Lec 7

Linear trend

$$Y_t = \delta + \beta t + \quad {}_t$$

$$d_t = Y_t - Y_{t-1} = (\delta + \beta t + x_t) - (\delta + \beta(t-1) + x_{t-1})$$

$$= \beta + X_t - x_{t-1}$$

$d_t \rightarrow$ stationary

Quad trend

$$d_t - d_{t-1} = (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2})$$

$$d_t = (\delta + \beta t + \gamma t^2 + X_t) - (\delta + \beta(t-1) + \gamma(t-1)^2 + X_{t-1})$$

$$= \cancel{\beta} + (\gamma \cancel{t^2} - \cancel{\gamma t^2} + 2\cancel{\gamma} t - \cancel{\gamma^2}) + (X_t - X_{t-1})$$

$$d_{t-1} = \cancel{\beta} + (2\gamma(\cancel{t}-1) - \cancel{\gamma^2}) + (X_{t-1} - X_{t-2})$$

$$= \gamma + X_t - 2X_{t-1} + X_{t-2}$$

AR(1)

$$Y_t = \delta + \phi Y_{t-1} + \epsilon_t \qquad \text{Assume } t \text{ is } \underline{\text{infinite}}$$

$$= \delta + \phi(\delta + \phi Y_{t-2} + \epsilon_{t-1}) + \epsilon_t$$

$$= (\delta + \phi\delta) + (\epsilon_t + \phi\epsilon_{t-1}) + \phi^2 Y_{t-2}$$

$$= (\delta + \phi\delta) + (\epsilon_t + \phi\epsilon_{t-1}) + \phi^2(\delta + \phi Y_{t-3} + \epsilon_{t-2})$$

$$= (\delta + \phi\delta + \phi^2\delta) + (\epsilon_t + \phi\epsilon_{t-1} + \phi^2\epsilon_{t-2}) + \phi^3 Y_{t-3}$$

$$= \delta(1 + \phi + \phi^2 + \phi^3 + \dots) + (\epsilon_t + \phi\epsilon_{t-1} + \phi^2\epsilon_{t-2} + \phi^3\epsilon_{t-3} + \dots)$$

$$E(Y_t) = \delta(1 + \phi + \phi^2 + \phi^3 + \dots) = \frac{\delta}{1-\phi}$$

$$Var(Y_t) = \sigma_w^2(1 + \phi^2 + \phi^4 + \phi^6 + \dots) = \frac{\sigma_w^2}{1-\phi^2}$$

Conds

1. $Var(Y_t) < \infty$ $\longrightarrow$ $\phi^2 < 1 \quad |\phi| < 1$
2. $E(Y_t) = \mu \quad \forall t$
3. $Cov(Y_t, Y_s) = Cov(Y_{t+k}, Y_{s+k})$

$$\gamma(h) = Cov(Y_t, Y_{t+h})$$

$$\gamma(0) = Cov(Y_t, Y_t) = Var(Y_t) = \frac{\sigma_\nu^2}{1-\phi^2}$$

$$\gamma(1) = Cov(Y_t, Y_{t-1}) = E((Y_t - \mu)(Y_{t+1} - \mu))$$

$$\boxed{\mu = \frac{\delta}{1-\phi} = (1+\phi+\phi^2+\dots)\delta}$$

$$= E\Big((\cancel{\epsilon_t} + \phi\epsilon_{t-1} + \phi^2\epsilon_{t-2} + \dots)(\epsilon_{t-1} + \phi\epsilon_{t-2} + \phi\epsilon_{t-3} + \dots)\Big)$$

$$= \phi E(\epsilon_{t-1}^2) + \phi^3(\epsilon_{t-2}^2) + \dots$$

$$= \phi\sigma_\nu^2 + \phi^3\sigma_\nu^2 + \phi^5\sigma_\nu^2 + \dots$$

$$= \phi\sigma_\nu^2(1 + \phi^2 + \phi^4 + \phi^6 + \dots) = \phi\frac{\sigma_\nu^2}{1-\phi^2} = \phi\gamma(0)$$

$$\gamma(2) = E\Big((\cancel{\epsilon_t} + \phi\cancel{\epsilon_{t-1}} + \phi^2\epsilon_{t-2} + \phi^3\epsilon_{t-3} + \dots)(\epsilon_{t-2} + \phi\epsilon_{t-3} + \dots)\Big)$$

$$= \phi^2\sigma_\nu^2 + \phi^3\sigma_\nu^2 + \dots = \phi^2\frac{\sigma_\nu^2}{1-\phi^2} = \phi^2\gamma(0)$$

$$\gamma(h) = \phi^h\gamma(0) = \phi^h\frac{\sigma_\nu^2}{1-\phi^2}$$