Seasonal Arima

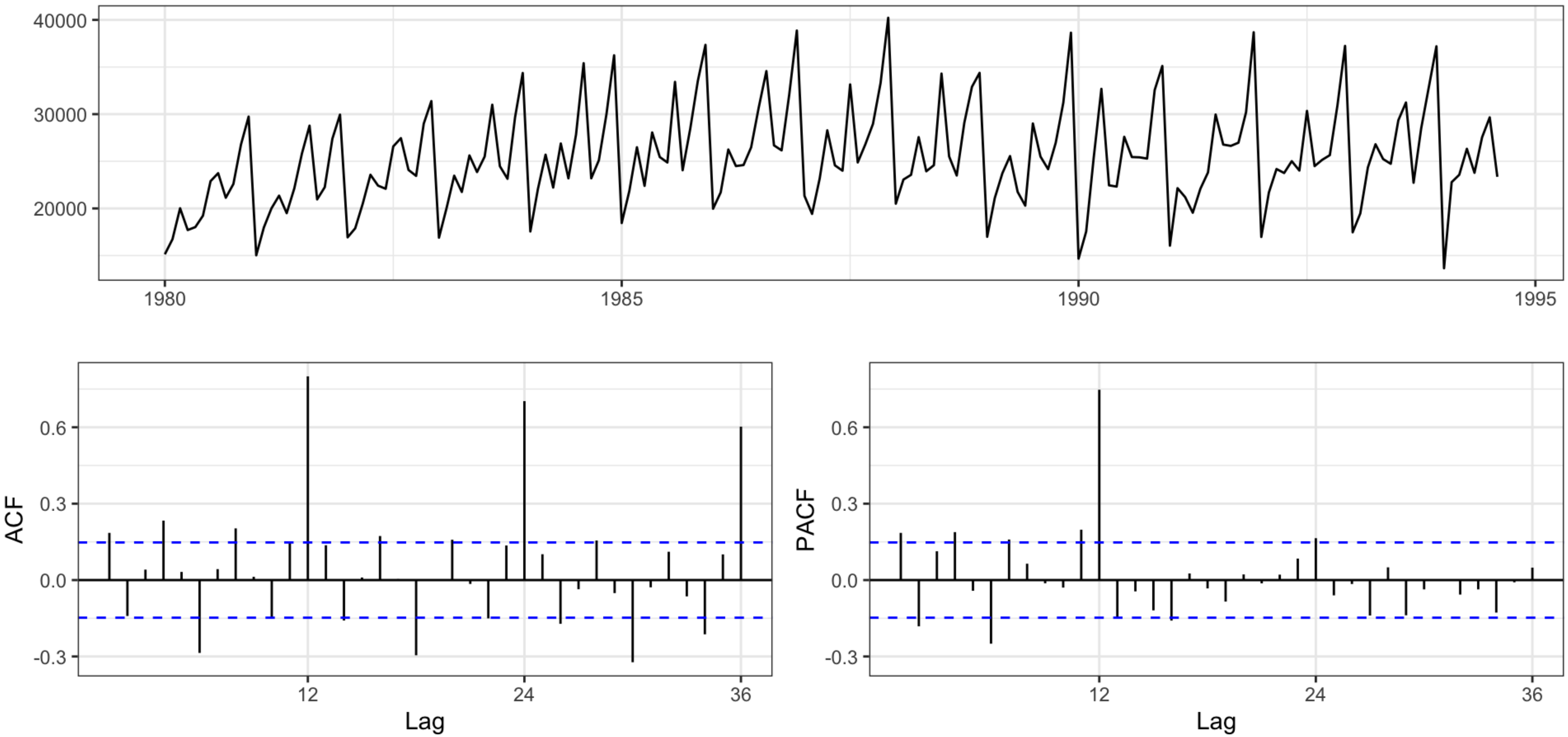
Lecture 11

Dr. Colin Rundel

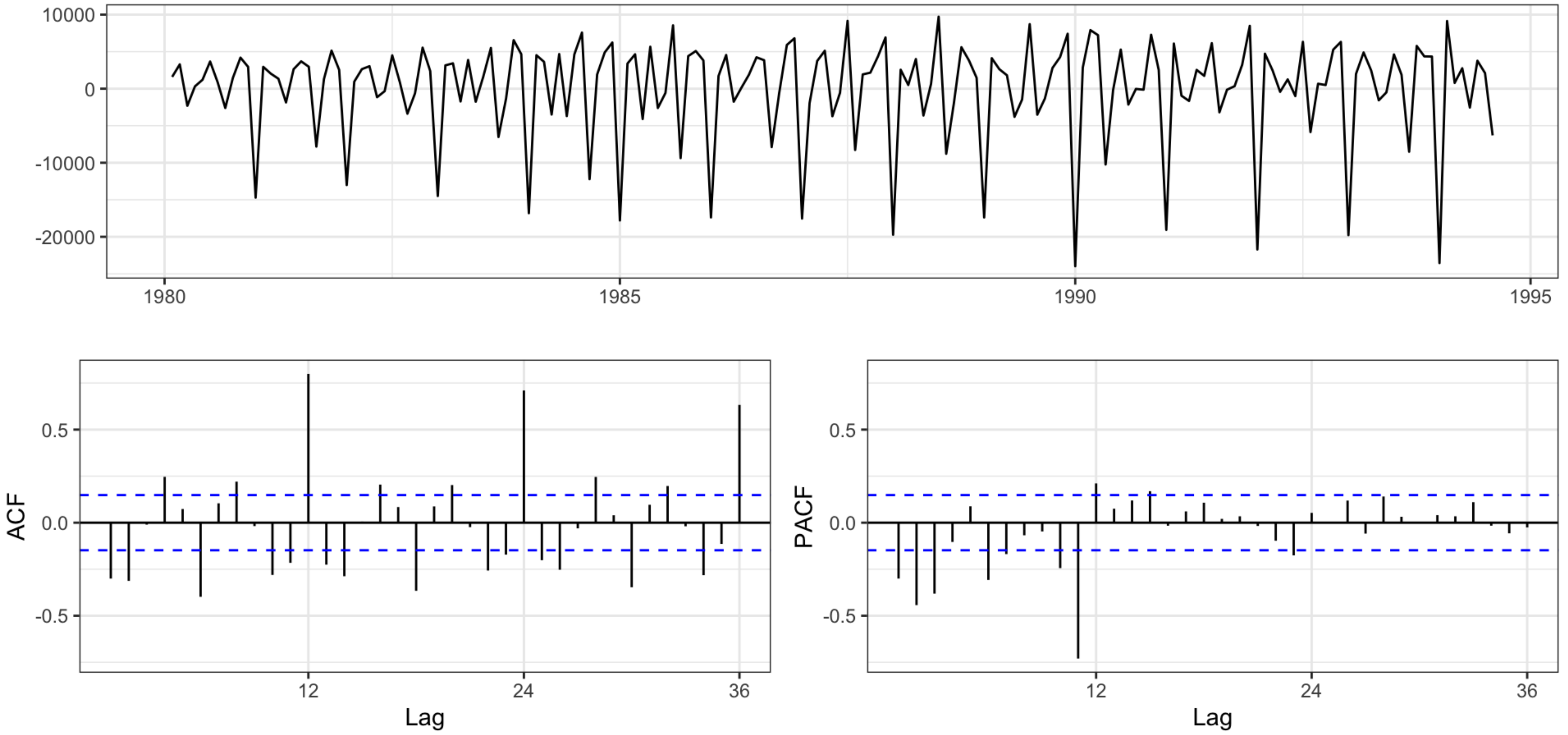
Seasonal Models

Australian Wine Sales Example

Australian total wine sales by wine makers in bottles <= 1 litre. Jan 1980 – Aug 1994.



Differencing



Seasonal Arima

We can extend the existing ARIMA model to handle these higher order lags (without having to include all of the intervening lags).

