Lec 7

$$Y_t = Y_{t-1} + \epsilon_t \qquad \epsilon_t \sim N(0,1)$$

$$Y_0 = 0$$

$$Y_0 = 0$$

$$Y_1 = \epsilon_1$$

$$Y_2 = \epsilon_2 + \epsilon_1$$

$$Y_3 = \epsilon_3 + \epsilon_2 + \epsilon_1$$

$$\vdots$$

$$Y_t = \sum_{i=1}^{t} \epsilon_i$$

$$E(Y_t) = E\left(\sum \epsilon_i\right) = \sum \left(E\epsilon_i\right) = \sum 0 = 0$$

$$\begin{aligned}
Cov(Y_t, Y_{t+k}) &= E\left((Y_t - 0)(Y_{t+k} - 0)\right) \\
&= E\left(Y_t \cdot Y_{t+k}\right) \\
&= E\left(\left(\sum_{i=1}^{t} \epsilon_i\right)\left(\sum_{j=1}^{t+k} \epsilon_j\right)\right)
\end{aligned}$$

if $i = j$

$$\begin{aligned}
E(\epsilon_i \epsilon_j) &= E(\epsilon_i^2) \\
&= Var(\epsilon_i) + E(\epsilon_i)^2 = 1 + 0 = 1
\end{aligned}$$

if $i \neq j$

$$\begin{aligned}
E(\epsilon_i \epsilon_j) &= E(\epsilon_i) E(\epsilon_j) \\
&= 0 \cdot 0 = 0
\end{aligned}$$

$$\begin{aligned}
&= E\left(\epsilon_i^2 + \epsilon_{i+1}^2 + \epsilon_{i+2}^2 + \dots\right) \\
&= t
\end{aligned}$$

Drift $Y_t = \delta + Y_{t-1} + w_t \quad Y_0 = 0$

$Y_0 = 0$

$Y_1 = \delta + w_1$

$Y_2 = 2\delta + w_2 + w_1$

$Y_3 = 3\delta + w_3 + w_2 + w_1$

$\vdots$

$Y_t = t\delta + \sum_{i=1}^{t} w_i$

$E(Y_t) = \delta t$

$Cov(Y_t, Y_{t+k}) = t$

MA

$$Y_t = w_{t-1} + w_t \qquad w_t \sim N(0,1)$$

$$Y_1 = w_0 + w_1$$

$$Y_2 = w_1 + w_2$$

$$Y_3 = w_2 + w_3$$

$$Y_4 = w_3 + w_4$$

$$\vdots$$

$$Y_t = w_{t-1} + w_t$$

$$E(Y_t) = E(w_{t-1}) + E(w_t)$$

$$= 0 + 0 = 0$$

$$Cov(Y_t, Y_{t+k})$$

$$= E\left((Y_t - 0)(Y_{t+k} - 0)\right)$$

$$= \frac{1}{2} E(Y_t Y_{t+k})$$

$$= \frac{1}{2} E\left((w_t + w_{t-1})(w_{t+k} + w_{t+k-1})\right)$$

$$= \frac{1}{2} E\left(\underbrace{w_t w_{t+k}}_{k=0} + \underbrace{w_t w_{t+k-1}}_{k=1} + \underbrace{w_{t-1} w_{t+k}}_{k=-1} + \underbrace{w_{t-1} w_{t+k-1}}_{k=0}\right)$$

$$= \frac{1}{2} \begin{cases} 2 & k = 0 \\ 1 & k = \pm 1 \\ 0 & |k| \geq 2 \end{cases}$$