Lec 7

$$Y_t = Y_{t-1} + \epsilon_t \qquad \epsilon_t \sim N(0,1)$$
$$Y_0 = 0$$

$$E(Y_t) = E\left(\sum \epsilon_i\right) = \sum\left(E\epsilon_i\right) = \sum 0 = 0$$

$Y_0 = 0$

$Y_1 = \epsilon_1$

$Y_2 = \epsilon_2 + \epsilon_1$

$Y_3 = \epsilon_3 + \epsilon_2 + \epsilon_1$

$\vdots$

$Y_t = \sum\limits_{i=1}^{t} \epsilon_i$

$$Cov(Y_t, Y_{t+k}) = E\left((Y_t - 0)(Y_{t+k} - 0)\right)$$

$$= E\left(Y_t \cdot Y_{t+k}\right)$$

$$= E\left(\left(\sum\limits_{i=1}^{t} \epsilon_i\right)\left(\sum\limits_{j=1}^{t+k} \epsilon_j\right)\right)$$

if $i = j$

$$E(\epsilon_i \epsilon_j) = E(\epsilon_i^2)$$
$$= Var(\epsilon_i) + E(\epsilon_i)^2 = 1 + 0 = 1$$

if $i \neq j$

$$E(\epsilon_i \epsilon_j) = E(\epsilon_i)E(\epsilon_j)$$
$$= 0 \cdot 0 = 0$$

$$= E\left(\epsilon_i^2 + \epsilon_{i+1}^2 + \epsilon_{i+2}^2 + \cdots\right)$$
$$= t$$

$$\text{Drift} \quad Y_t = \delta + Y_{t-1} + W_t \qquad Y_0 = 0$$

$$Y_0 = 0$$

$$Y_1 = \delta + V_1$$

$$Y_2 = 2\delta + V_2 + V_1$$

$$Y_3 = 3\delta + V_3 + V_2 + W_1$$

$$\vdots$$

$$Y_t = t\delta + \sum_{i=1}^{t} V_i$$

$$E(Y_t) = \delta t$$

$$Cov(Y_t, Y_{t+k}) = t$$

MA

$$y_t = v_{t-1} + w_t \qquad v_t \sim N(0,1)$$

$$y_1 = v_0 + w_1$$

$$y_2 = w_1 + v_2$$

$$y_3 = w_2 + w_3$$

$$y_4 = v_3 + v_4$$

$$\vdots$$

$$y_t = v_{t-1} + v_k$$

$$E(y_t) = E(w_{t-1}) + E(v_t)$$

$$= 0 + 0 = 0$$

$$Cov(y_t, y_{t+k})$$

$$= E\big((y_t - 0)(y_{t+k} - 0)\big)$$

$$= E(y_t \, y_{t+k})$$

$$= E\Big(\big(w_t + v_{t-1}\big)\big(v_{t+k} + v_{t+k-1}\big) \Big)$$

$$= E\Big(\underbrace{w_t \, v_{t+k}}_{k=0} + \underbrace{v_t \, v_{t+k-1}}_{k=1} + \underbrace{v_{t-1} \, v_{t+k}}_{k=-1} + \underbrace{v_{k-1} \, w_{t+k-1}}_{k=0} \Big)$$

$$= \begin{cases} 2 & k = 0 \\ 1 & k = \pm 1 \\ 0 & |k| \geq 2 \end{cases}$$