Seasonal $\text{ARIMA}\,(p, d, q) \times (P, D, Q)_s$:

$$\Phi_P(L^s)\,\phi_p(L)\,\Delta_s^D\,\Delta^d\,y_t = \delta + \Theta_Q(L^s)\,\theta_q(L)\,w_t$$

where

$$\begin{aligned}
\phi_p(L) &= 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p \\
\theta_q(L) &= 1 + \theta_1 L + \theta_2 L^2 + \ldots + \theta_p L^q \\
\Delta^d &= (1 - L)^d
\end{aligned}$$

$$\begin{aligned}
\Phi_P(L^s) &= 1 - \Phi_1 L^s - \Phi_2 L^{2s} - \ldots - \Phi_P L^{Ps} \\
\Theta_Q(L^s) &= 1 + \Theta_1 L + \Theta_2 L^{2s} + \ldots + \theta_p L^{Qs} \\
\Delta_s^D &= (1 - L^s)^D
\end{aligned}$$

Seasonal ARIMA - AR

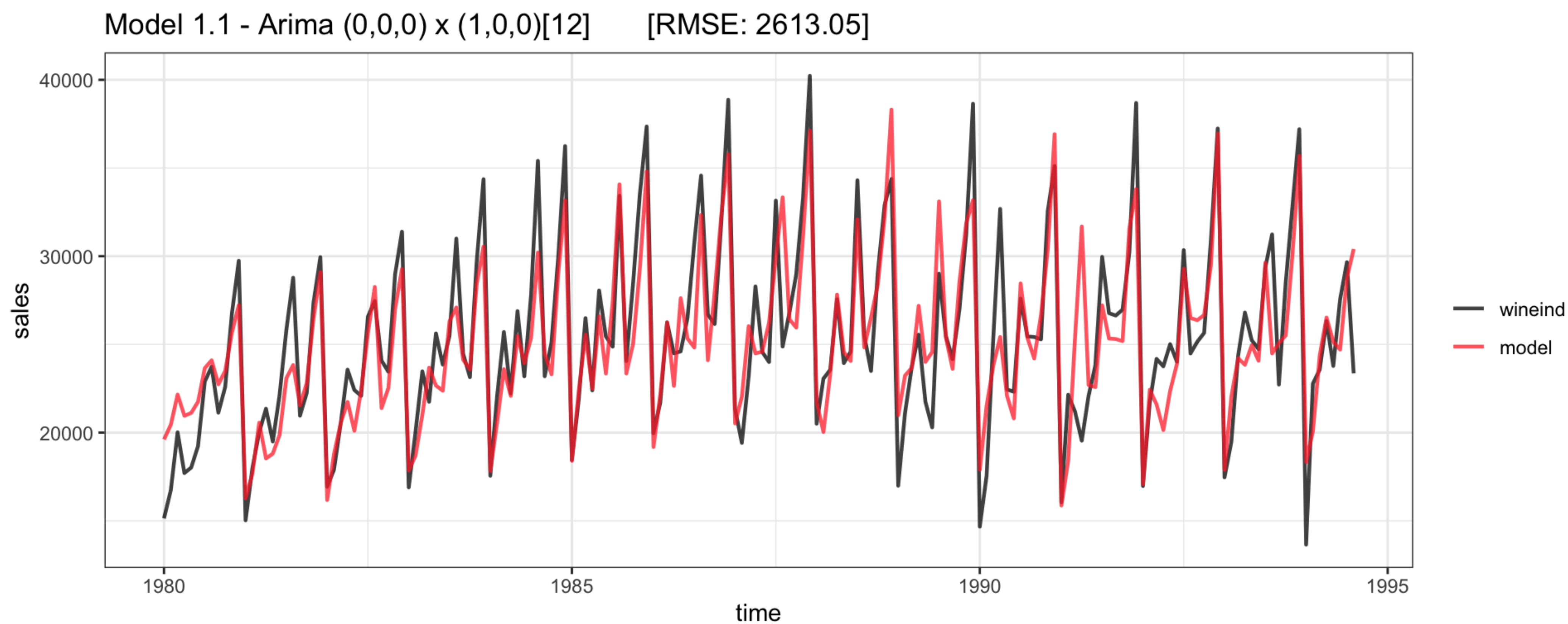
Lets consider an $\text{ARIMA}(0,0,0) \times (1,0,0)_{12}$:

$$(1 - \Phi_1 L^{12})\, y_t = \delta + w_t$$
$$y_t = \Phi_1 y_{t-12} + \delta + w_t$$

```r
(m1.1 = forecast::Arima(wineind, seasonal=list(order=c(1,0,0), period=12
```

```
Series: wineind
ARIMA(0,0,0)(1,0,0)[12] with non-zero mean

Coefficients:
        sar1        mean
      0.8780   24489.243
s.e.  0.0314    1154.487

sigma^2 = 6906536:  log likelihood = -1643.39
AIC=3292.78   AICc=3292.92   BIC=3302.29
```

Fitted model



Seasonal Arima - Diff

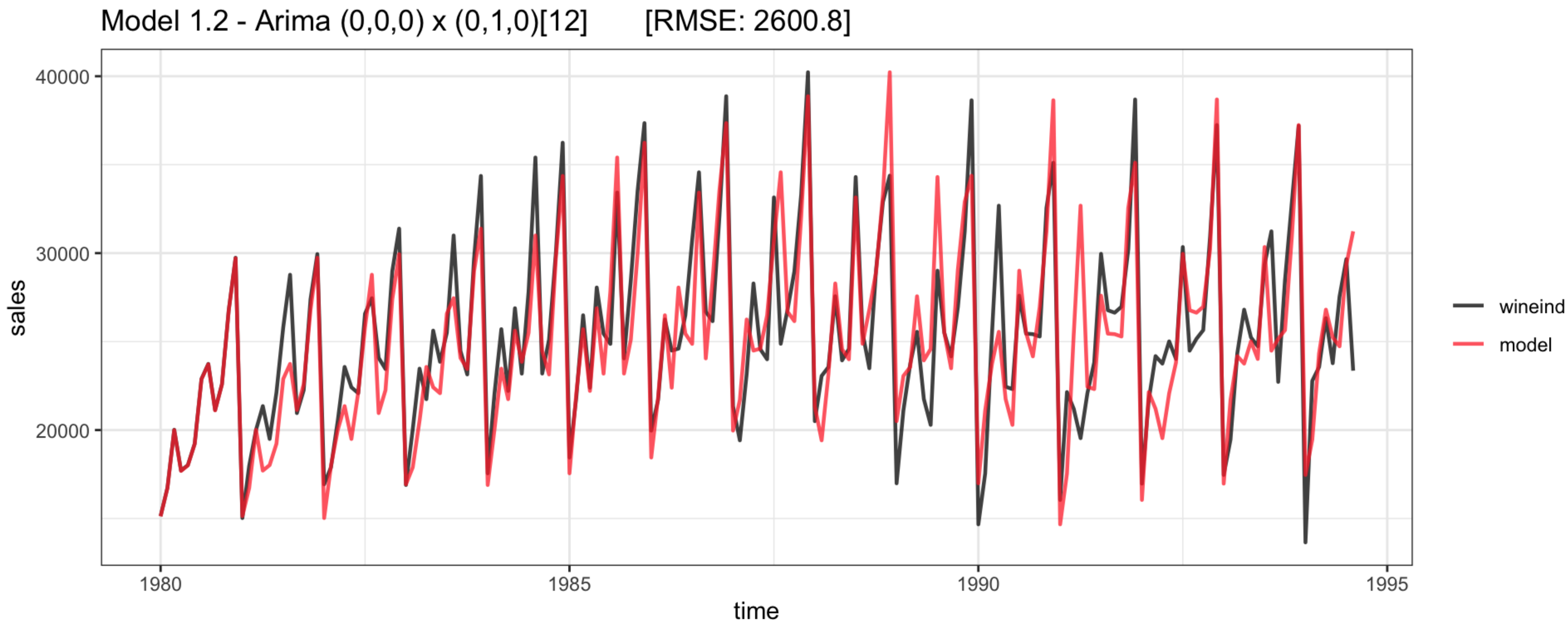
Lets consider an $\text{ARIMA}(0,0,0) \times (0,1,0)_{12}$:

$$
\begin{aligned}
(1 - L^{12})\, y_t &= \delta + w_t \\
y_t &= y_{t-12} + \delta + w_t
\end{aligned}
$$

