GPs for GLMs + Spatial Data

Lecture 17

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Sta 344 - Fall 2022

GPs and GLMs

Logistic Regression

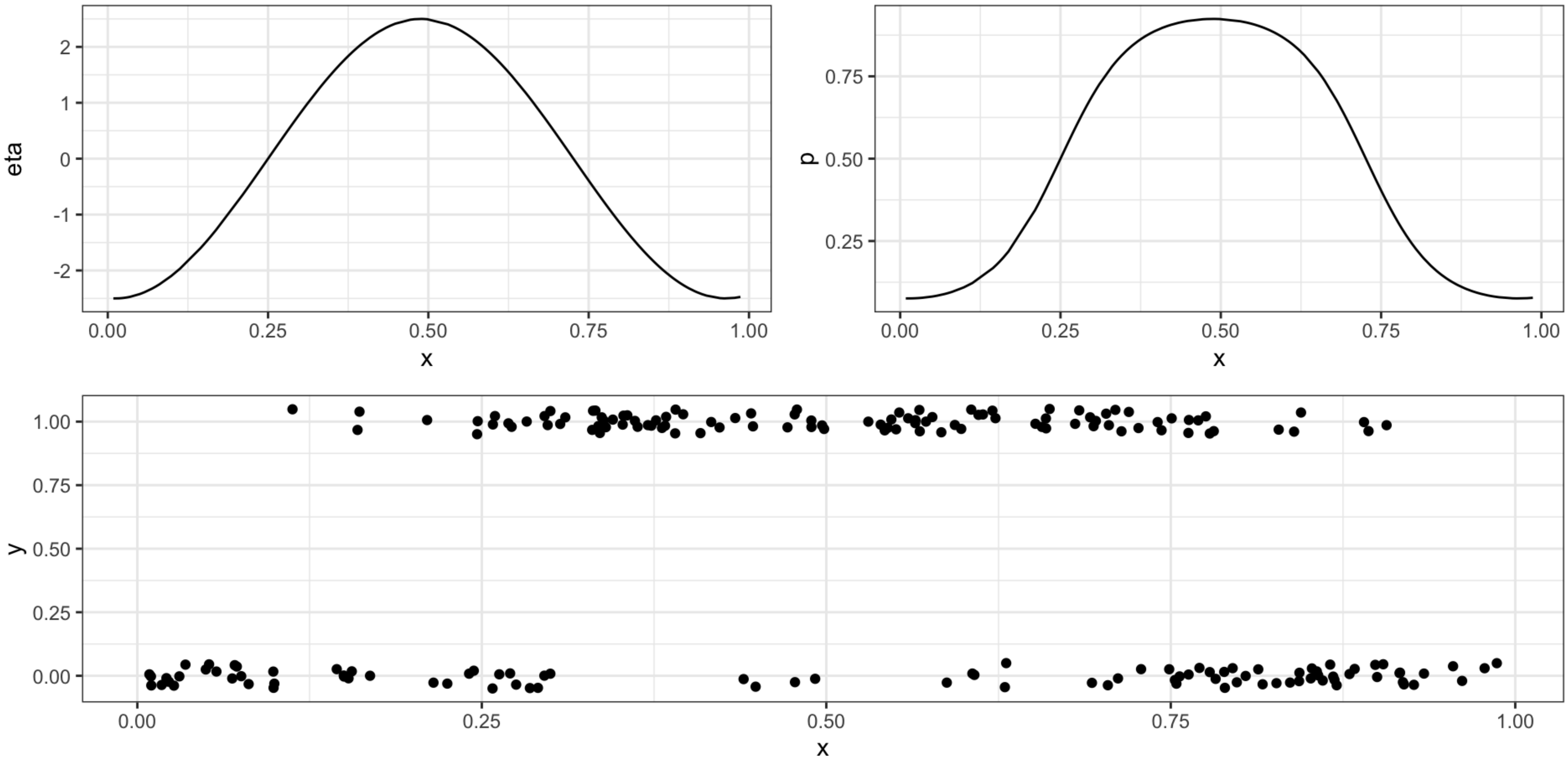
A typical logistic regression problem uses the following model,

$$
\begin{aligned}
y_i &\sim \text{Bern}(p_i) \\
\text{logit}(p_i) &= \boldsymbol{X}\boldsymbol{\beta} \\
&= \beta_0 + \beta_1\, x_{i1} + \cdots + \beta_k\, x_{ik}
\end{aligned}
$$

there is no reason that the linear equation above can’t contain thing like random effects or GPs

$$
\begin{aligned}
y_i &\sim \text{Bern}(p_i) \\
\text{logit}(p_i) &= \eta_i = \boldsymbol{X}\boldsymbol{\beta} + w(\boldsymbol{x}) \\
w(\boldsymbol{x}) &\sim N(0, \Sigma)
\end{aligned}
$$

A toy example

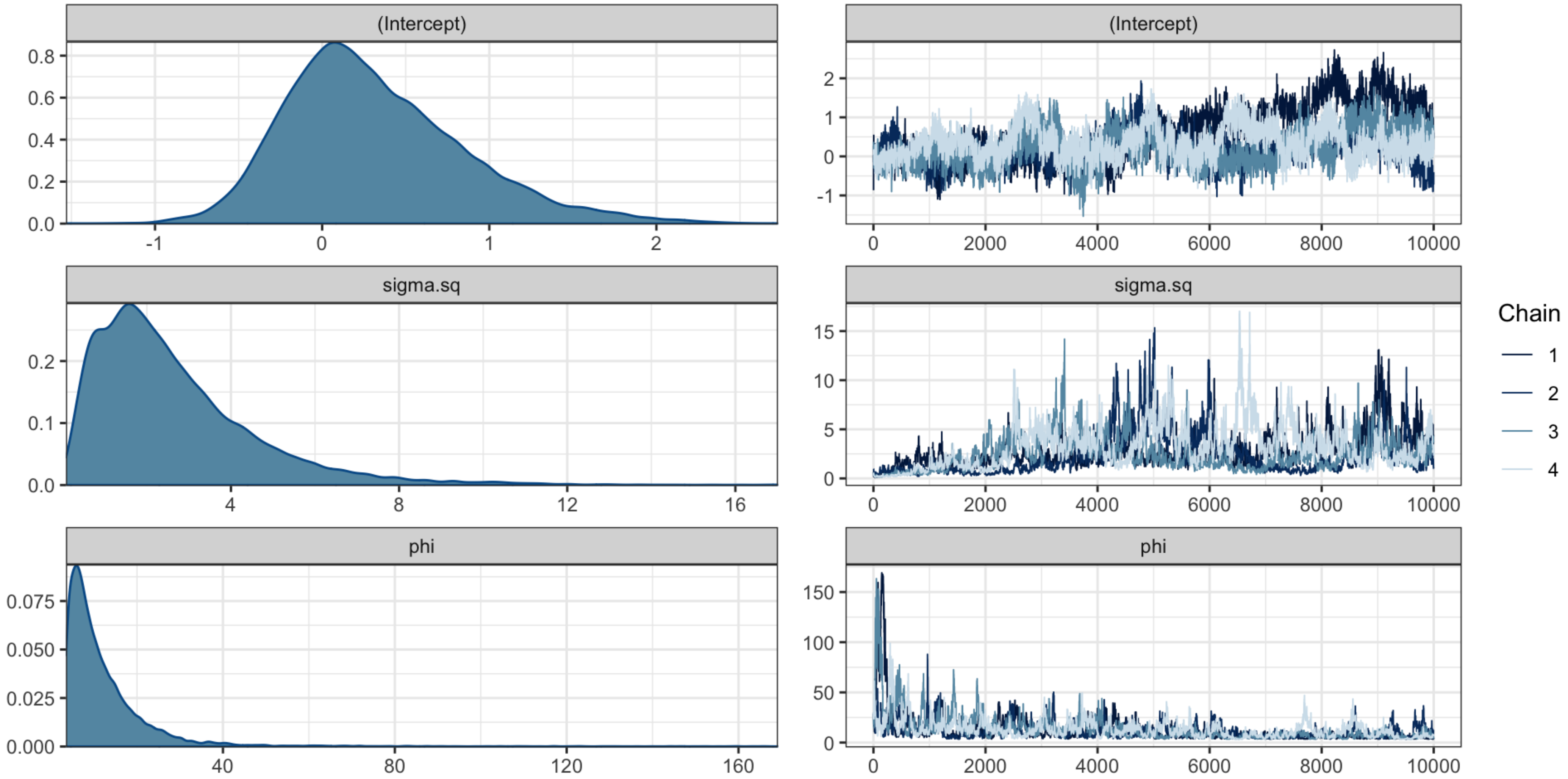


Model fitting

```r
m = gpglm(
  y~1, family="binomial",
  data = d, coords = c("x"),
  cov_model = "exponential",
  starting=list(
    "beta"=0, "phi"=3/0.1, "sigma.sq"=1, "w"=0
  ),
  tuning=list(
    "beta"=0.5, "phi"=0.5, "sigma.sq"=0.5, "w"=0.5
  ),
  priors=list(
    "beta.Normal"=list(0,1),
    "phi.unif"=c(3/0.5, 3/0.01),
    "sigma.sq.ig"=c(2, 1)
  ),
  n_batch = 100,
  batch_len = 100,
  verbose = FALSE
```

