 3. | $p \wedge r$ | assumption |
| 4. | $p$ | $\wedge e_1$ 3 |
| 5. | $r$ | $\wedge e_2$ 3 |
| 6. | $q$ | $\to e$ 4,1 |
| 7. | $s$ | $\to e$ 5,2 |
| 8. | $q \wedge s$ | $\wedge i$ 6,7 |
| 9. | $p \wedge r \to q \wedge s$ | $\to i$ 3-8 |
| 10. | $(r \to s) \to (p \wedge r \to q \wedge s)$ | $\to i$ 2-9 |
| 11. | $(p \to q) \to ((r \to s) \to (p \wedge r \to q \wedge s))$ | $\to i$ 1-10 |

qed

3.e

$\vdash \neg p \to (p \to (p \to q))$

| | | |
|---|---|---|
| 1. | $\neg p$ | assumption |
| 2. | $p$ | assumption |
| 3. | $\bot$ | $\neg_e$ 2,1 |
| 4. | $p \to q$ | $\bot_e$ 3 |
| 5. | $p \to (p \to q)$ | $\to_i$ 2-4 |
| 6. | $\neg p \to (p \to (p \to q))$ | $\to_i$ 1-5 |

qed

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3.f

$q \vdash (p \wedge q) \vee (\neg p \wedge q)$

| | | |
|---|---|---|
| 1. | $q$ | premise |
| 2. | $\neg((p \wedge q) \vee (\neg p \wedge q))$ | assumption |
| 3. | $p$ | assumption |
| 4. | $p \wedge q$ | $\wedge_i$ 3,1 |
| 5. | $((p \wedge q) \vee (\neg p \wedge q))$ | $\vee i_1$ 4 |
| 6. | $\bot$ | $\neg_e$ 5,2 |
| 7. | $\neg p$ | $\neg_i$ 3-6 |
| 8. | $\neg p \wedge q$ | $\wedge_i$ 7,1 |
| 9. | $((p \wedge q) \vee (\neg p \wedge q))$ | $\vee i_2$ 8 |
| 10. | $\bot$ | $\neg_e$ 9,2 |
| 11. | $\neg\neg((p \wedge q) \vee (\neg p \wedge q))$ | $\neg_i$ 2-10 |
| 12. | $(p \wedge q) \vee (\neg p \wedge q)$ | $\neg\neg_e$ 11 |

qed

4.a

$p \wedge \neg p \vdash \neg(r \to q) \wedge (r \to q)$

| | | |
|---|---|---|
| 1. | $p \wedge \neg p$ | premise |
| 2. | $p$ | $\wedge_{e_1}$ 1 |
| 3. | $\neg p$ | $\wedge_{e_2}$ 2 |
| 4. | $\bot$ | $\neg_e$ 2,3 |
| 5. | $\neg(r \to q) \wedge (r \to q)$ | $\bot_e$ 4 |

qed

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4.b

$\neg(\neg p \vee \neg q) \vdash p \wedge q$

| | | |
|---|---|---|
| 1. | $\neg(\neg p \vee \neg q)$ | premise |
| 2. | $\neg p$ | assumption |
| 3. | $\neg p \vee \neg q$ | $\vee_{i_1}$ 2 |
| 4. | $\bot$ | $\neg_e$ 3,1 |
| 5. | $\neg\neg p$ | $\neg_i$ 2-4 |
| 6. | $p$ | $\neg\neg_e$ 5 |
| 7. | $\neg q$ | assumption |
| 8. | $\neg p \vee \neg q$ | $\vee_{i_2}$ 7 |
| 9. | $\bot$ | $\neg_e$ 8,1 |
| 10. | $\neg\neg q$ | $\neg_i$ 7-9 |
| 11. | $q$ | $\neg\neg_e$ 10 |
| 12. | $p \wedge q$ | $\wedge_i$ 6,11 |

qed

4.c

$(p \to q) \to r,\ s \to \neg p,\ t,\ (\neg s \wedge t) \to q \vdash r$

| | | |
|---|---|---|
| 1. | $(p \to q) \to r$ | premise |
| 2. | $s \to \neg p$ | premise |
| 3. | $t$ | premise |
| 4. | $\neg s \wedge t \to q$ | premise |
| 5. | $p$ | assumption |
| 6. | $\neg\neg p$ | $\neg\neg_i$ 5 |
| 7. | $\neg s$ | MT 2,6 |
| 8. | $\neg s \wedge t$ | $\wedge_i$ 7,3 |
| 9. | $q$ | $\to_e$ 8,4 |
| 10. | $p \to q$ | $\to_i$ 5–9 |
| 11. | $r$ | $\to_e$ 10,1 |