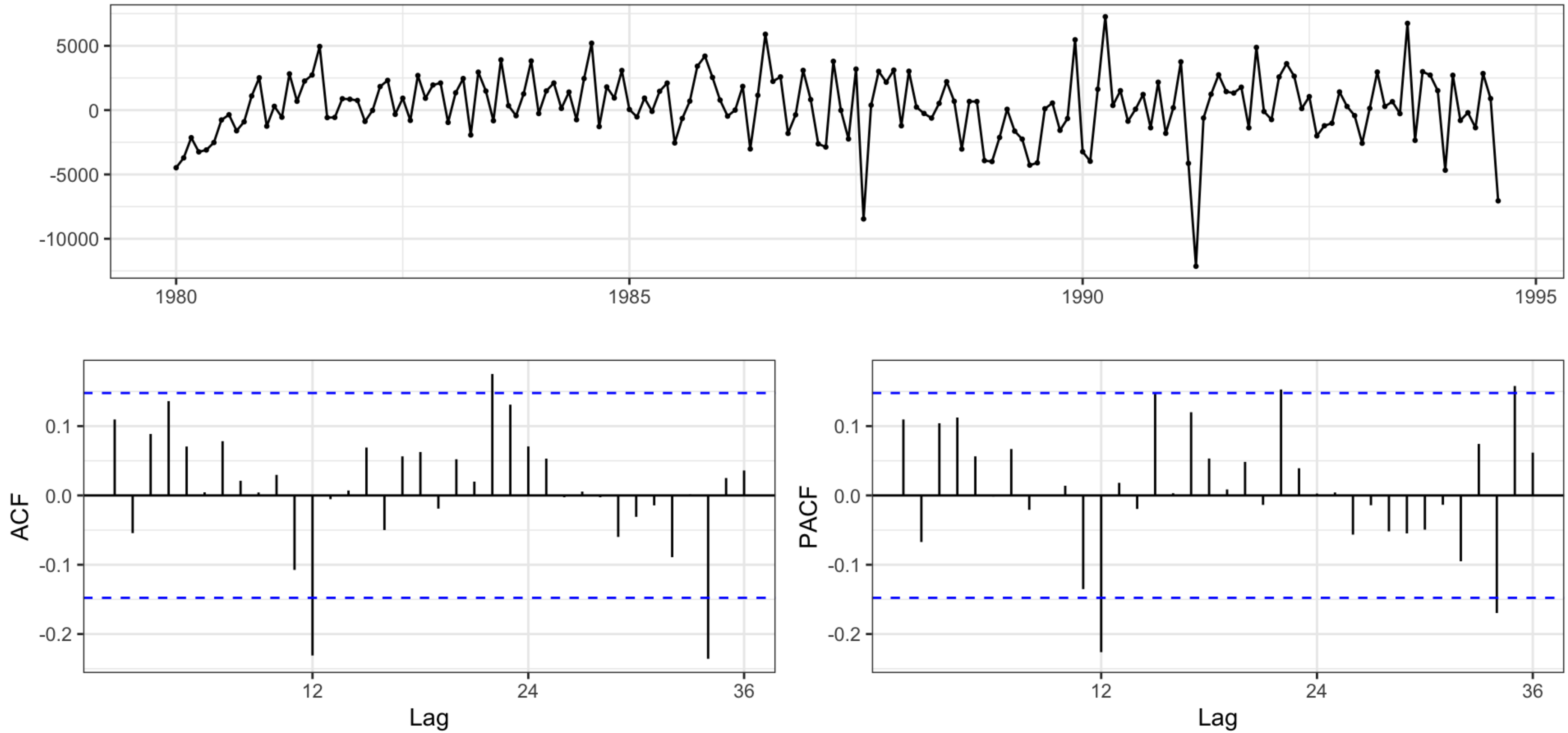
```r
(m1.2 = forecast::Arima(wineind, seasonal=list(order=c(0,1,0), period=12
```

```
Series: wineind
ARIMA(0,0,0)(0,1,0)[12]

sigma^2 = 7259076:  log likelihood = -1528.12
AIC=3058.24   AICc=3058.27   BIC=3061.34
```