Model diagnostics

```r
plot(m)
```



Model predictions

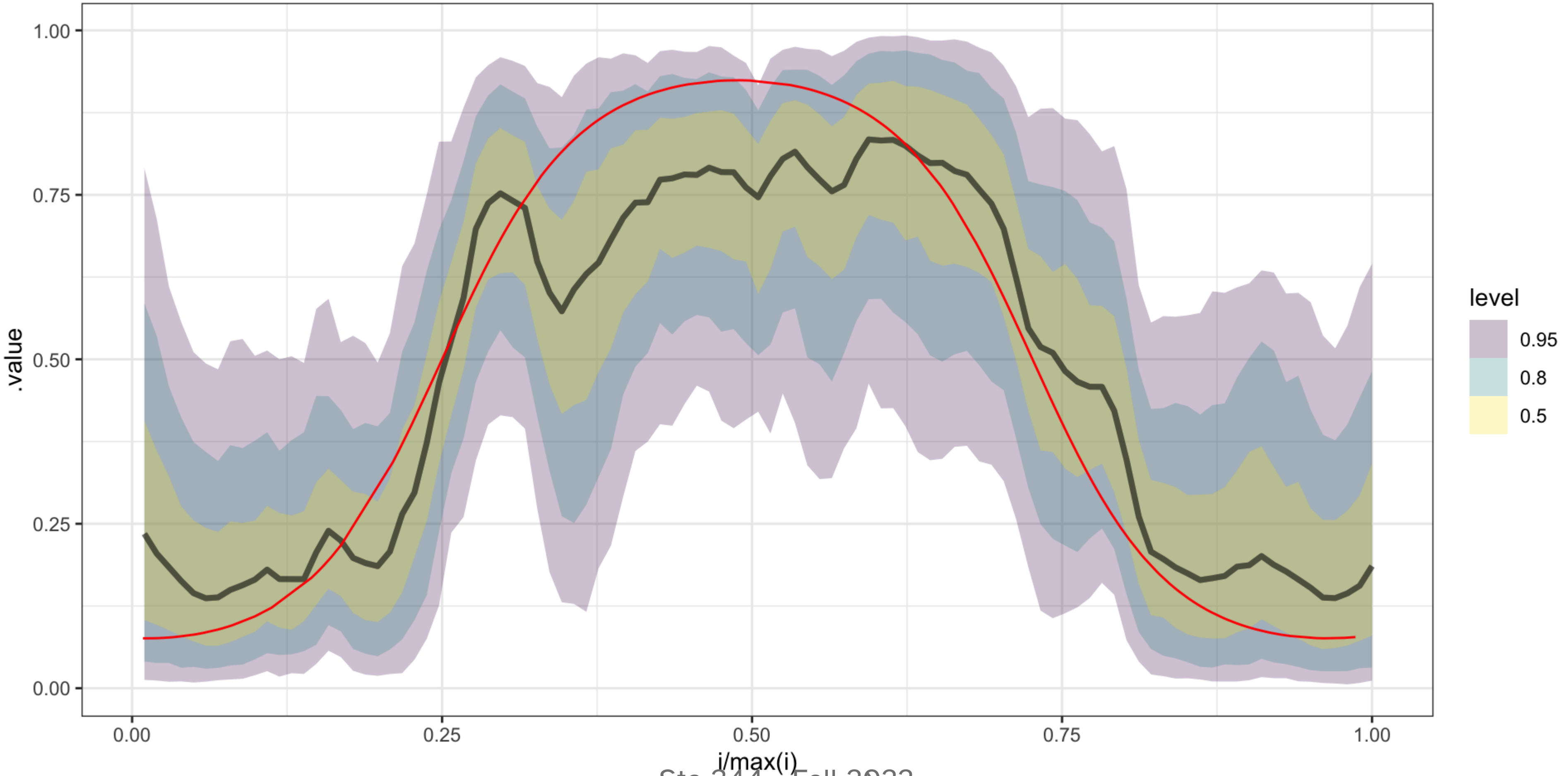
```r
newdata = data.frame(
  x=seq(0,1,length.out=101)
)

(p = predict(m, newdata=newdata, coords="x", thin=50))
```

```
# A draws_matrix: 200 iterations, 4 chains, and 202 variables
    variable
draw  w[1]     w[2]   w[3]   w[4]   w[5]   w[6]  w[7]
  1  -1.18 -0.22201 -0.354  0.072  0.380 -0.226 -0.45
  2  -0.22 -0.00092 -0.167 -0.079  0.170  0.149 -0.29
  3  -0.64  0.09967 -0.345  0.357 -0.801 -0.339 -0.44
  4  -0.82  1.25431 -0.542 -1.391 -0.637 -0.033 -0.30
  5   0.19 -0.90628 -0.969 -0.561 -0.188 -1.444 -0.94
  6  -0.58 -1.01544  0.036 -0.611 -1.297 -1.107 -0.77
  7  -1.74  0.64735  0.036 -0.357 -1.268 -0.870 -0.70
  8  -0.25 -0.01433 -0.460  0.312 -0.046 -0.902 -1.68
  9  -0.83 -2.72657 -2.128 -1.316 -1.648 -2.682 -2.54
  10 -0.51 -1.15691 -1.570 -0.998 -1.074 -1.079 -1.63
```

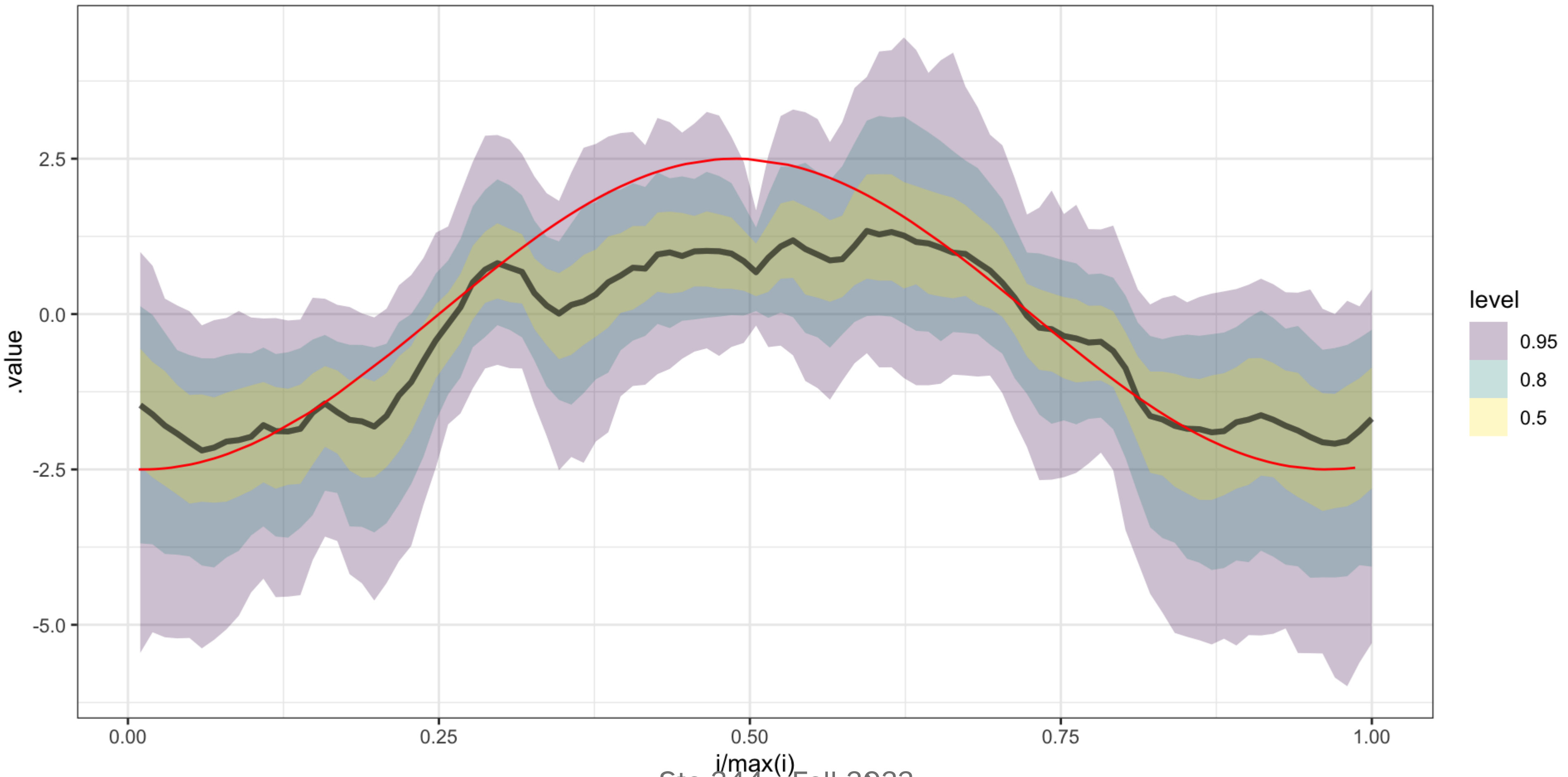
Predicted y

```
p |>
  tidybayes::gather_draws(y[i]) |>
  ggplot2::ggplot(ggplot2::aes(x=i/max(i),y=.value)) +
    tidybayes::stat_lineribbon(alpha=0.25) +
    geom_line(data=d |> arrange(x), aes(x=x, y=p), color='red')
```



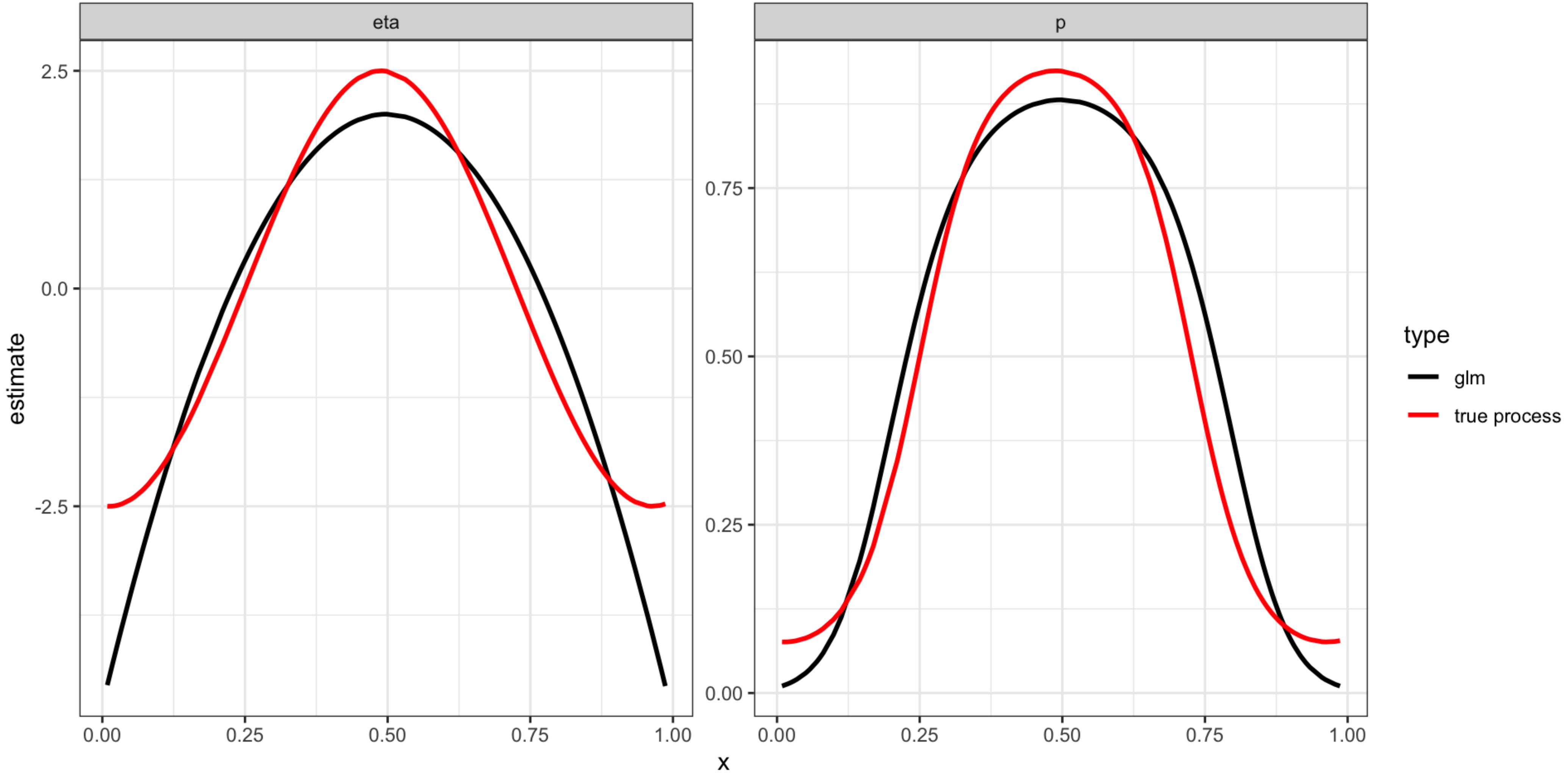
Predicted w

```r
p |>
  tidybayes::gather_draws(w[i]) |>
  ggplot2::ggplot(ggplot2::aes(x=i/max(i),y=.value)) +
    tidybayes::stat_lineribbon(alpha=0.25) +
    geom_line(data=d |> arrange(x), aes(x=x, y=eta), color='red')
```



