Lec 9

Invertability

MA(1) w/ $\delta = 0$

$$y_t = v_t + \theta v_{t-1}$$

$$v_t = y_t - \theta v_{t-1}$$

$$= y_t - \theta \left(y_{t-1} - \theta v_{t-2} \right)$$

$$= y_t - \theta y_{t-1} + \theta^2 v_{t-2}$$

$$= y_t - \theta y_{t-1} + \theta^2 y_{t-2} - \theta^3 v_{t-3}$$

$$= y_t + \sum_{i=1}^{p} (-\theta)^i y_{t-i} + (-\theta)^{p+1} w_{t-p+1}$$

Invertable iff $|\theta| < 1$

Ex 1

$$y_t = \mu_t + w_t \qquad \mu_t = \mu_{t-1} + v_t$$

w/ w_t, v_t stationary

$E(w_t) = 0 \quad Var(w_t) = \sigma_w^2$

$E(v_t) = 0 \quad Var(v_t) = \sigma_v^2$

$$y_t = (\mu_{t-1} + v_t) + w_t$$

$$= (\mu_{t-1} + v_t) + v_t + w_t$$

$$= v_t + \sum_{i=1}^{\infty} v_t$$

$$E(y_t) = E\left(w_t + \sum_{i=1}^{\infty} v_t\right) = 0$$

$$Var(y_t) = Var\left(v_t + \sum_{i=1}^{\infty} v_t\right)$$

$$= \sigma_w^2 + \sum_{i=1}^{\infty} \sigma_v^2 = \infty$$

$$\Delta y_t = (\mu_t + v_t) - (\mu_{t-1} + v_{t-1})$$

$$= (\mu_{t-1} + v_t) - \mu_{t-1} + (w_t - v_{t-1})$$

$$= v_t + \Delta w_t$$

Ex 2

$$y_t = \mu_t + v_t$$

$$\mu_t = \mu_{t-1} + v_t$$

$$v_t = v_{t-1} + e_t$$

$$\Delta y_t = (\mu_t + v_t) - (\mu_{t-1} + v_{t-1})$$

$$= (\mu_{t-1} + v_t + v_t) - (\mu_{t-1} + v_{t-1})$$

$$= v_t + v_t - v_{t-1} = v_t + \Delta v_t$$

$$\Delta^2 y_t = (v_t + \Delta v_t) - (v_{t-1} + \Delta v_{t-1})$$

$$= e_t + \Delta^2 v_t$$