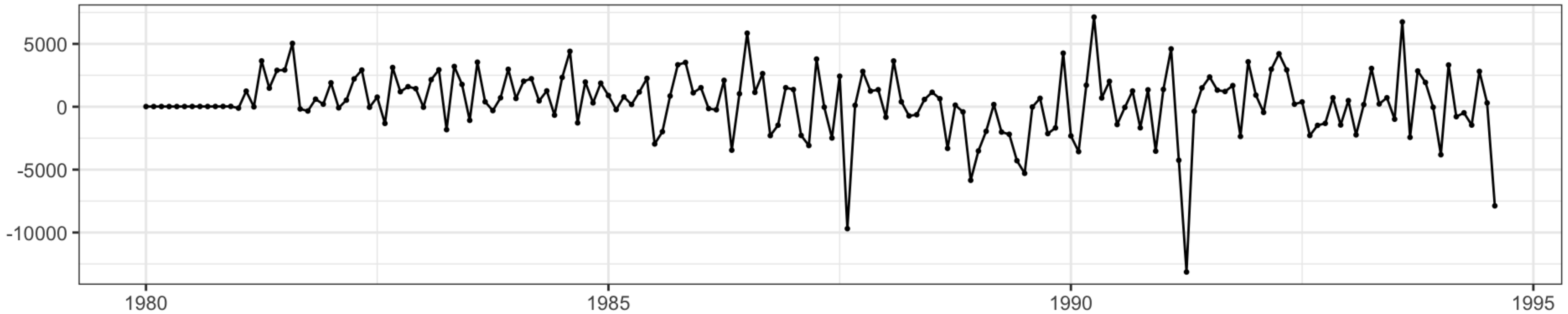
Fitted model

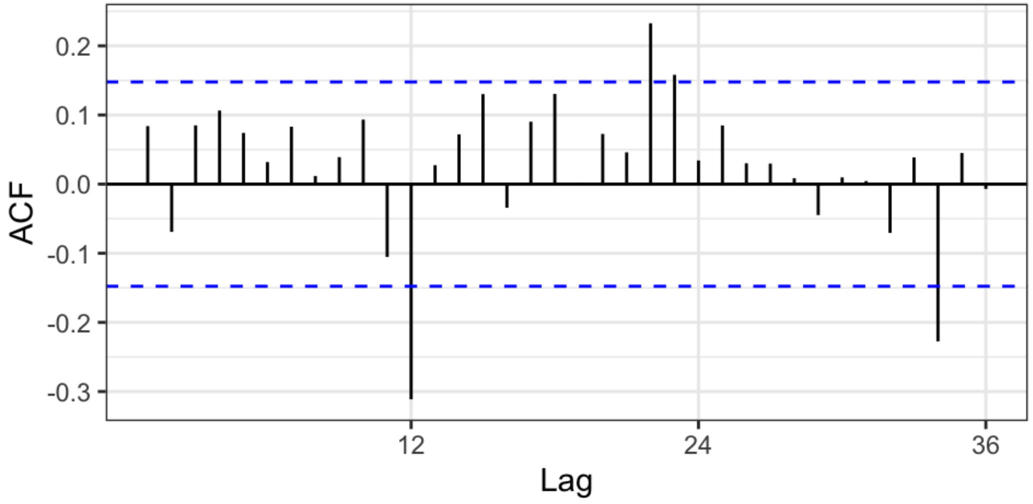


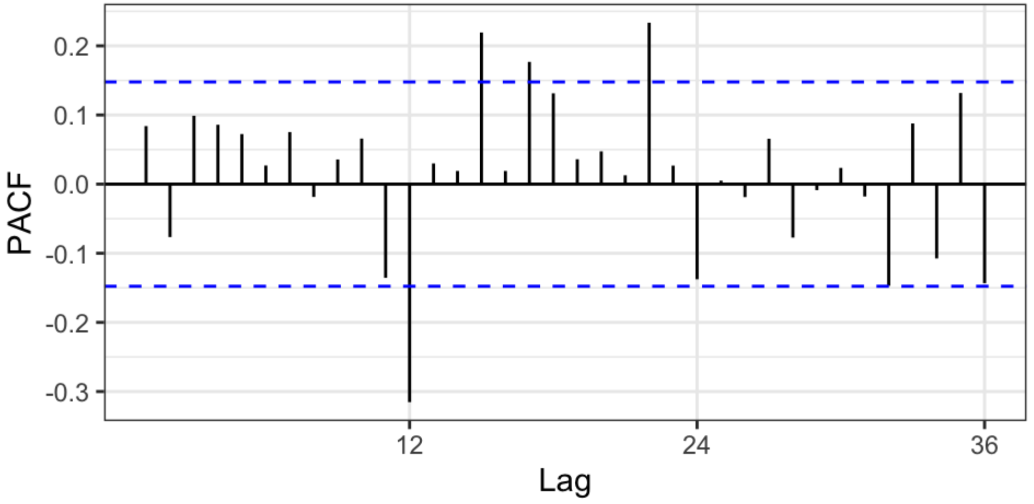
Residuals - Model 1.1



Residuals - Model 1.2







Model 2

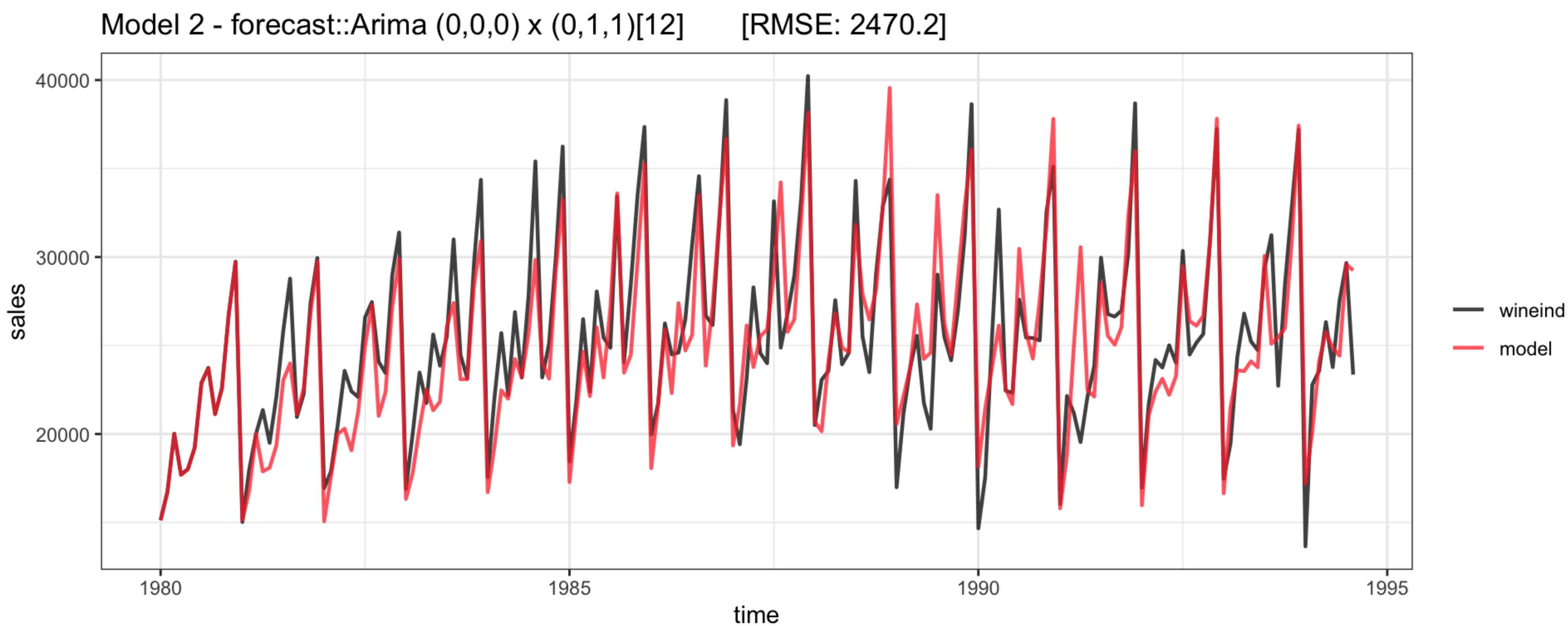
$\text{ARIMA}(0,0,0) \times (0,1,1)_{12}:$

$$
\begin{aligned}
(1 - L^{12}) y_t &= \delta + (1 + \Theta_1 L^{12}) w_t \\
y_t &= \delta + y_{t-12} + w_t + \Theta_1 w_{t-12}
\end{aligned}
$$

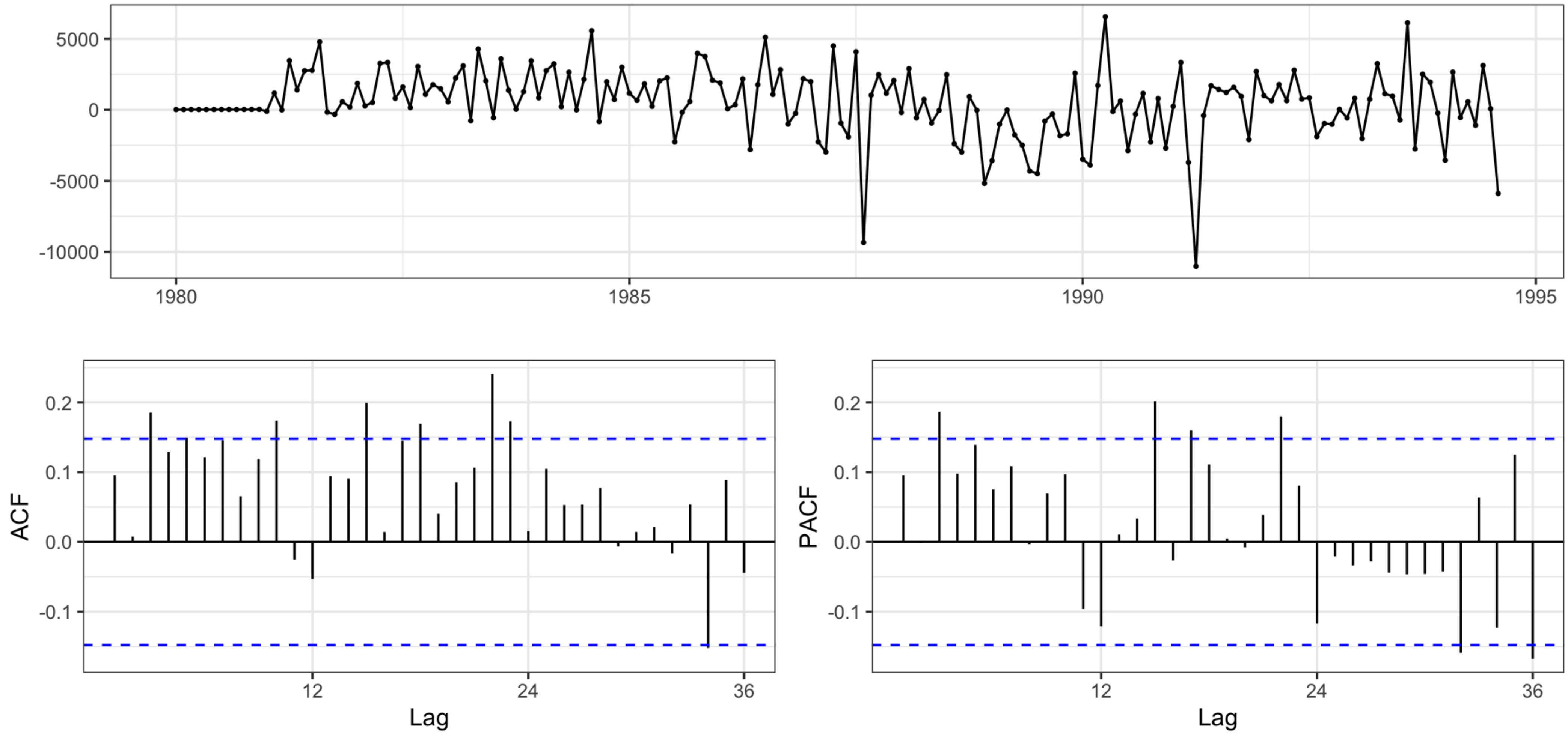
```r
(m2 = forecast::Arima(wineind, order=c(0,0,0),
                      seasonal=list(order=c(0,1,1), period=12)))
```

```
Series: wineind
ARIMA(0,0,0)(0,1,1)[12]

Coefficients:
         sma1
      -0.3246
s.e.   0.0807

sigma^2 = 6588531:  log likelihood = -1520.34
AIC=3044.68   AICc=3044.76   BIC=3050.88
```

Fitted model



Residuals



5000
0
-5000
-10000
1980
1985
1990
1995
0.2
0.1
0.0
-0.1
ACF
12
24
36
Lag
0.2
0.1
0.0
-0.1
PACF
12
24
36
Lag

Model 3

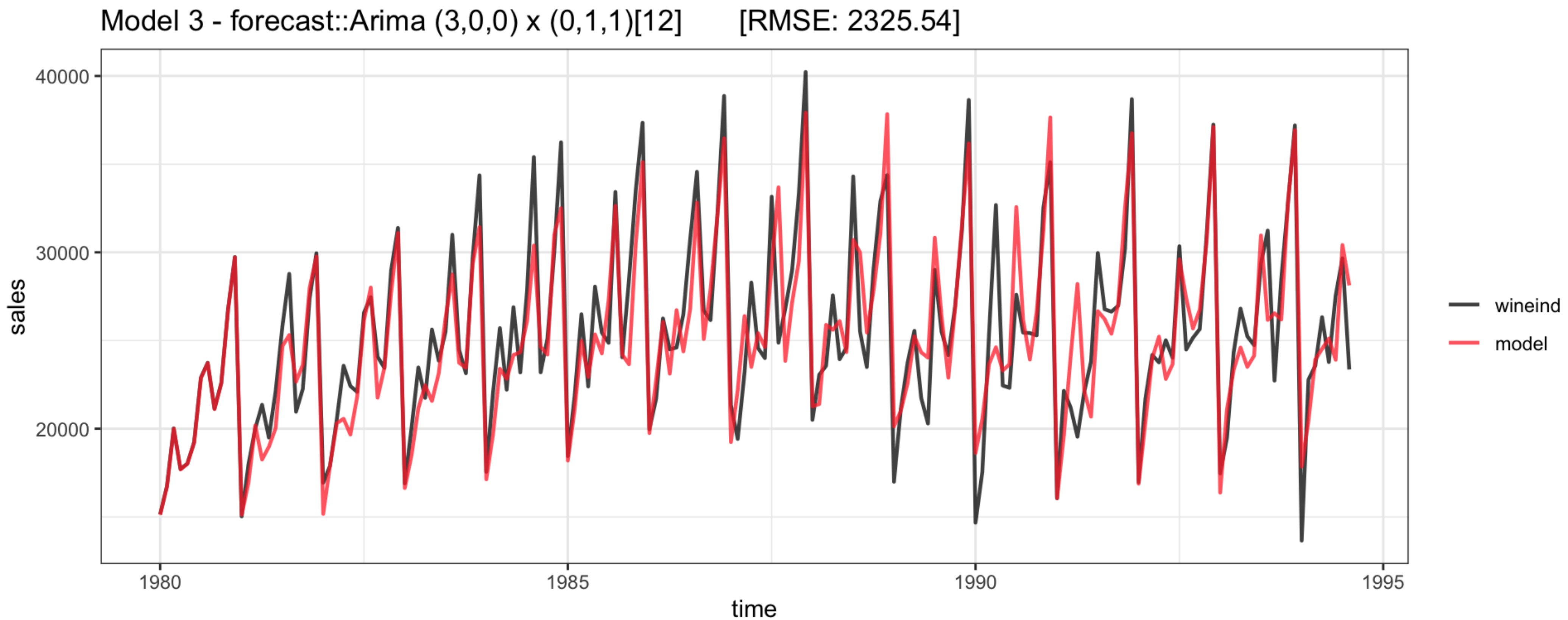
$\text{ARIMA}(3,0,0) \times (0,1,1)_{12}$

