# Lec 10

MA(1) Invertibility

$$Y_t = W_t + \theta V_{t-1}$$

$$V_t = Y_t - \theta V_{t-1}$$

$$V_t = Y_t - \theta \left( Y_{t-1} - \theta V_{t-2} \right)$$

$$= Y_t - \theta Y_{t-1} + \theta^2 V_{t-2}$$

$$= Y_t + \sum_{i=1}^{\infty} (-\theta)^i Y_{t-i}$$

$$= Y_t + \sum_{i=1}^{p} (-\theta)^i Y_{t-i} + (-\theta)^{p+1} Y_{t-(p+1)}$$

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$$Y_t = \mu_t + W_t \qquad \mu_t = \mu_{t-1} + V_t$$

$$\mu_0 = 0$$

$$\mu_1 = V_1$$

$$\mu_2 = V_1 + V_2$$

$$\vdots$$

$$\mu_t = \sum_{i=1}^{t} V_i$$

$$Y_0 = 0$$

$$Y_1 = V_1 + W_1$$

$$Y_2 = V_1 + V_2 + V_2$$

$$\vdots$$

$$Y_t = \sum_{i=1}^{t} V_i + W_t$$

$$E(Y_t) = 0$$

~~$$Var(Y_t) = t\sigma_v^2 + \sigma^2$$~~

$$\Delta Y_t = Y_t - Y_{t-1} = (\mu_t + w_t) - (\mu_{t-1} + w_{t-1})$$

$$= \sum_{i=1}^{t} v_i + w_t - \sum_{i=1}^{t-1} v_i - w_{t-1}$$

$$= V_t + w_t - w_{t-1}$$

$$E(\Delta Y_t) = 0$$

$$Var(\Delta Y_t) = \sigma_v^2 + 2\sigma_w^2$$

$$Cov(\Delta Y_t, \Delta Y_{t-h}) = E\left( (V_t + w_t + w_{t-1})(V_{t-1} + w_{t-1} + w_{t-2}) \right)$$

$$= \begin{cases} \sigma_v^2 + 2\sigma_w^2 & \text{if } h = 0 \\ \sigma_w^2 & \text{if } h = 1 \\ 0 & \text{if } h > 1 \end{cases}$$

$$Y_t = \mu_t + v_t \qquad \mu_t = \mu_{t-1} + v_t \qquad v_t = v_{t-1} + e_t$$

$v_0 = 0$ $\qquad \mu_0 = 0$

$v_1 = e_1$ $\qquad \mu_1 = e_1$

$v_2 = e_1 + e_2$ $\qquad \mu_2 = e_1 + v_2 = 2e_1 + e_2$

$v_3 = e_1 + e_2 + e_3$ $\qquad \mu_3 = 2e_1 + e_2 + (e_1 + e_2 + e_3)$

$\qquad = 3e_1 + 2e_2 + e_3$

$\vdots$ $\qquad \vdots$

$$v_t = \sum_{i=1}^{t} e_i \qquad \mu_t = \sum_{i=1}^{t} (t+1-i)\, e_i$$

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$$\Delta Y_t = (\mu_t + v_t) - (\mu_{t-1} + v_{t-1})$$

$$= (\mu_t - \mu_{t-1}) + (v_t - v_{t-1})$$

$$= (\mu_{t-1} + v_t - \mu_{t-1}) + (w_t - w_{t-1})$$

$$= v_t + (w_t - w_{t-1})$$

$$= \sum_{i=1}^{t} e_i + (w_t - w_{t-1})$$

$E(\Delta Y_t) = 0$ $\qquad$ $Var(\Delta Y_t) = t\sigma_e^2 + 2\sigma_w^2$ ✗

$$\Delta^2 y_t = \left[v_t + (w_t - w_{t-1})\right] - \left[v_{t-1} + (w_{t-1} - w_{t-2})\right]$$

$$= v_{t-1} + e_t - v_{t-1} + w_t - 2w_{t-1} + w_{t-2}$$

$$= e_t + w_t - 2w_{t-1} + w_{t-2}$$

$$E(\Delta^2 y_t) \simeq 0$$

$$V_{ar}(\Delta^2 y_t) = \sigma_e^2 + 6\sigma_w^2$$