glm

```r
g = glm(y~poly(x,2), data=d, family="binomial")
```

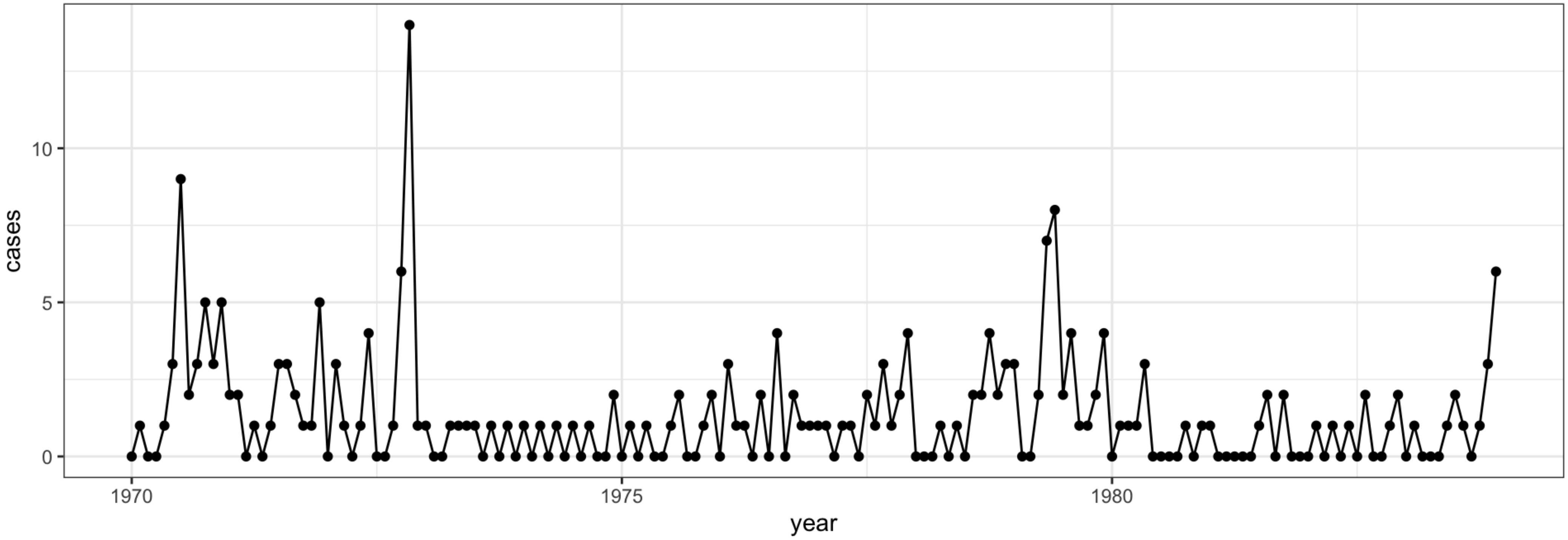


Count data

Polio cases

`Polio` from the `glarma` package.

> This data set gives the monthly number of cases of poliomyelitis in the U.S. for the years 1970–1983 as reported by the Center for Disease Control.



Polio Model

Model:

$$y_i \sim \text{Pois}(\lambda_i)$$
$$\log(\lambda_i) = \beta_0 + w(\boldsymbol{t})$$

$$w(\boldsymbol{t}) \sim N(0, \Sigma)$$
$$\{\boldsymbol{\Sigma}\}_{ij} = \sigma^2 \exp(-|l\, d_{ij}|)$$

. . .

Priors:

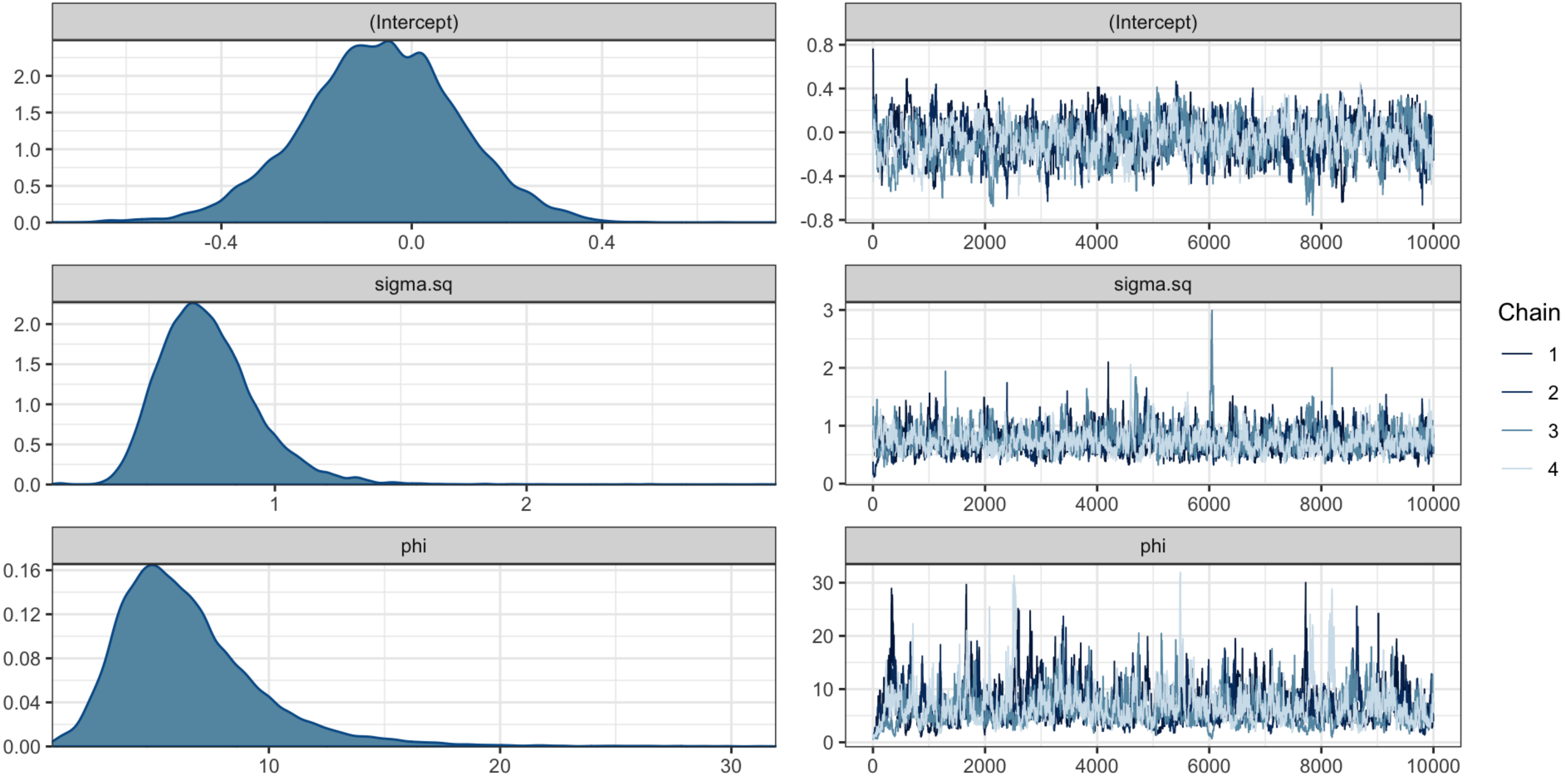
$$\beta_0 \sim N(0, 1)$$
$$\phi \sim \text{Unif}\left(\frac{3}{6}, \frac{3}{1/12}\right)$$
$$\sigma^2 \sim \text{Inv-Gamma}(2, 1)$$

Model fitting

```r
m = gpglm(
  cases~1, family="poisson",
  data = polio, coords = c("year"),
  cov_model = "exponential",
  starting=list(
    "beta"=0, "phi"=3/2, "sigma.sq"=1, "w"=0
  ),
  tuning=list(
    "beta"=0.5, "phi"=0.5, "sigma.sq"=0.5, "w"=0.5
  ),
  priors=list(
    "beta.Normal"=list(0,1),
    "phi.unif"=c(3/6, 3/(1/12)),
    "sigma.sq.ig"=c(2, 1)
  ),
  n_batch = 100,
  batch_len = 100,
  verbose = FALSE
```