$$
\begin{aligned}
(1-\phi_1 L - \phi_2 L^2 - \phi_3 L^3)\,(1-L^{12}) y_t &= \delta + (1 + \Theta_1 L) w_t \\
(1-\phi_1 L - \phi_2 L^2 - \phi_3 L^3)\,(y_t - y_{t-12}) &= \delta + w_t + w_{t-12} \\
y_t = \delta + \sum_{i=1}^{3} \phi_i y_{t-1} + y_{t-12} - \sum_{i=1}^{3} \phi_i y_{t-12-i} &+ w_t + w_{t-12}
\end{aligned}
$$

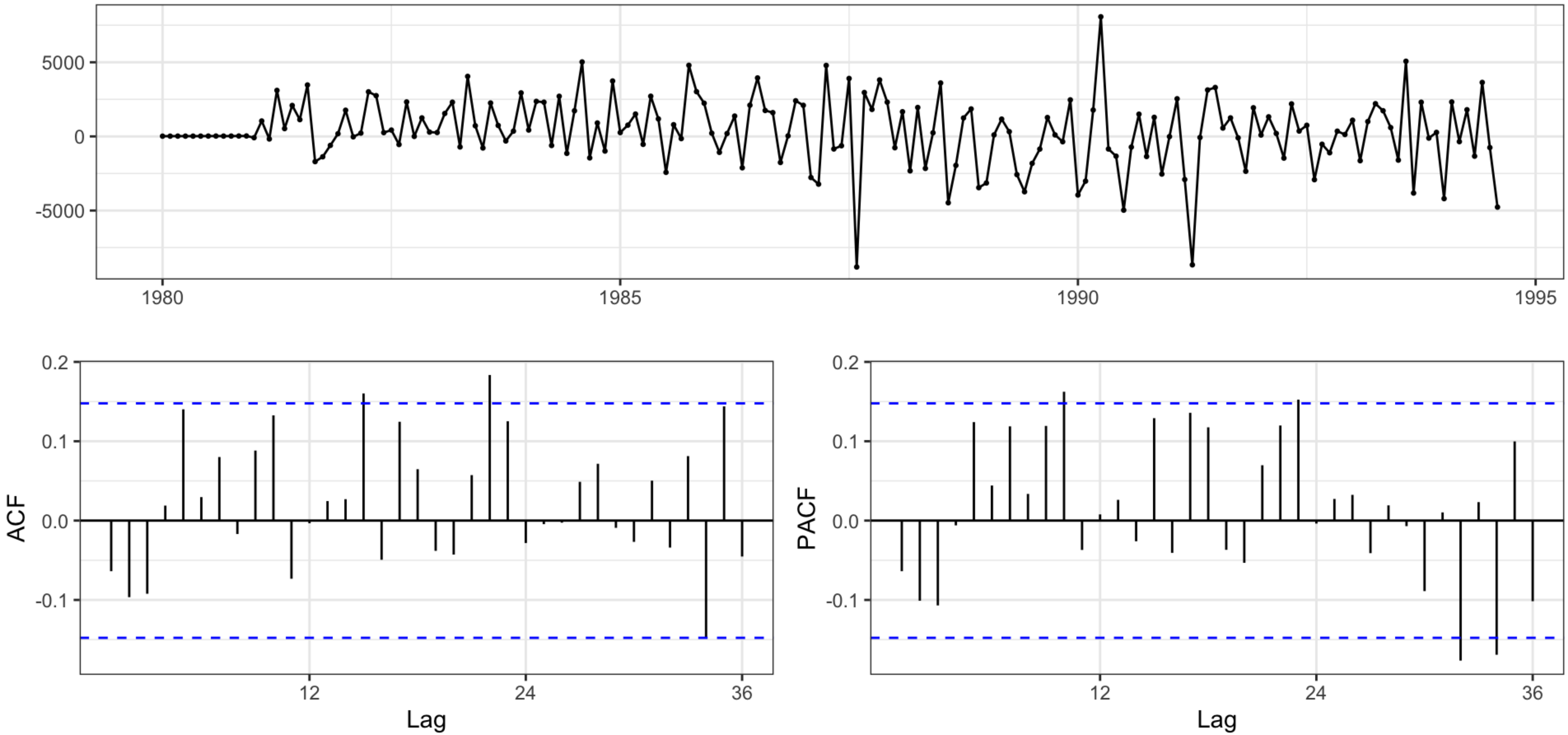
```r
(m3 = forecast::Arima(wineind, order=c(3,0,0),
                      seasonal=list(order=c(0,1,1), period=12)))
```

```
Series: wineind
ARIMA(3,0,0)(0,1,1)[12]

Coefficients:
         ar1     ar2     ar3     sma1
      0.1402  0.0806  0.3040  -0.5790
s.e.  0.0755  0.0813  0.0823   0.1023
```

Fitted model



Model - Residuals

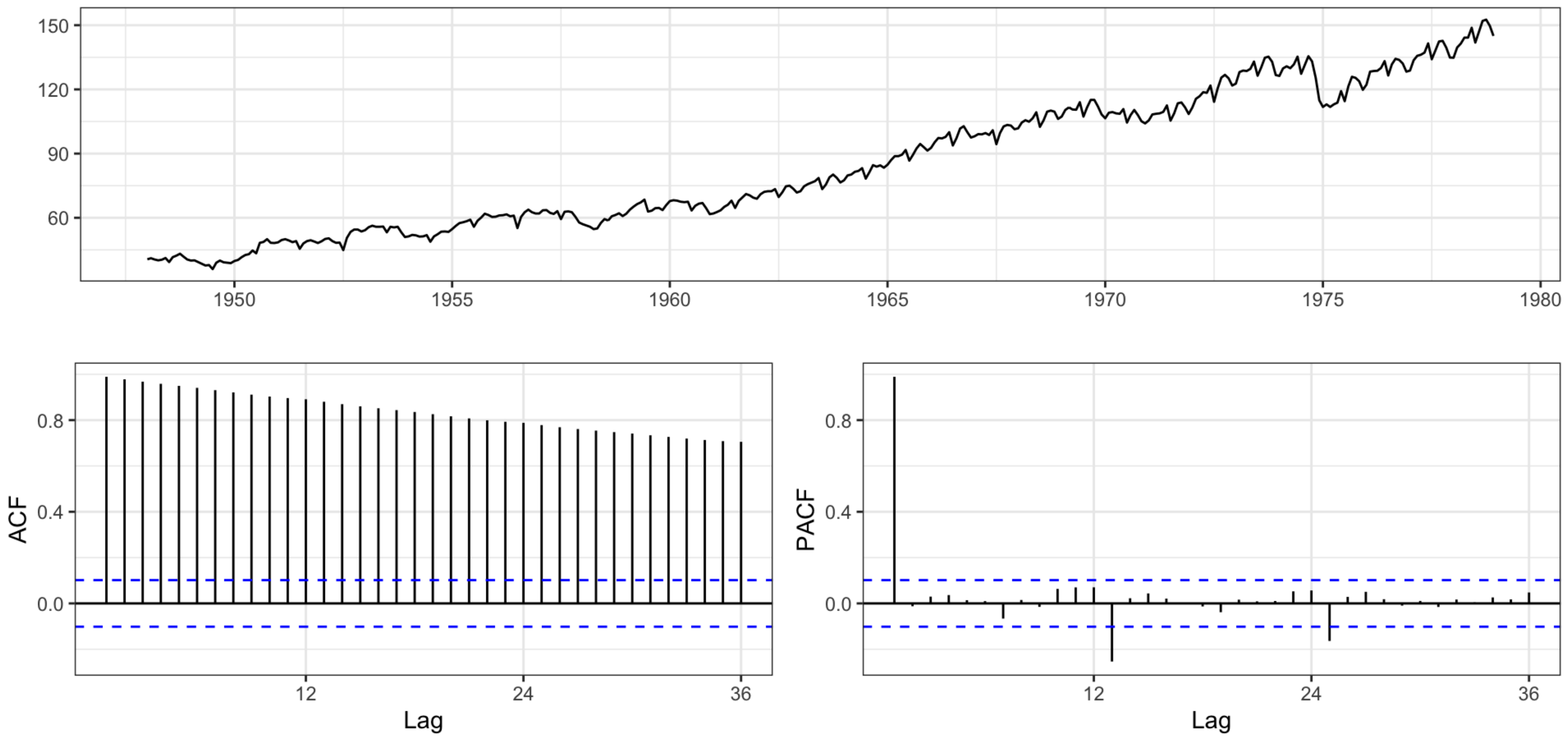


Federal Reserve Board Production Index

from the package

Monthly Federal Reserve Board Production Index (1948-1978)

