BIL 133 HW5

Murat Sahin
191101010

1 THINK RECURSIVELY

Let a_n be the number of n-length sequences ending with 0, b_n be the number of n-length sequences ending with 1 but not 11, c_n be the number of n-length sequences ending with 11 but not 111.

This three sequence sets contains all valid n-length sequences. So,

$$F_n = a_n + b_n + c_n \ (1)$$

How to find a_{n+1}, b_{n+1} and c_{n+1}?

Part 1 : We can take any of the sequence of the valid sequences of length n and add 0 to end of it and it will be a valid sequence of length $n+1$. Since it ends with 0, it is in a_{n+1}. So,

$$a_{n+1} = a_n + b_n + c_n \ (2)$$

Part 2 : We can take any of the sequence of the valid sequences of length n that ends with 0 and add 1 to end of it and it will be a valid sequence of length $n+1$. Since it ends with 1 and not 11, it is in b_{n+1}. So,

$$b_{n+1} = a_n \ (3)$$

Part 3 : We can take any of the sequence of the valid sequences of length n that ends with 01 and add 1 to end of it and it will be a valid sequence of length $n+1$. Since it ends with 11, it is in c_{n+1}. So,

$$c_{n+1} = b_n = a_{n-1} \ (4)$$

from above explanation and eq3.

Substitute b_n and c_n from eq3 and eq4 in eq2, we will find this equation:

$$a_{n+1} = a_n + a_{n-1} + a_{n-2}(5)$$

From equation 1 and 2 we can say that $F_n = a_{n+1}$. From this equality and eq5:

$$F_n = F_{n-1} + F_{n-2} + F_{n-3} \text{ with the base case } F_1 = 2,\ F_2 = 4, F_1 = 7$$

2 SIZES OF SETS

If A and B are countably-infinite sets they are two of the subsets of $\mathbb{N}$. So, we can define bijection between A, B to $\mathbb{N}$.

Define $a(n) = 2n$ from $\mathbb{N}$ to A.
Define $b(n) = 3n$ from $\mathbb{N}$ to B.

Now, lets define A' and B'.

Define $c(n) = 4n$ from $\mathbb{N}$ to A'.
Define $d(n) = 6n$ from $\mathbb{N}$ to B'.

Now, $A' \subset A$ and $B' \subset B$. A, A', B and B' are all countably infinite sets. We can define a bijection from A' to B **(f)** and B' to A **(g)** by mapping smallest element of first set to smallest element of second set then mapping second smallest element of first set to second smallest element of second set and so forth.

All conditions are satisfied.

3 SHOOTING THE DAMNED RABBIT

Lets create a matrix that has rows and columns with that order:

" 0 1 -1 2 -2 ..."

Rows will represent rabbit's position in time **t** and columns will represent where hunter shoots in time **t+1**. Then, we will traverse this matrix by dovetailing (Just like how we proved rational numbers are countable). Independent from s and z, we will reach correct row and column in a finite time. It is guaranteed that hunter will shoot rabbit in a finite time. Which means, hunter should shoot in a order like "0 0 1 0 1 -1 0 1 -1 2 ...".

4 LANGUAGES

Lets say L and $\overline{L}$ are not decidable. L is recursively enumerable and it can return true in a finite time but it can not return false in a finite time , $\overline{L}$ is recursively enumerable and it can return true in a finite time too but if something is true for $\overline{L}$ it is false for L. So L can return true and false in a finite time. This is a contradiction, one of them or both should be decidable language.

Lets say L is decidable. Then, it can return true in a finite time. True for L is false for $\overline{L}$. So, $\overline{L}$ can return false in a finite time, it is decidable. We can assume $\overline{L}$ is decidable too, we will reach L is decidable by following nearly same steps.

L and $\overline{L}$ are both decidable.

5 NUMBER OF SQUARE INTEGER MATRICES

Lets define a matrix with rows representing sorted positive integers and columns representing sorted non-negative integers. Elements of matrix are set of matrices contains m x m matrices with maximum element of n (m,n are representing current value of row and column). We will traverse this matrix by dovetailing. Elements of our matrix are countable because there are finite matrices that m x m with maximum element of n. With this way, we will look all of the square integer matrices. Thus, they are countable.

6 AN EQUIVALENCE RELATION

We need to show that $\sim$ relation satisfies following three conditions: reflexive, symmetric and transitive. We will show that our relation satisfies all of them. This implies it is an equivalence relation.

Reflexive: We can define a bijection $f : S_1 \to S_1$ by mapping each element to itself.

Symmetric: Lets say bijection $f : S_1 \to S_2$ exists. Since it is a bijection, it has an inverse f^{-1}. $f^{-1} : S_2 \to S_1$ and It is mapping elements symmetric from f so it is a bijection too.

Transitive: Lets say bijections $f : S_1 \to S_2$ and $g : S_2 \to S_3$ exist. For each element k that mapped from S_1 to S_2 , we will map that element to S_3's element $g(f(k))$. Thus, it will be a bijection $h : S_1 \to S_3$.

We showed that $\sim$ relation is reflexive, symmetric and transitive. So, it is an equivalence relation.

7 SPACE CONSIDERATIONS FOR MERGE SORT

For this question, I am considering the implementation that we did on class.

In the combine step of the merge sort, which uses divide and conquer approach, we need to use **two n/2 sized external arrays** that combines our calculations in the conquer step.

We said space differs from time in reusability. Combine steps do not execute parallel, so all we need to find is that maximum usage of space in conquer steps. At the last stage of the algorithm, halves of the first array are sorted and needs to get combined in combine step. In order to do that, we need **two n/2 sized external arrays**. We can use these arrays in all of the steps too. So we need **n** extra space.