Model diagnostics

```r
plot(m)
```



Model fit

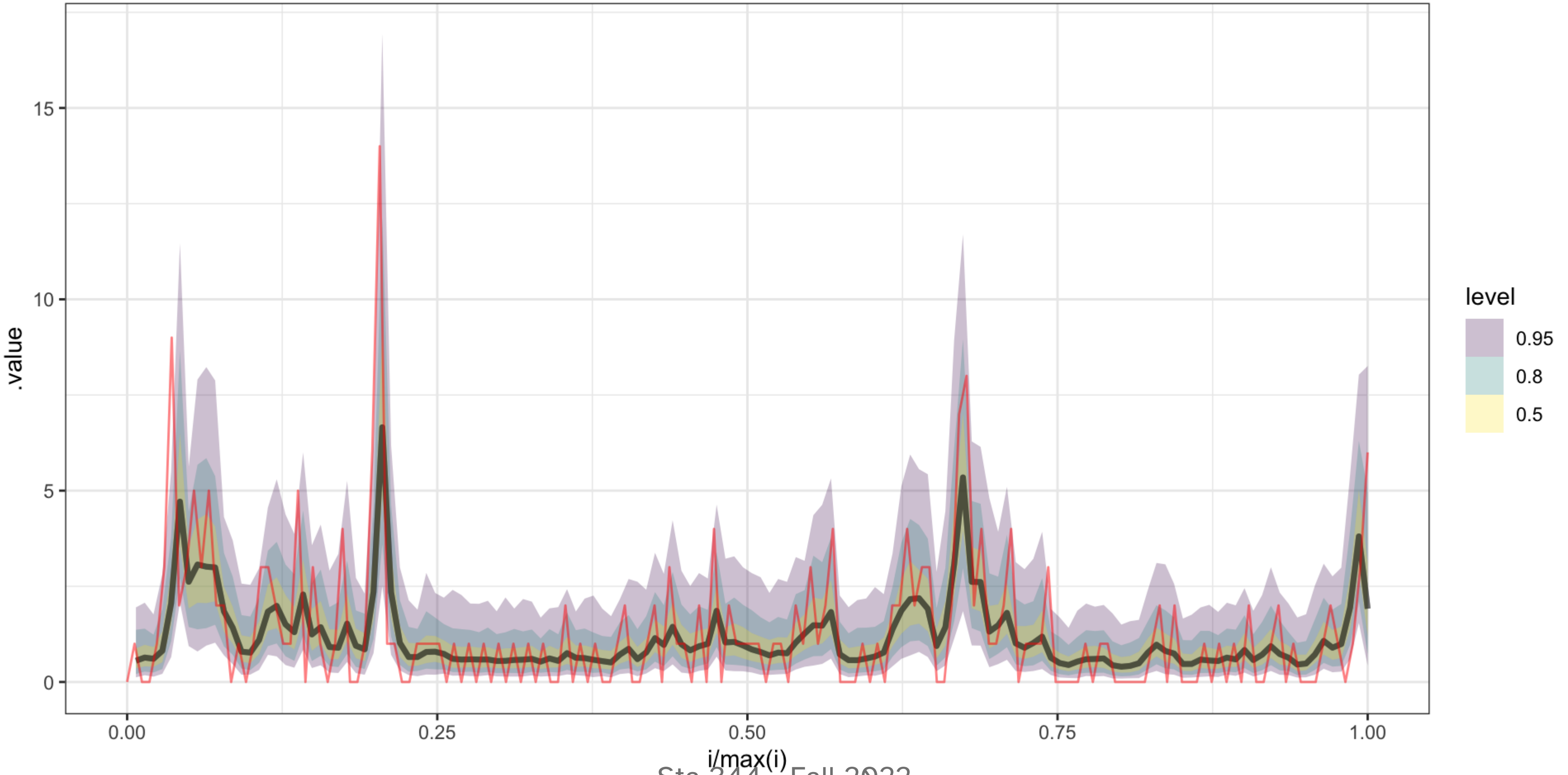
```r
newdata = data.frame(
  year = seq(1970, 1984, by=0.1) |> jitter()
)

(p = predict(m, newdata=newdata, coords="year", thin=50))
```

```
# A draws_matrix: 200 iterations, 4 chains, and 282 variables
    variable
draw   w[1]   w[2]    w[3]    w[4]  w[5] w[6]    w[7]  w[8]
  1  -0.498  0.065  0.3183  0.2519 -0.52 0.32 -0.0085 -0.22
  2   0.190 -0.115 -0.0028  0.0917  1.12 1.62  1.0786  1.34
  3  -1.322 -0.651 -0.5380 -0.4450  0.94 1.80  1.5415  1.80
  4   0.112  0.075  0.3479  0.6895  1.05 1.68  1.6113  0.79
  5  -0.043 -0.088 -0.2408 -0.2396  2.15 1.79 -0.0989  1.04
  6  -0.528 -0.995 -0.9240 -0.0067  1.50 2.00  0.9907  1.63
  7  -1.779 -0.426 -1.2744  0.9432  0.51 0.90  1.2267  0.04
  8  -0.594 -0.119 -0.9112 -0.0822  1.69 1.28  0.2578  0.88
  9   0.259  0.157  0.3803  0.4777  0.30 1.23  0.3618  0.35
  10 -1.880 -1.996 -0.5955 -0.5550  1.45 1.43  0.6164  0.61
```

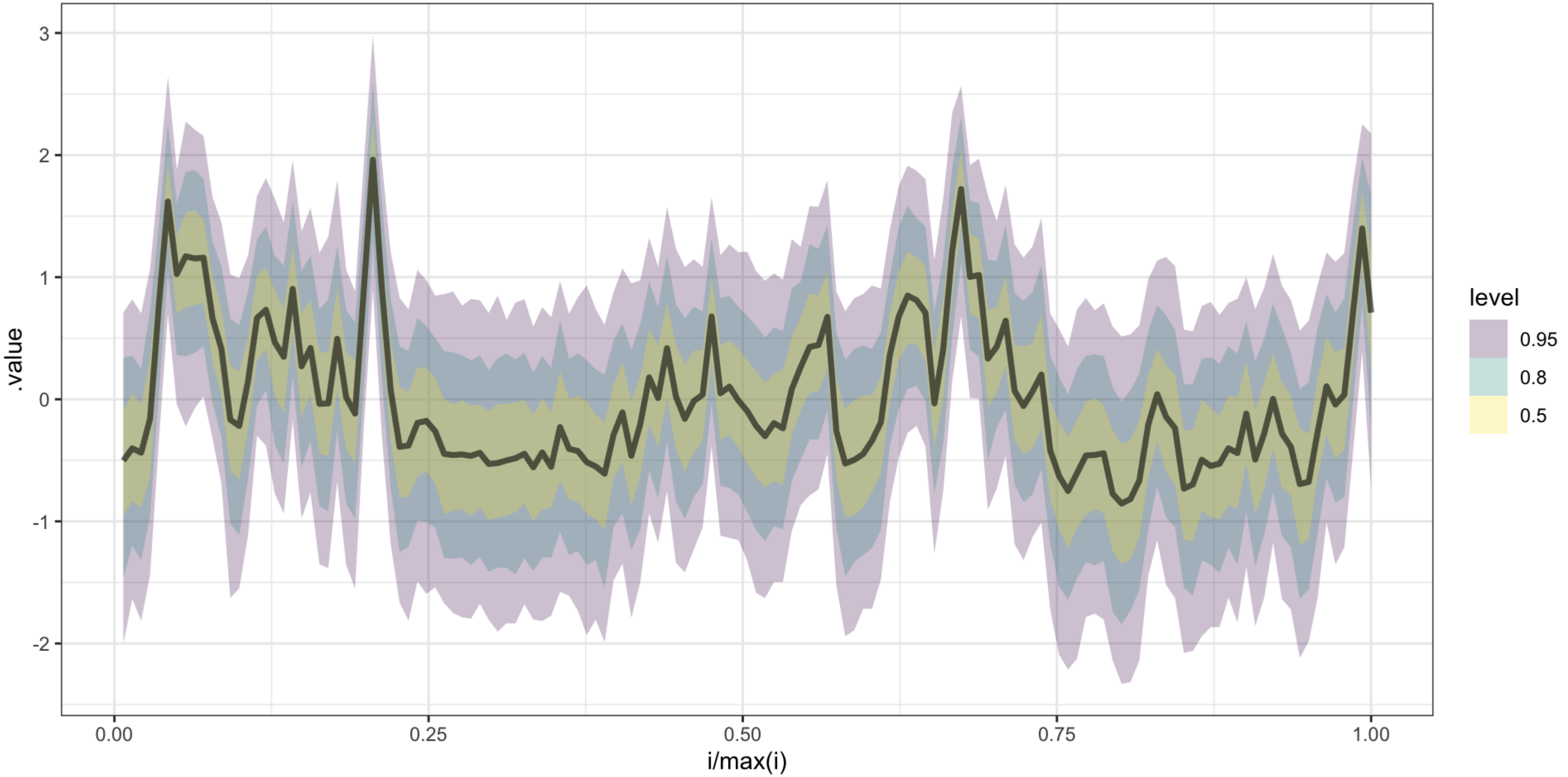
Predicted y

```r
p |>
  tidybayes::gather_draws(y[i]) |>
  ggplot2::ggplot(ggplot2::aes(x=i/max(i),y=.value)) +
    tidybayes::stat_lineribbon(alpha=0.25) +
    geom_line(data=polio, aes(x=(year-min(year))/(max(year)-min(year)), y=cases), color='red', a
```



Predicted w

```r
p |>
  tidybayes::gather_draws(w[i]) |>
  ggplot2::ggplot(ggplot2::aes(x=i/max(i),y=.value)) +
    tidybayes::stat_lineribbon(alpha=0.25)
```



Spatial data in R

Packages for geospatial data in R

R has a rich package ecosystem for read/writing, manipulating, and analyzing geospatial data.

Some core packages:

- `sp` - core classes for handling spatial data, additional utility functions - **Deprecated**
- `rgdal` - R interface to `gdal` (Geospatial Data Abstraction Library) for reading and writing spatial data - **Deprecated**
- `rgeos` - R interface to `geos` (Geometry Engine Open Source) library for querying and manipulating spatial data. Reading and writing WKT. - **Deprecated**
- `sf` - Combines the functionality of `sp`, `rgdal`, and `rgeos` into a single package based on tidy simple features.
- `raster` - classes and tools for handling spatial raster data.
- `stars` - Reading, manipulating, writing and plotting spatiotemporal arrays (rasters)

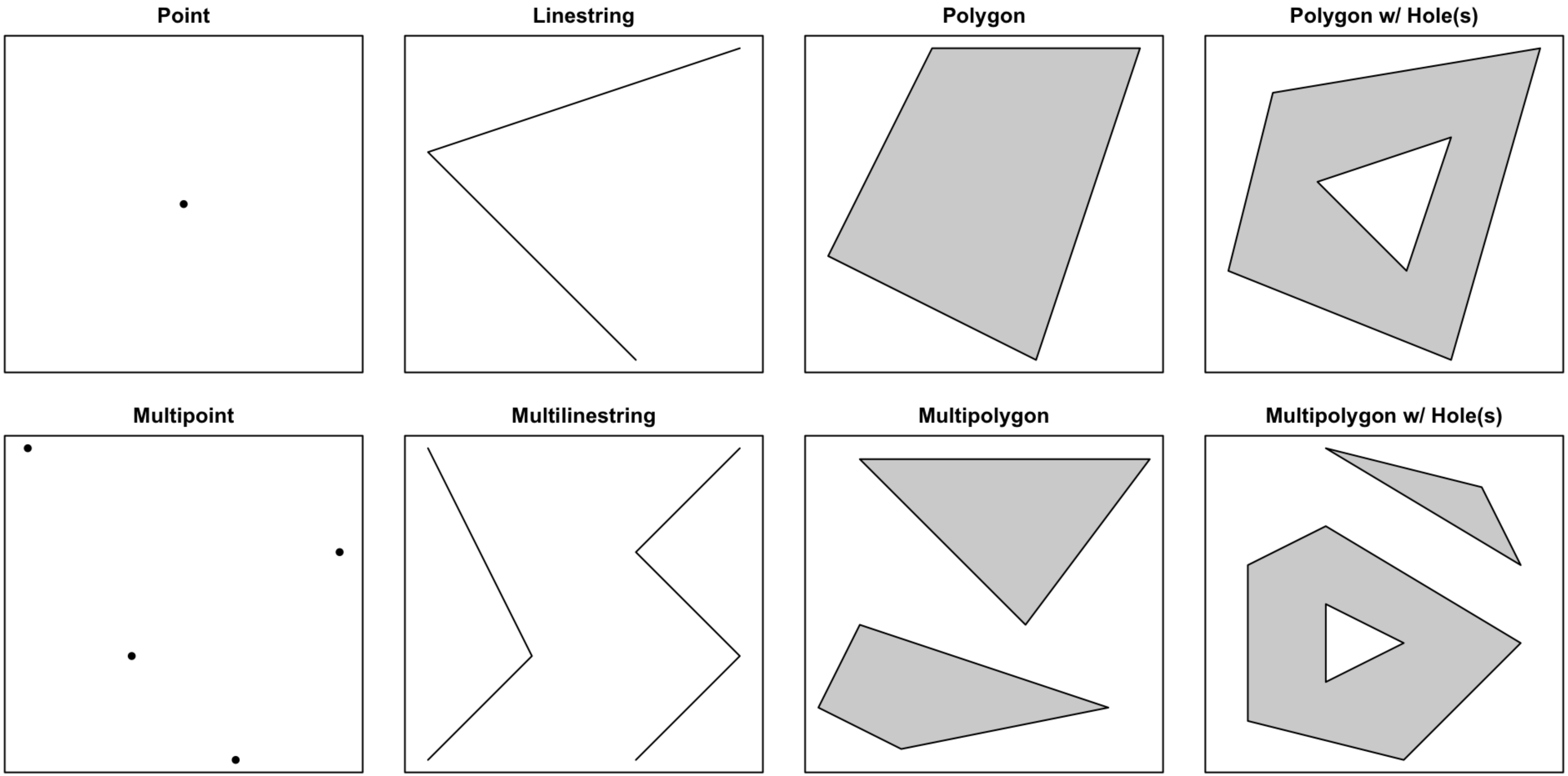
Installing `sf`

This is the hardest part of using the `sf` package, difficulty comes from is dependence on several external libraries (`geos`, `gdal`, and `proj`).

- *Windows* - installing from source works when Rtools is installed (system requirements are downloaded from rwinlib)
- *MacOS* - install dependencies via homebrew: `gdal2`, `geos`, `proj`.
- *Linux* - Install development packages for GDAL (>= 2.0.0), GEOS (>= 3.3.0) and Proj.4 (>= 4.8.0) from your package manager of choice.

More specific details are included in the README on github.

Simple Features



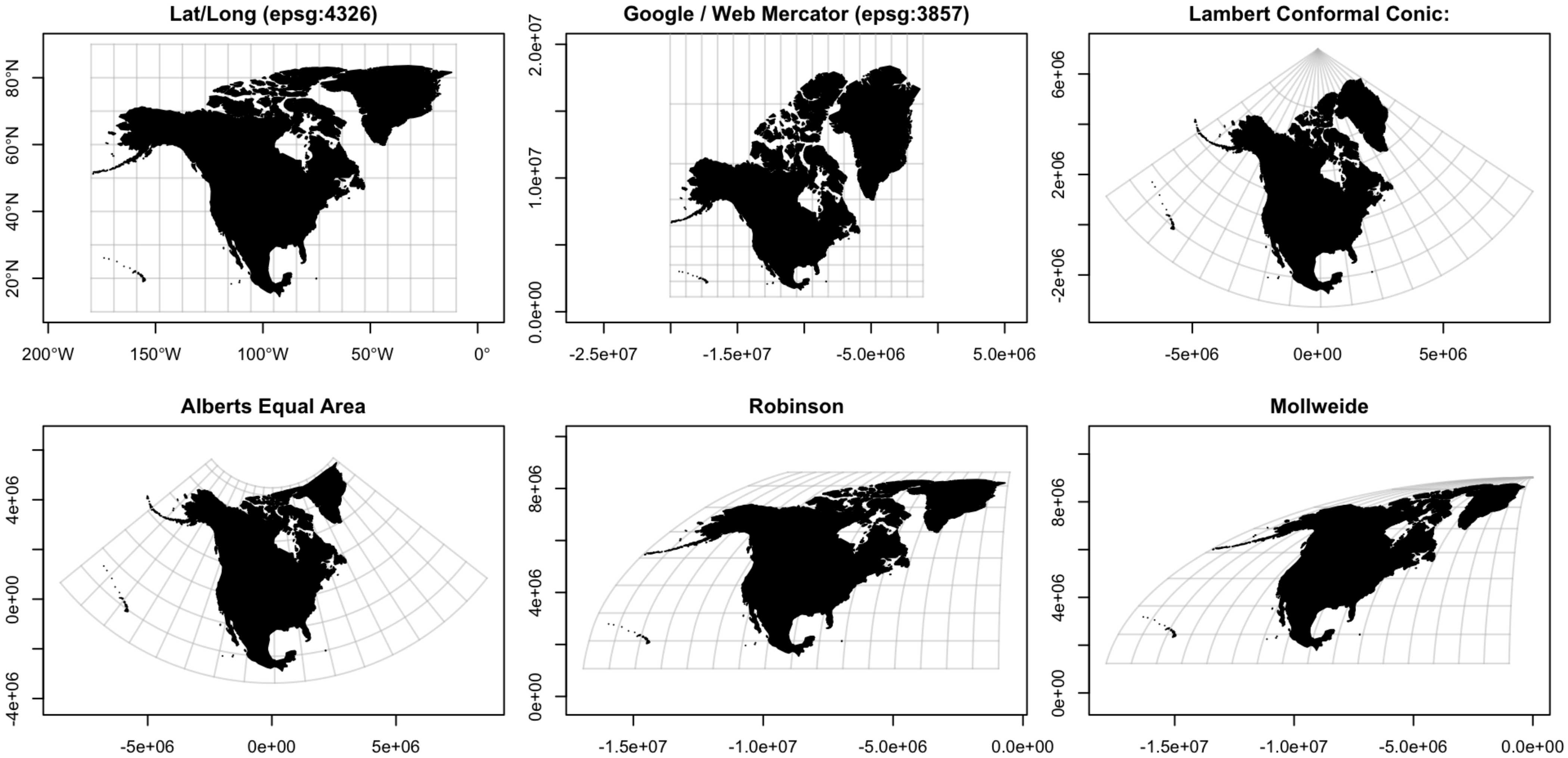
Reading, writing, and converting

- `sf`
  - `st_read()` / `st_write()` - Shapefile, GeoJSON, KML, ...
  - `read_sf()` / `write_sf()` - Same, supports tibbles ...
  - `st_as_sfc()` / `st_as_wkt()` - sf <-> WKT
  - `st_as_sfc()` / `st_as_binary()` - sf <-> WKB
  - `st_as_sfc()` / `as(x, "Spatial")` - sf <-> sp

See sf vignette #2 - Reading, Writing and Converting Simple Features.

Geospatial data in the real world

Projections



Lat/Long (epsg:4326)

Google / Web Mercator (epsg:3857)

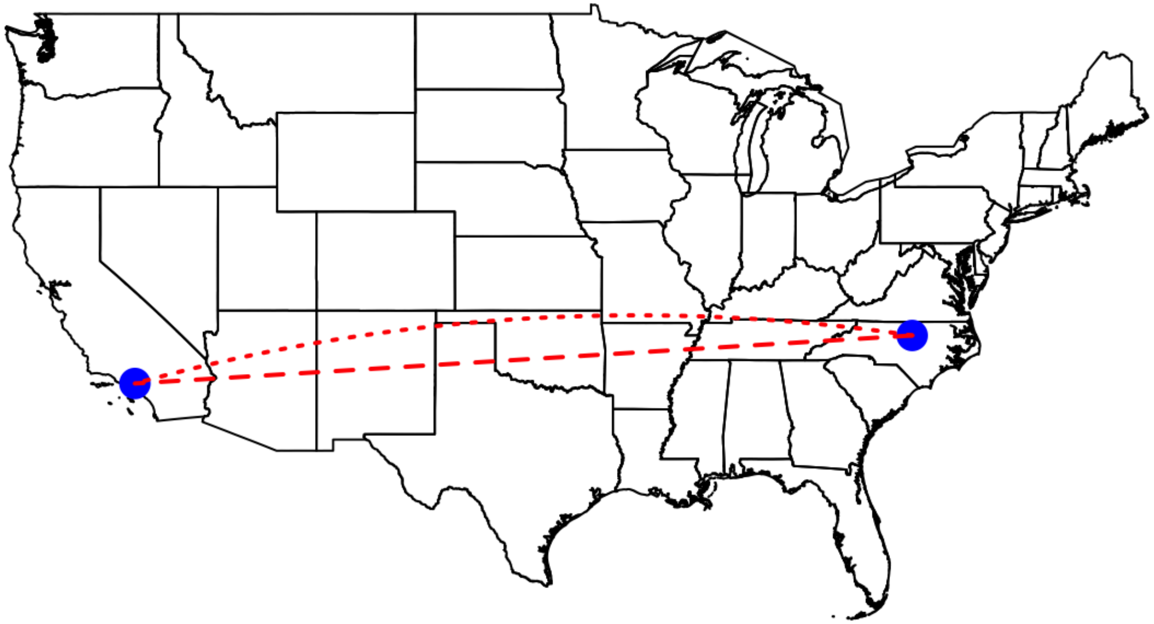
Lambert Conformal Conic:

Alberts Equal Area

Robinson

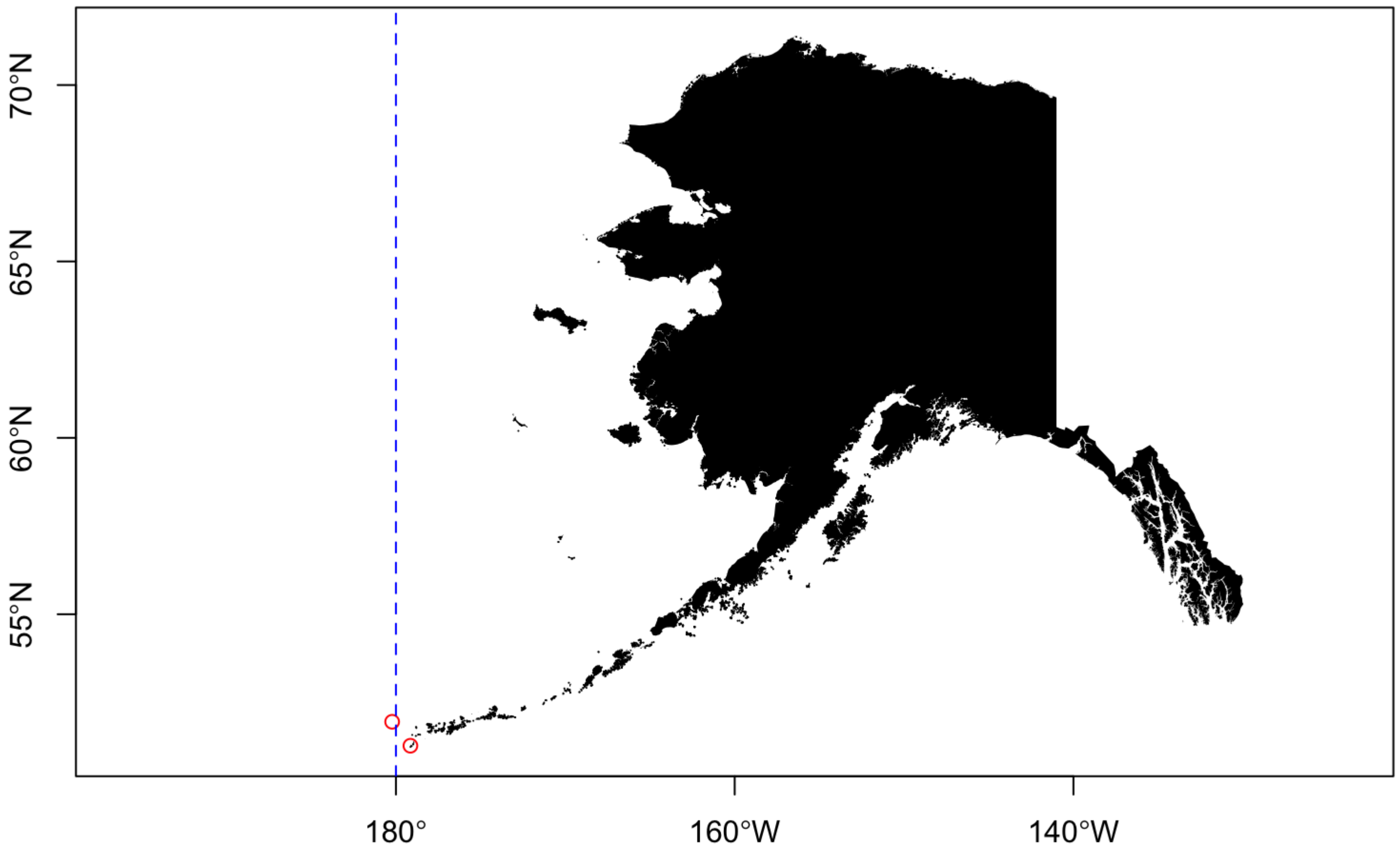
Mollweide

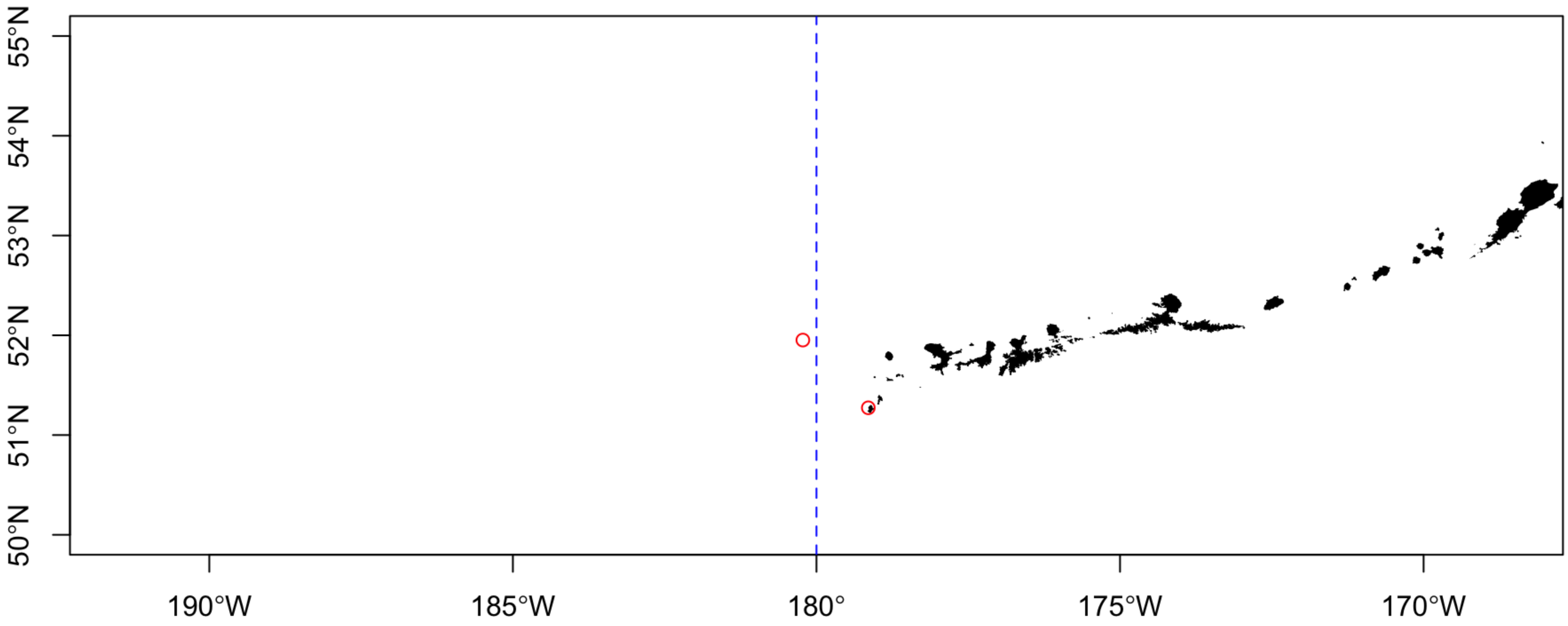
Distance on a Sphere



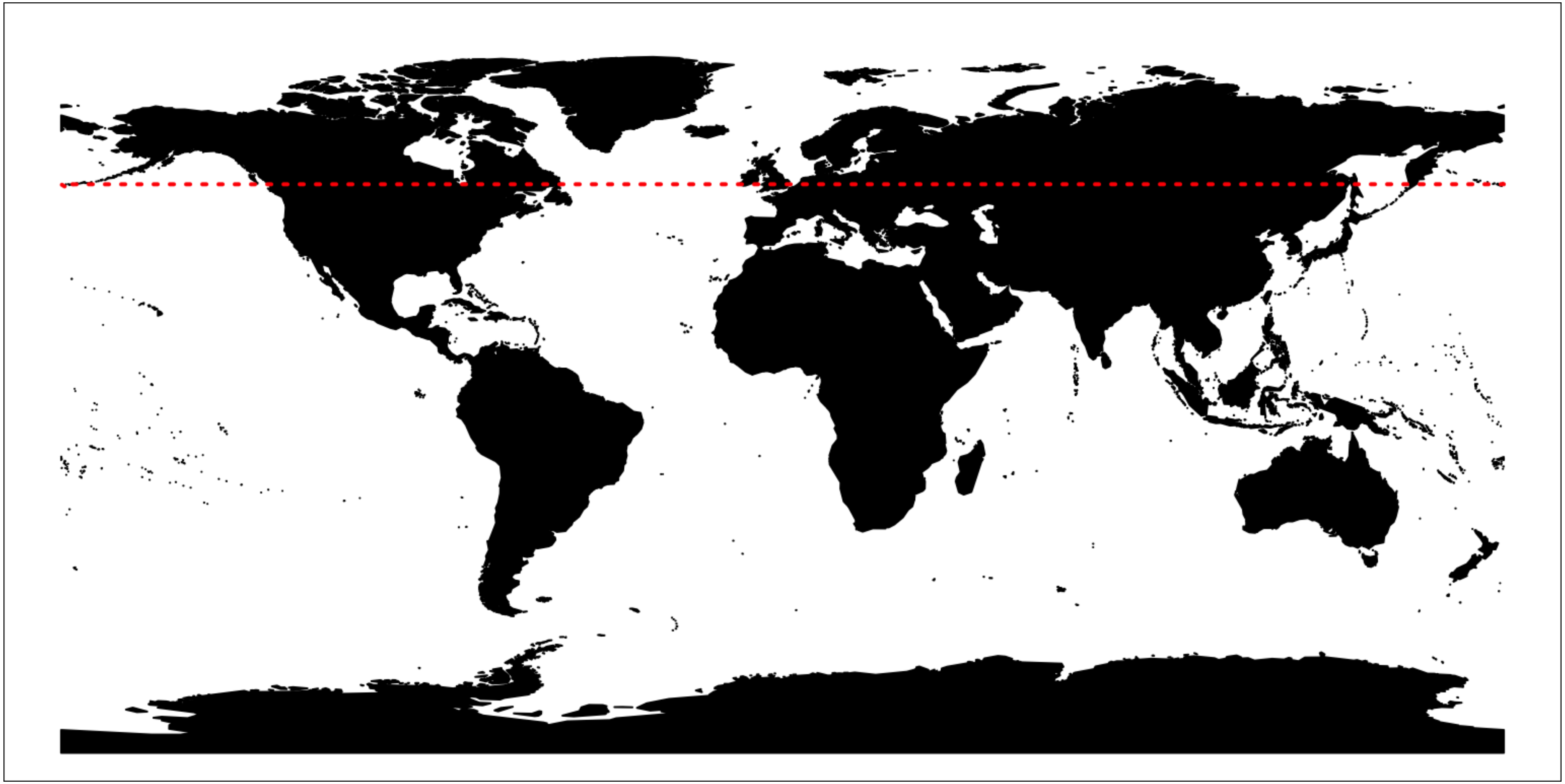
Dateline

How long is the flight between the Western most and the Eastern most points in the US?




55°N
54°N
53°N
52°N
51°N
50°N
190°W
185°W
180°
175°W
170°W

```r
path = geosphere::gcIntermediate(
  c(179.776, 51.952), c(-179.146, 51.273),
  n=50, addStartEnd=TRUE
)
```



Using `sf`

Example data

```r
nc  = read_sf("data/gis/nc_counties/", quiet=TRUE)
air = read_sf("data/gis/airports/", quiet=TRUE)
hwy = read_sf("data/gis/us_interstates/", quiet=TRUE)
```

```r
nc
```

```
Simple feature collection with 100 features and 8 fields
Geometry type: MULTIPOLYGON
Dimension:     XY
Bounding box:  xmin: -84.32186 ymin: 33.84175 xmax: -75.46003 ymax: 36.58815
Geodetic CRS:  NAD83
# A tibble: 100 × 9
     AREA PERIM…¹ COUNT…² STATE COUNTY FIPS  STATE…³ SQUAR…⁴
    <dbl>   <dbl>   <dbl> <chr> <chr>  <chr> <chr>     <dbl>
 1 0.112     1.61    1994 NC    Ashe … 37009 37         429.
 2 0.0616    1.35    1996 NC    Alleg… 37005 37         236.
 3 0.140     1.77    1998 NC    Surry… 37171 37         539.
 4 0.0891    1.43    1999 NC    Gates… 37073 37         342.
 5 0.0687    4.43    2000 NC    Curri… 37053 37         264.
 6 0.119     1.40    2001 NC    Stoke… 37169 37         456.
 7 0.0626    2.11    2002 NC    Camde… 37029 37         241.
 8 0.115     1.46    2003 NC    Warre… 37185 37         444.
```

```r
air
```

```
Simple feature collection with 940 features and 16 fields
Geometry type: POINT
Dimension:     XY
Bounding box:  xmin: -176.646 ymin: 17.70156 xmax: -64.80172 ymax: 71.28545
Geodetic CRS:  NAD83
# A tibble: 940 × 17
   AIRPRTX…¹ FEATURE ICAO  IATA  AIRPT…² CITY  STATE STATE…³
       <dbl> <chr>   <chr> <chr> <chr>   <chr> <chr> <chr>
 1         0 AIRPORT KGON  GON   GROTON… GROT… CT    09
 2         3 AIRPORT K6S5  6S5   RAVALL… HAMI… MT    30
 3         4 AIRPORT KMHV  MHV   MOJAVE… MOJA… CA    06
 4         6 AIRPORT KSEE  SEE   GILLES… SAN … CA    06
 5         7 AIRPORT KFPR  FPR   ST LUC… FORT… FL    12
 6         8 AIRPORT KRYY  RYY   COBB C… ATLA… GA    13
 7        10 AIRPORT KMKL  MKL   MC KEL… JACK… TN    47
 8        11 AIRPORT KCCR  CCR   BUCHAN… CONC… CA    06
```

```r
hwy
```

```
Simple feature collection with 233 features and 3 fields
Geometry type: MULTILINESTRING
Dimension:     XY
Bounding box:  xmin: -7472582 ymin: 2911107 xmax: 2443707 ymax: 8208428
Projected CRS: NAD83 / UTM zone 15N
# A tibble: 233 × 4
   ROUTE_NUM DIST_MILES DIST_KM                        geometry
   <chr>          <dbl>   <dbl>        <MULTILINESTRING [m]>
 1 I10          2449.   3941.   ((-1881200 4072307, -187992…
 2 I105           20.8    33.4  ((-1910156 5339585, -191013…
 3 I110           41.4    66.6  ((1054139 3388879, 1054287 …
 4 I115            1.58    2.55 ((-1013796 5284243, -101313…
 5 I12            85.3   137.   ((680741.7 3366581, 682709.…
 6 I124            1.73    2.79 ((1201467 3906285, 1201643 …
 7 I126            3.56    5.72 ((1601502 3829718, 1602136 …
 8 I129            3.1     4.99 ((217446 4705389, 217835.1 …
```

sf structure

```r
str(nc)
```

```
sf [100 × 9] (S3: sf/tbl_df/tbl/data.frame)
 $ AREA      : num [1:100] 0.1118 0.0616 0.1402 0.0891 0.0687 ...
 $ PERIMETER : num [1:100] 1.61 1.35 1.77 1.43 4.43 ...
 $ COUNTYP010: num [1:100] 1994 1996 1998 1999 2000 ...
 $ STATE     : chr [1:100] "NC" "NC" "NC" "NC" ...
 $ COUNTY    : chr [1:100] "Ashe County" "Alleghany County" "Surry County" "Gates County" ...
 $ FIPS      : chr [1:100] "37009" "37005" "37171" "37073" ...
 $ STATE_FIPS: chr [1:100] "37" "37" "37" "37" ...
 $ SQUARE_MIL: num [1:100] 429 236 539 342 264 ...
 $ geometry  :sfc_MULTIPOLYGON of length 100; first list element: List of 1
  ..$ :List of 1
  .. ..$ : num [1:1030, 1:2] -81.7 -81.7 -81.7 -81.6 -81.6 ...
  ..- attr(*, "class")= chr [1:3] "XY" "MULTIPOLYGON" "sfg"
 - attr(*, "sf_column")= chr "geometry"
 - attr(*, "agr")= Factor w/ 3 levels "constant","aggregate",..: NA NA NA NA NA NA NA NA
  ..- attr(*, "names")= chr [1:8] "AREA" "PERIMETER" "COUNTYP010" "STATE" ...
```

sf classes

```r
class(nc)
```

```
[1] "sf"         "tbl_df"     "tbl"        "data.frame"
```

```r
class(nc$geometry)
```

```
[1] "sfc_MULTIPOLYGON" "sfc"
```

```r
class(nc$geometry[[1]])
```

```
[1] "XY"           "MULTIPOLYGON" "sfg"
```

Projections

```r
st_crs(nc)
```

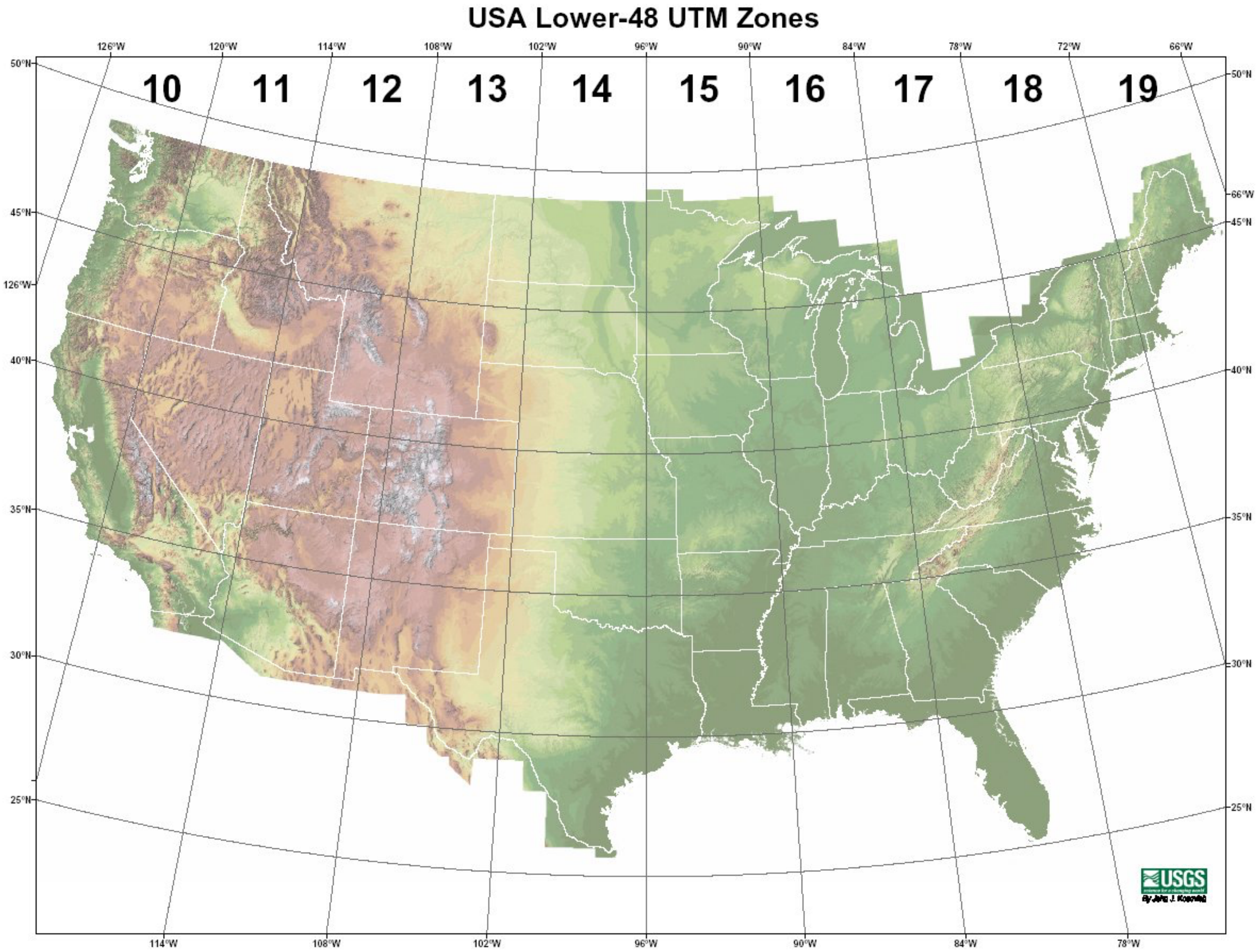
```
Coordinate Reference System:
  User input: NAD83 
  wkt:
GEOGCRS["NAD83",
    DATUM["North American Datum 1983",
        ELLIPSOID["GRS 1980",6378137,298.257222101,
            LENGTHUNIT["metre",1]]],
    PRIMEM["Greenwich",0,
        ANGLEUNIT["degree",0.0174532925199433]],
    CS[ellipsoidal,2],
        AXIS["latitude",north,
            ORDER[1],
            ANGLEUNIT["degree",0.0174532925199433]],
        AXIS["longitude",east,
            ORDER[2],
            ANGLEUNIT["degree",0.0174532925199433]],
```