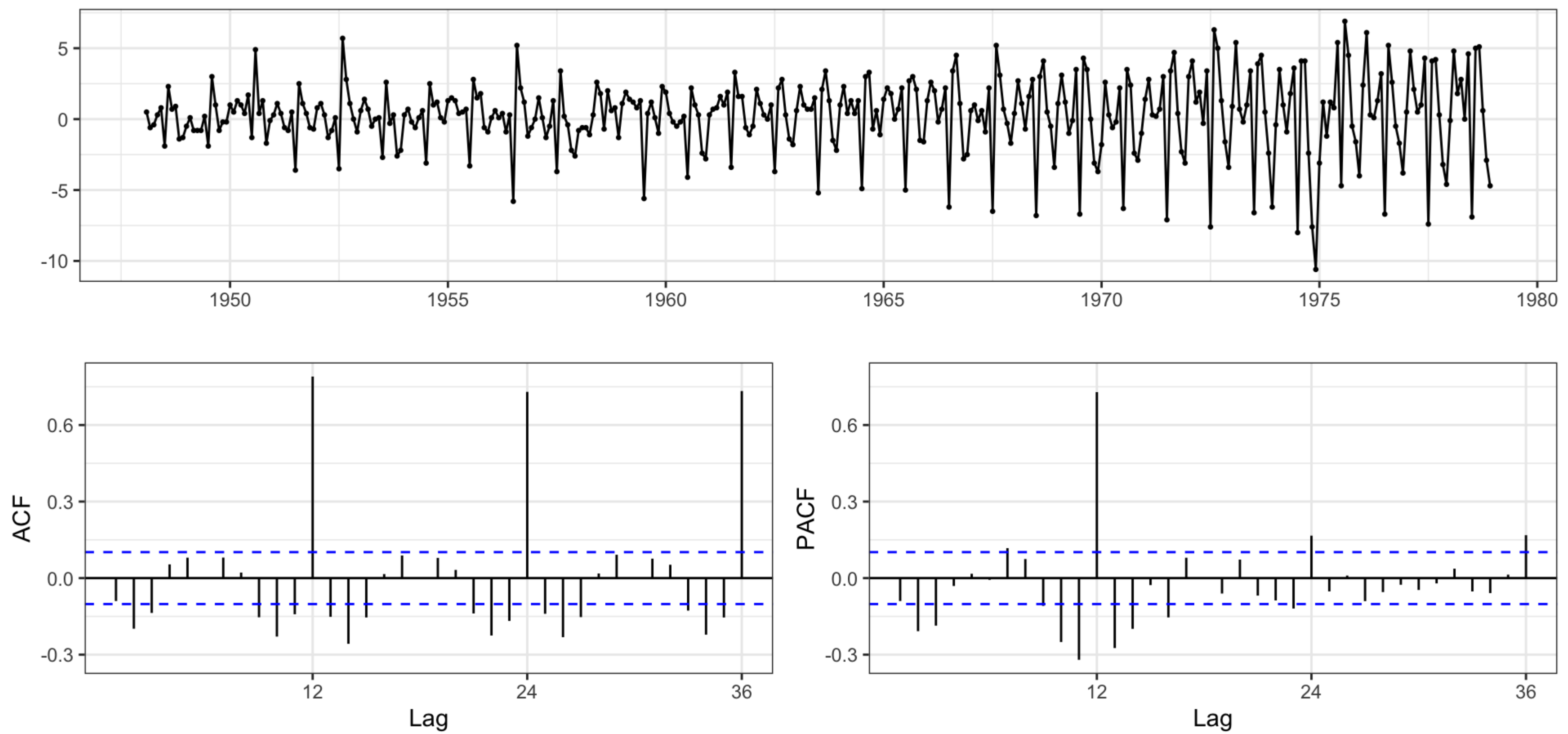
```r
data(prodn, package="astsa"); forecast::ggtsdisplay(prodn, points = FALS
```



Differencing

Based on the ACF it seems like standard differencing may be required

```r
forecast::ggtsdisplay(diff(prodn))
```



Differencing + Seasonal Differencing

Additional seasonal differencing also seems warranted

```
(fr_m1 = forecast::Arima(
  prodn, order = c(0,1,0),
  seasonal = list(order=c(0,0,0), period=12)
))
```

```
Series: prodn
ARIMA(0,1,0)

sigma^2 = 7.147:  log likelihood = -891.26
AIC=1784.51   AICc=1784.52   BIC=1788.43
```

```
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m1$fitted %>% unclass()
)
```

```
[1] 2.669854
```

```
(fr_m2 = forecast::Arima(
  prodn, order = c(0,1,0),
  seasonal = list(order=c(0,1,0), period=12)
))
```

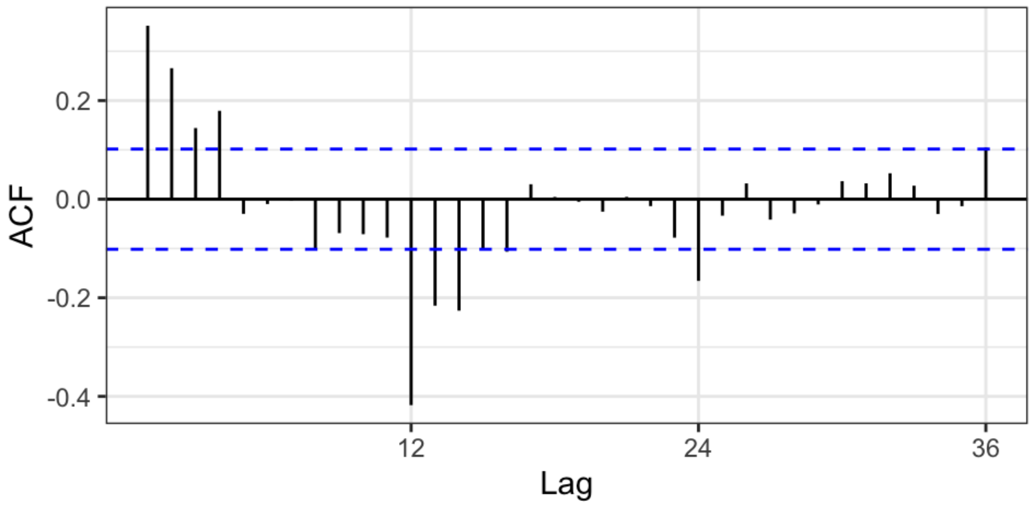
```
Series: prodn
ARIMA(0,1,0)(0,1,0)[12]

sigma^2 = 2.52:  log likelihood = -675.29
AIC=1352.58   AICc=1352.59   BIC=1356.46
```

```
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m2$fitted %>% unclass()
)
```

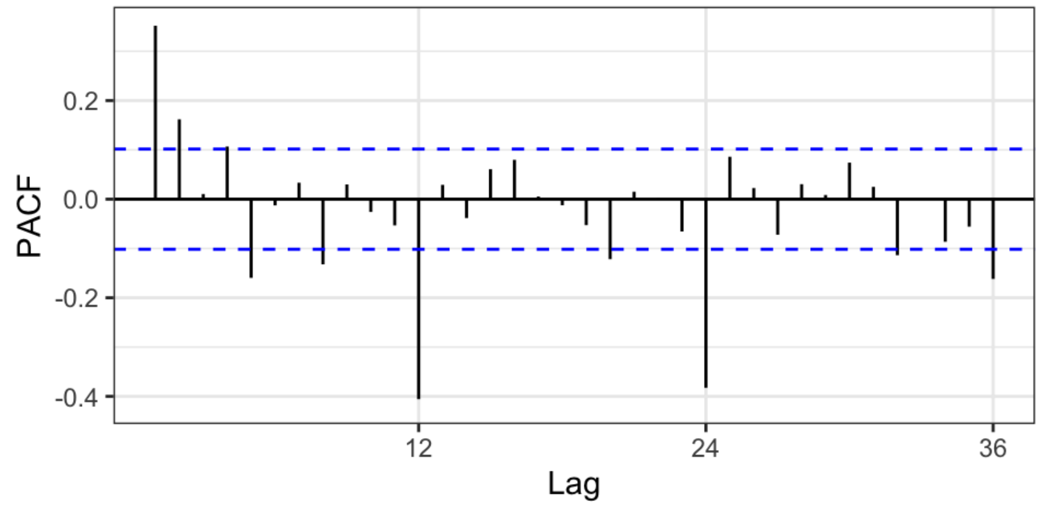
```
[1] 1.559426
```

Residuals

4
0
-4
-8
1950
1955
1960
1965
1970
1975
1980



ACF
0.2
0.0
-0.2
-0.4
12
24
36
Lag



PACF
0.2
0.0
-0.2
-0.4
12
24
36
Lag

Adding Seasonal MA

```r
(fr_m3.1 = forecast::Arima(
  prodn, order = c(0,1,0),
  seasonal = list(order=c(0,1,1), period=12)
))
```

```
Series: prodn
ARIMA(0,1,0)(0,1,1)[12]

Coefficients:
         sma1
      -0.7151
s.e.   0.0317

sigma^2 = 1.616:  log likelihood = -599.29
AIC=1202.57   AICc=1202.61   BIC=1210.34
```

```r
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m3.1$fitted %>% unclass()
)
```

```
[1] 1.246885
```

```r
(fr_m3.2 = forecast::Arima(
  prodn, order = c(0,1,0),
  seasonal = list(order=c(0,1,2), period=12)
))
```

```
Series: prodn
ARIMA(0,1,0)(0,1,2)[12]

Coefficients:
         sma1    sma2
      -0.7624  0.0520
s.e.   0.0689  0.0666

sigma^2 = 1.615:  log likelihood = -598.98
AIC=1203.96   AICc=1204.02   BIC=1215.61
```

```r
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m3.2$fitted %>% unclass()
)
```

```
[1] 1.245104
```

Adding Seasonal MA (cont.)

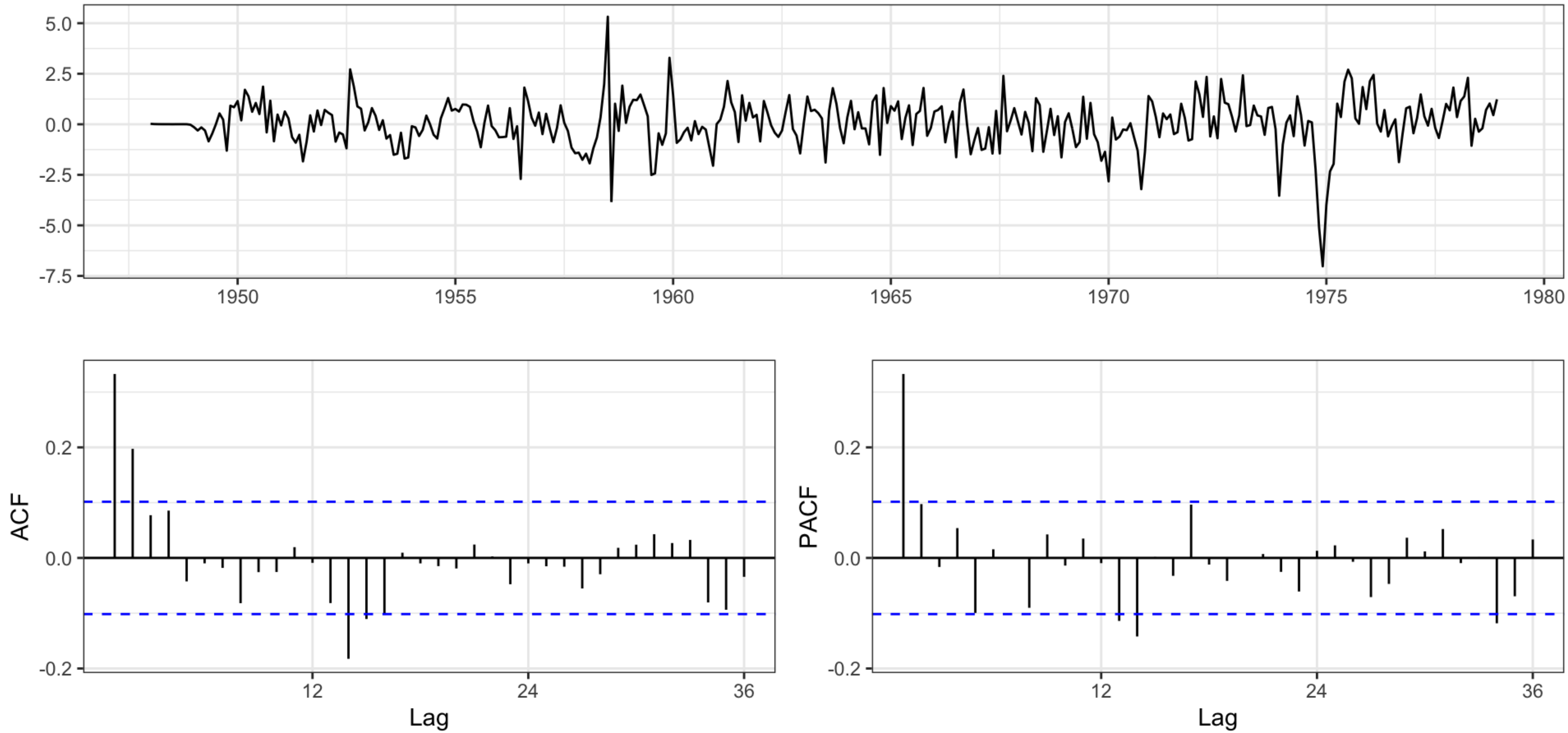
```
(fr_m3.3 = forecast::Arima(
  prodn, order = c(0,1,0),
  seasonal = list(order=c(0,1,3), period=12)
))
```

```
Series: prodn
ARIMA(0,1,0)(0,1,3)[12]

Coefficients:
         sma1     sma2    sma3
      -0.7853  -0.1205  0.2624
s.e.   0.0529   0.0644  0.0529

sigma^2 = 1.506:  log likelihood = -587.58
AIC=1183.15   AICc=1183.27   BIC=1198.69
```

```
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m3.3$fitted %>% unclass()
)
```

```
[1] 1.200592
```

Residuals - Model 3.3



Adding AR

```r
(fr_m4.1 = forecast::Arima(
  prodn, order = c(1,1,0),
  seasonal = list(order=c(0,1,3)
))
```

```
Series: prodn
ARIMA(1,1,0)(0,1,3)[12]

Coefficients:
         ar1     sma1     sma2
sma3
      0.3393  -0.7619  -0.1222
0.2756
s.e.  0.0500   0.0527   0.0646
0.0525

sigma^2 = 1.341:  log likelihood =
```

```r
(fr_m4.2 = forecast::Arima(
  prodn, order = c(2,1,0),
  seasonal = list(order=c(0,1,3)
))
```

```
Series: prodn
ARIMA(2,1,0)(0,1,3)[12]

Coefficients:
         ar1     ar2     sma1
sma2    sma3
      0.3038  0.1077  -0.7393
-0.1445  0.2815
s.e.  0.0526  0.0538   0.0539
0.0653  0.0526

sigma^2 = 1.331:  log likelihood =
```

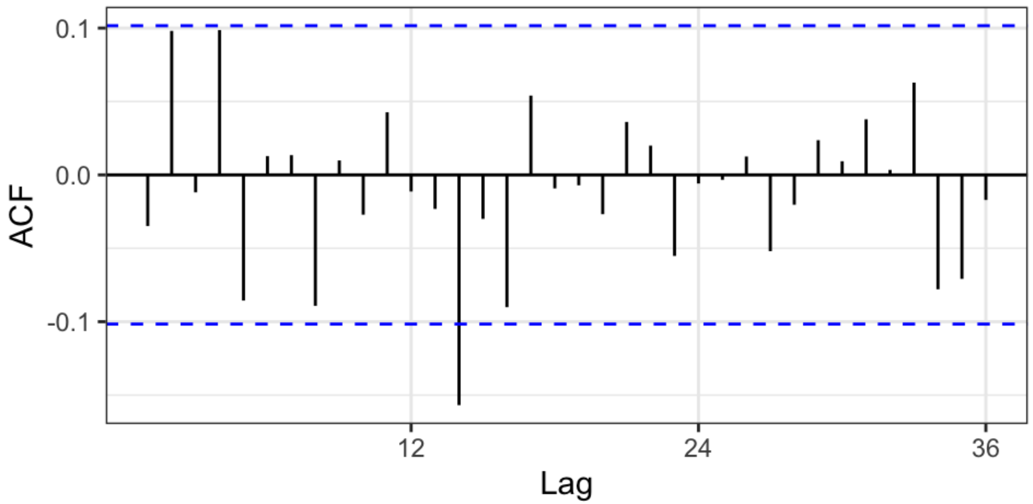
```r
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m4.1$fitted %>% unclass()
)
```