qed

4.d

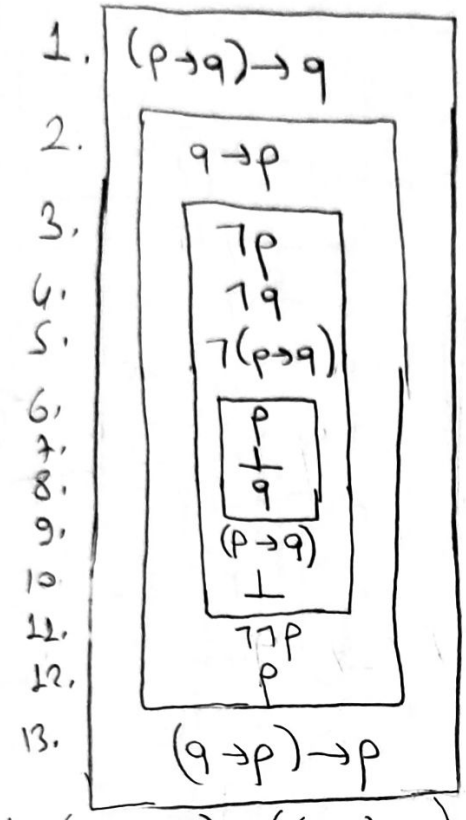
$\vdash ((p \to q) \wedge (q \to p)) \to ((p \vee q) \to (p \wedge q))$

| | | |
|---|---|---|
| 1. | $(p \to q) \wedge (q \to p)$ | assumption |
| 2. | $(p \to q)$ | $\wedge e_1$ 1 |
| 3. | $(q \to p)$ | $\wedge e_2$ 1 |
| 4. | $p \vee q$ | assumption |
| 5. | $p$ | assumption |
| 6. | $q$ | $\to_e$ 5,2 |
| 7. | $p \wedge q$ | $\wedge_i$ 5,6 |
| 8. | $q$ | assumption |
| 9. | $p$ | $\to_e$ 8,3 |
| 10. | $p \wedge q$ | $\wedge_i$ 9,10 |
| 11. | $p \wedge q$ | $\vee e$ 4,5–7,8–10 |
| 12. | $p \vee q \to p \wedge q$ | $\to_i$ 4–11 |
| 13. | $((p \to q) \wedge (q \to p)) \to ((p \vee q) \to (p \wedge q))$ | $\to_i$ 1–12 |

qed

4.e

$\vdash ((p \to q) \to q) \to ((q \to p) \to p)$

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5

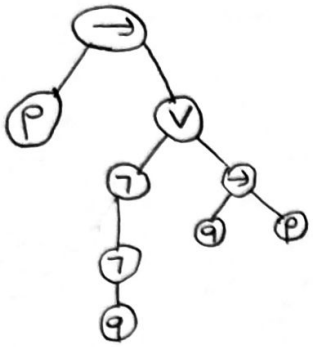
There is a problem in the inductive step. If $n=2$ (which means if there are only two horses), there is no middle horses. It is true that when we remove one horse, rest has the same colour which is the only one horse. But, we cannot conclude that all horses' in $n=2$ sitiation have the same color.

There is a step missing, proof is wrong.

6

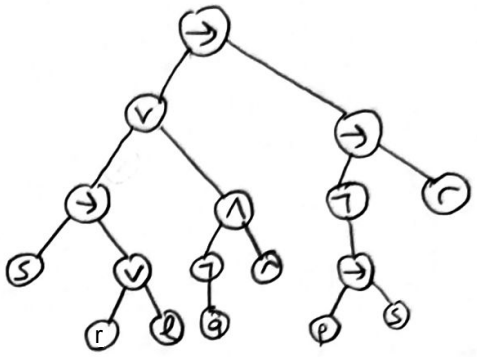
6.a

$p \to (\neg\neg q \vee (q \to p))$



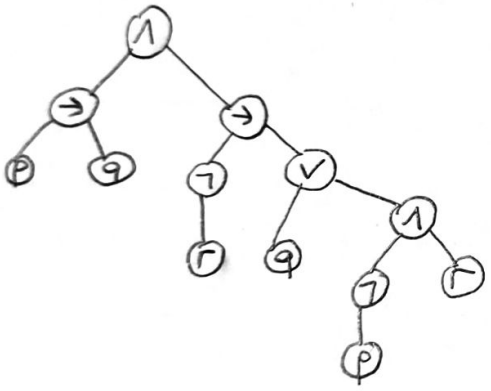
6.b

$(((s \to (r \vee 1)) \vee ((\neg q) \wedge r)) \to ((\neg(p \to s)) \to r))$



6.c

$(p \to q) \wedge (\neg r \to (q \vee (\neg p \wedge r)))$



Since formula is not always true, it is not valid.
But in some cases it is true, so it is satisfiable.

7

| p | q | r | ¬p | q→¬p | r∨q | p→r∨q | (q→¬p)∧(p→r∨q) | ¬((q→¬p)∧(p→r∨q)) |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |