TOBB ETU COMPUTER ENGINEERING DEPARTMENT

BİL 133 Combinatorics and Graph Theory

HOMEWORK 1 (35 Points)

Due Date: May 24, 2020

1 [4 POINTS] CONVERTING DECLARATIVE SENTENCES TO THE LANGUAGE OF PROPOSITIONAL LOGIC

The aim of logic in computer science is to develop languages to model situations we encounter, in such a way that we can *reason* (construct arguments) about them *formally* so that the arguments are *valid* and can be executed on a machine. This question test your ability to express several real-life scenarios in the language of propositional logic.

Use $\neg, \wedge, \vee$, and $\rightarrow$ to express the following declarative sentences in propositional logic; in each case state what your respective propositional atoms p, q, etc. mean:

- $[1 Point]$ If Alice is a friend of Bob, and Bob is a friend of Carol then Alice is a friend of Carol.
- $[1 Point]$ If you have read the lecture notes and if you have done the first three homework assignments, then you should be in good shape for the first exam; otherwise, you will have a problem.
- $[1 Point]$ Cancer will not be cured unless its cause is determined and a new drug for cancer is found.
- $[1 Point]$ If Smith has installed central heating, then he has sold his car, or he has not paid his mortgage.

2 [8 POINTS] CHOOSING AN OUTFIT FOR A PARTY WITH PROPOSITIONAL LOGIC

Banu and Cansu are getting ready for a themed party, where attendees have to wear either red, green or blue outfits. Banu and Cansu have the following preferences which have to be satisfied:

(1) If Banu wears red, then Cansu wears green.

(2) If Banu wears green, then Cansu wears blue.

(3) If Cansu does not wear red, then Banu wears blue.

At the party, what is the color of Banu's outfit?

(a) First, explain your reasoning informally.

(b) Then, given the following boolean variables:

B_r = *"Banu wears red"*
B_g = *"Banu wears green"*
B_b = *"Banu wears blue"*

C_r = *"Cansu wears red"*
C_g = *"Cansu wears green"*
C_b = *"Cansu wears blue"*

Specify the premises that you need for formalizing your reasoning in (a).

(c) Using the premises you have given in (b) and the natural deduction rules, prove what color Banu's outfit is.

3 [8 POINTS] ON NATURAL DEDUCTION

Prove the following sequents by using the calculus of propositional logic.

- $[1 Points]\ (p \to r) \land (q \to r) \vdash p \land q \to r$
- $[1 Points]\ q \to r \vdash (p \to q) \to (p \to r)$
- $[1 Points]\ p \to q \vdash \big((p \land q) \to p\big) \land \big(p \to (p \land q)\big)$
- $[1 Points] \vdash (p \to q) \to ((r \to s) \to (p \land r \to q \land s))$
- $[2 Points] \vdash \neg p \to (p \to (p \to q))$
- $[2 Points]\ q \vdash (p \land q) \lor (\neg p \land q)$

4 [5 Points] More Propositional Logic Sequents to be Proven

Prove the following sequents by using the natural deduction rules:

- $[1Point]$ $p \wedge \neg p \vdash \neg(r \rightarrow q) \wedge (r \rightarrow q)$
- $[1Point]$ $\neg(\neg p \vee \neg q) \vdash p \wedge q$
- $[1Point]$ $(p \rightarrow q) \rightarrow r, s \rightarrow \neg p, t, (\neg s \wedge t) \rightarrow q \vdash r$
- $[1Point]$ $\vdash ((p \rightarrow q) \wedge (q \rightarrow p)) \rightarrow ((p \vee q) \rightarrow (p \wedge q))$
- $[1Point]$ $\vdash ((p \rightarrow q) \rightarrow q) \rightarrow ((q \rightarrow p) \rightarrow p)$

5 [5 Points] Avoid Pitfalls when Using Mathematical Induction

This question is designed to check whether you still remember the material from BIL 132. Let us prove below that all horses are the same color by mathematical induction on the number of horses in a given set.

Basis: If there is just one horse then it's the same color as itself, so the basis is trivial.

Inductive Hypothesis: Horses $1, \ldots, n-1$ are the same color, and similarly horses $2, \ldots, n$ are the same color.

Inductive Step: The middle horses, $\{2, \ldots, n-1\}$, can't change color when they are in different groups. So, horses 1 and n must be the same color as well, by transitivity. □

What, if anything, is wrong with this proof?

6 [3 Points] Drawing Parse Trees of Propositional Logic Formulas

Draw the corresponding parse trees of the following propositional logic formulas:

- $[1Point]$ $p \rightarrow (\neg\neg q \vee (q \rightarrow p))$
- $[1Point]$ $(((s \rightarrow (r \vee l)) \vee ((\neg q) \wedge r)) \rightarrow ((\neg(p \rightarrow s)) \rightarrow r))$
- $[1Point]$ $(p \rightarrow q) \wedge (\neg r \rightarrow (q \vee (\neg p \wedge r)))$

7 [2 POINTS] VALIDITY AND SATISFIABILITY OF FORMULAS

We call a formula **valid** if it always computes T(true), no matter which truth values we choose for propositional atoms. We call a formula **satisfiable** if it computes T for at least one set of truth values for propositional atoms.
Is the formula $\neg((q \to \neg p) \wedge (p \to r \vee q))$ valid? Is it satisfiable?