EPSG? See here

```r
st_crs(hwy)
```

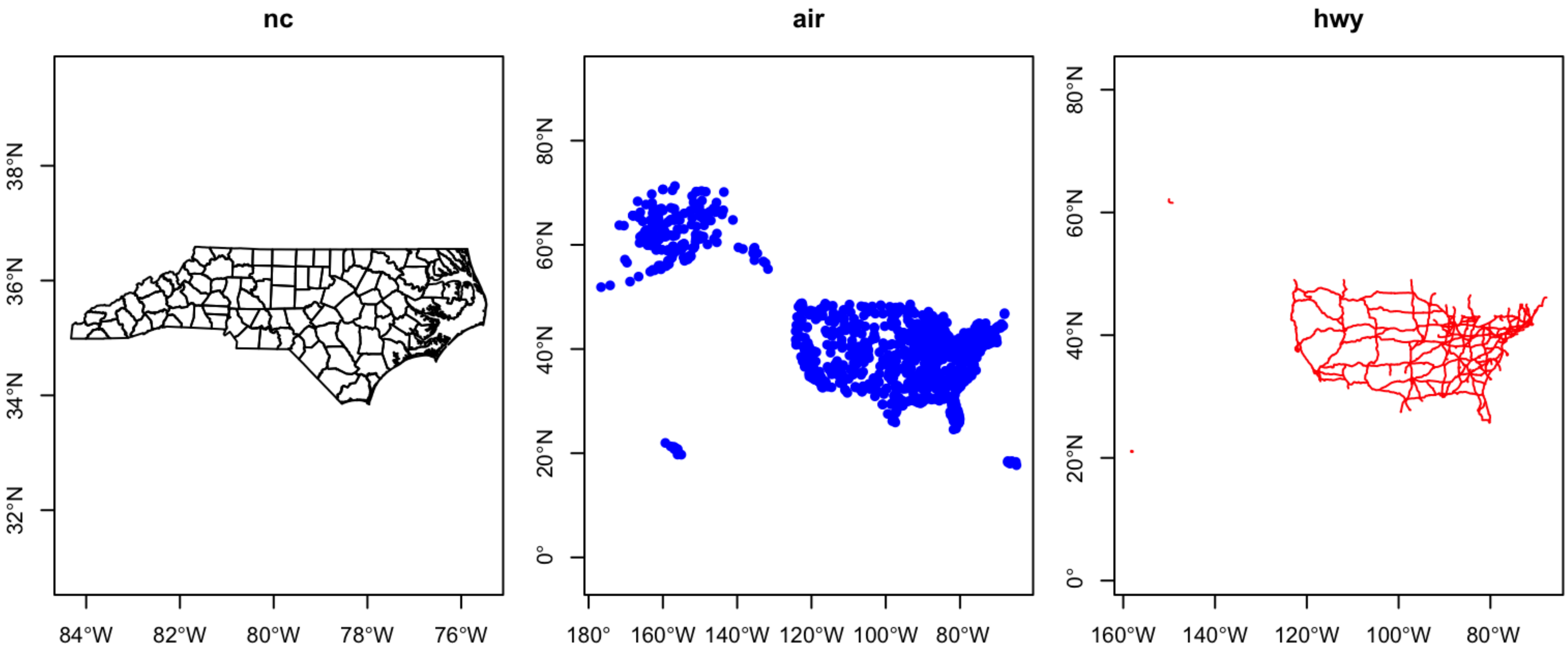
```
Coordinate Reference System:
  User input: NAD83 / UTM zone 15N
  wkt:
PROJCRS["NAD83 / UTM zone 15N",
    BASEGEOGCRS["NAD83",
        DATUM["North American Datum 1983",
            ELLIPSOID["GRS 1980",6378137,298.257222101,
                LENGTHUNIT["metre",1]]],
        PRIMEM["Greenwich",0,
            ANGLEUNIT["degree",0.0174532925199433]],
        ID["EPSG",4269]],
    CONVERSION["UTM zone 15N",
        METHOD["Transverse Mercator",
            ID["EPSG",9807]],
        PARAMETER["Latitude of natural origin",0,
            ANGLEUNIT["Degree",0.0174532925199433],
```

UTM Zones


USA Lower-48 UTM Zones
126°W
120°W
114°W
108°W
102°W
96°W
90°W
84°W
78°W
72°W
66°W
10
11
12
13
14
15
16
17
18
19
50°N
45°N
126°W
40°N
35°N
30°N
25°N
50°N
66°W
45°N
40°N
35°N
30°N
25°N
114°W
108°W
102°W
96°W
90°W
84°W
78°W
USGS

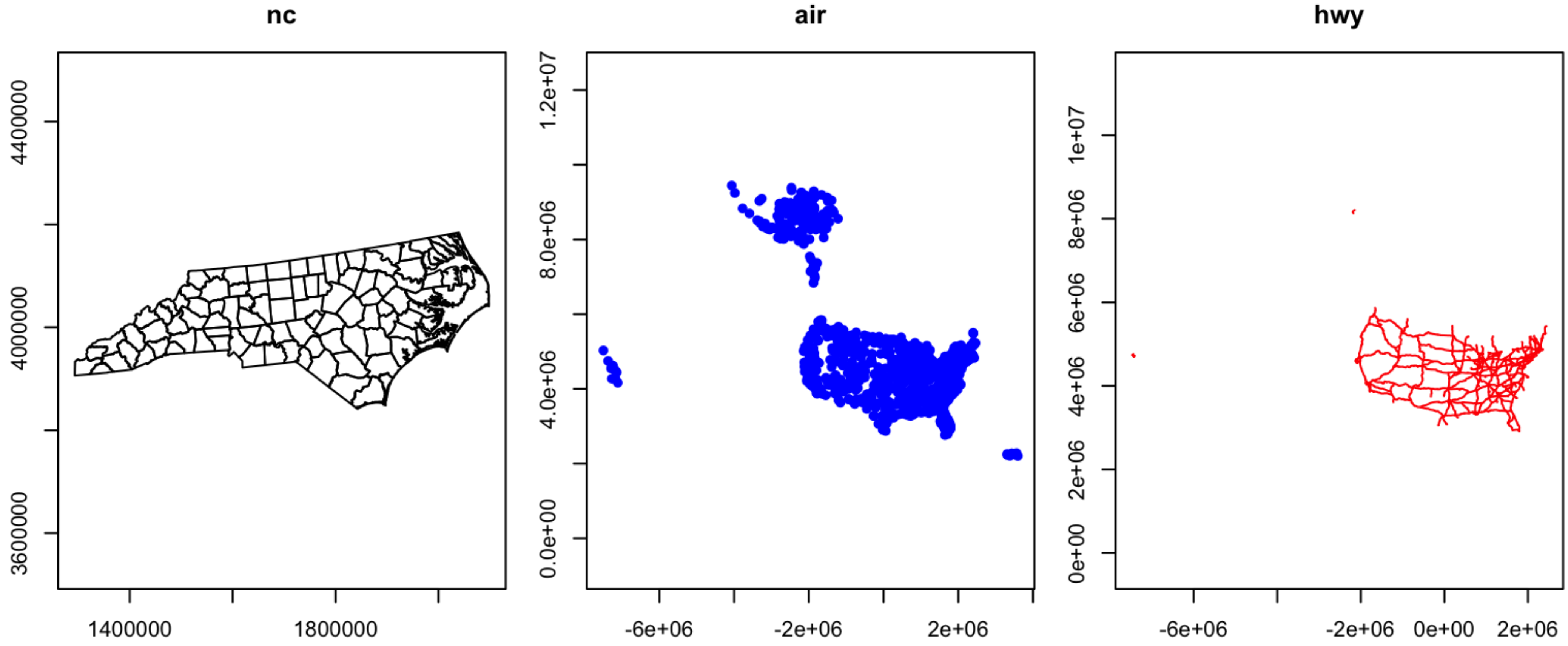
Lat/Long

```
nc_ll = nc
air_ll = air
hwy_ll = st_transform(hwy, st_crs(nc))
```

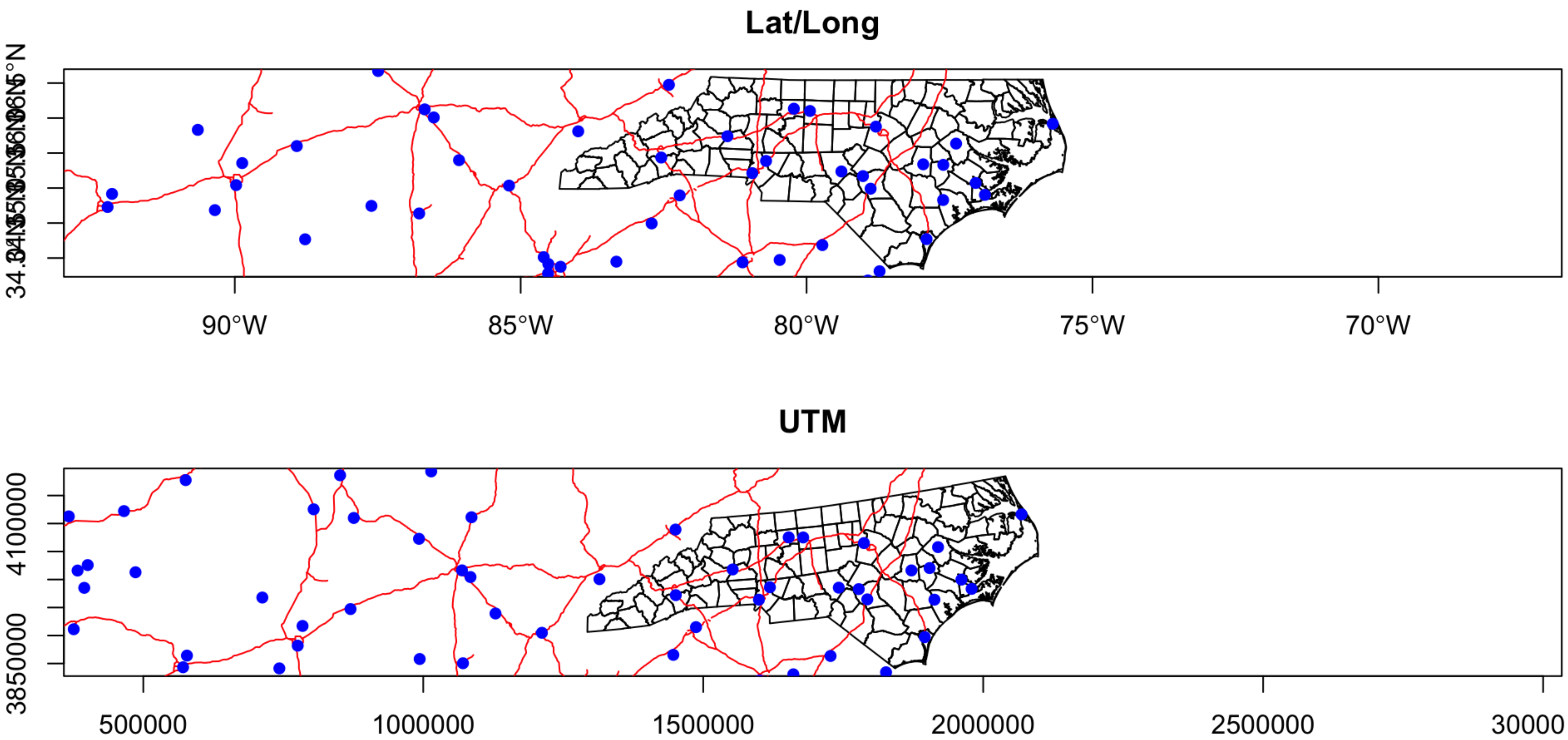


UTM

```
nc_utm = st_transform(nc, st_crs(hwy))
air_utm = st_transform(air, st_crs(hwy))
hwy_utm = hwy
```



Comparison



Lat/Long

UTM