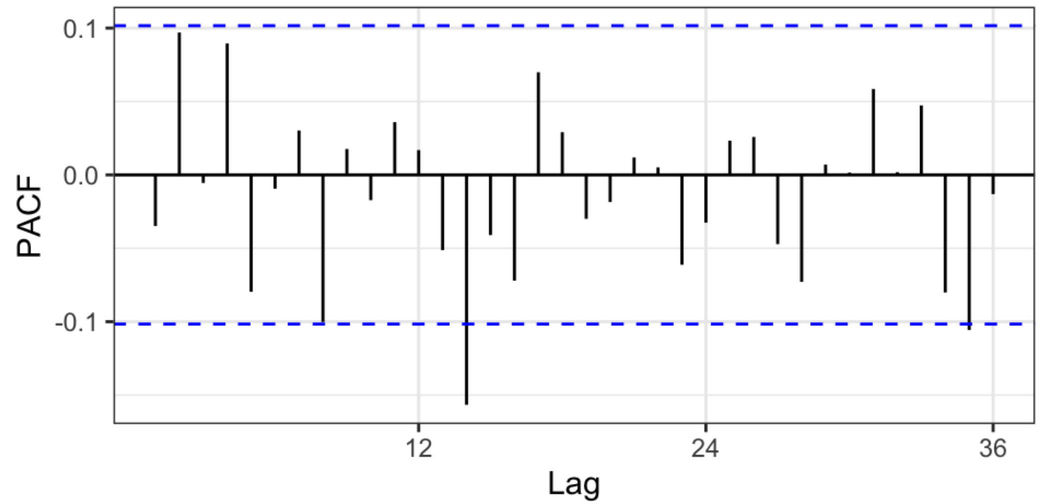
```
[1] 1.131115
```

```r
yardstick::rmse_vec(
  prodn %>% unclass(),
  fr_m4.2$fitted %>% unclass()
)
```

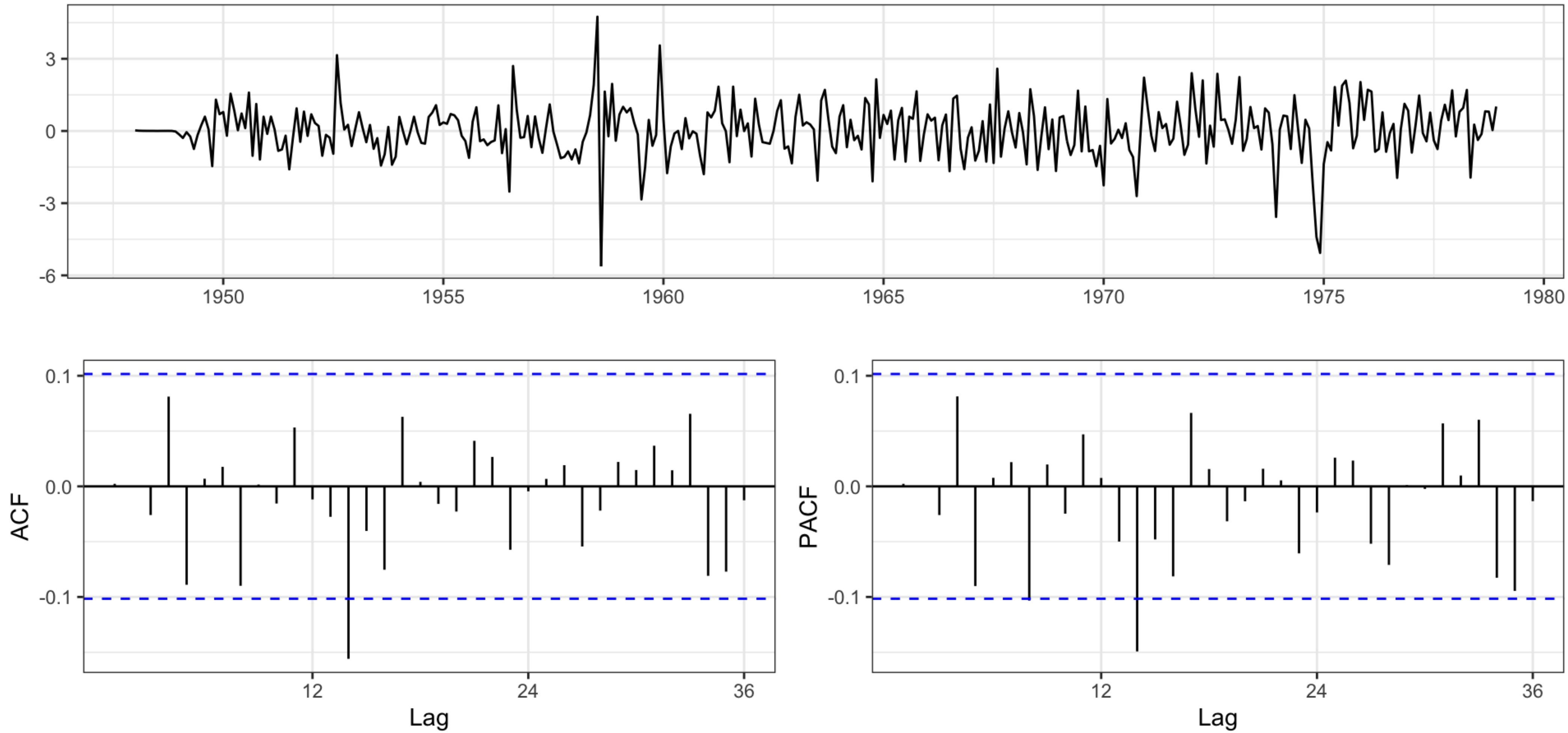
```
[1] 1.125322
```

Residuals - Model 4.1

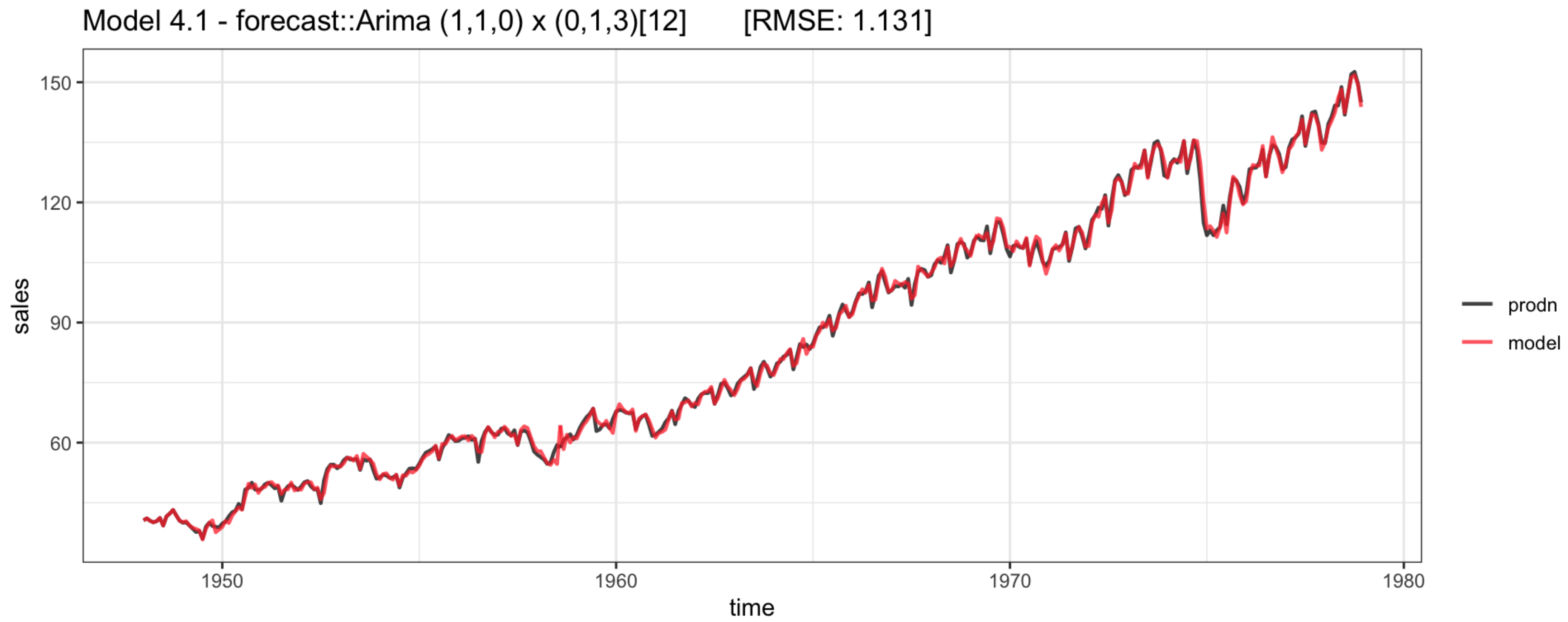




Residuals - Model 4.2



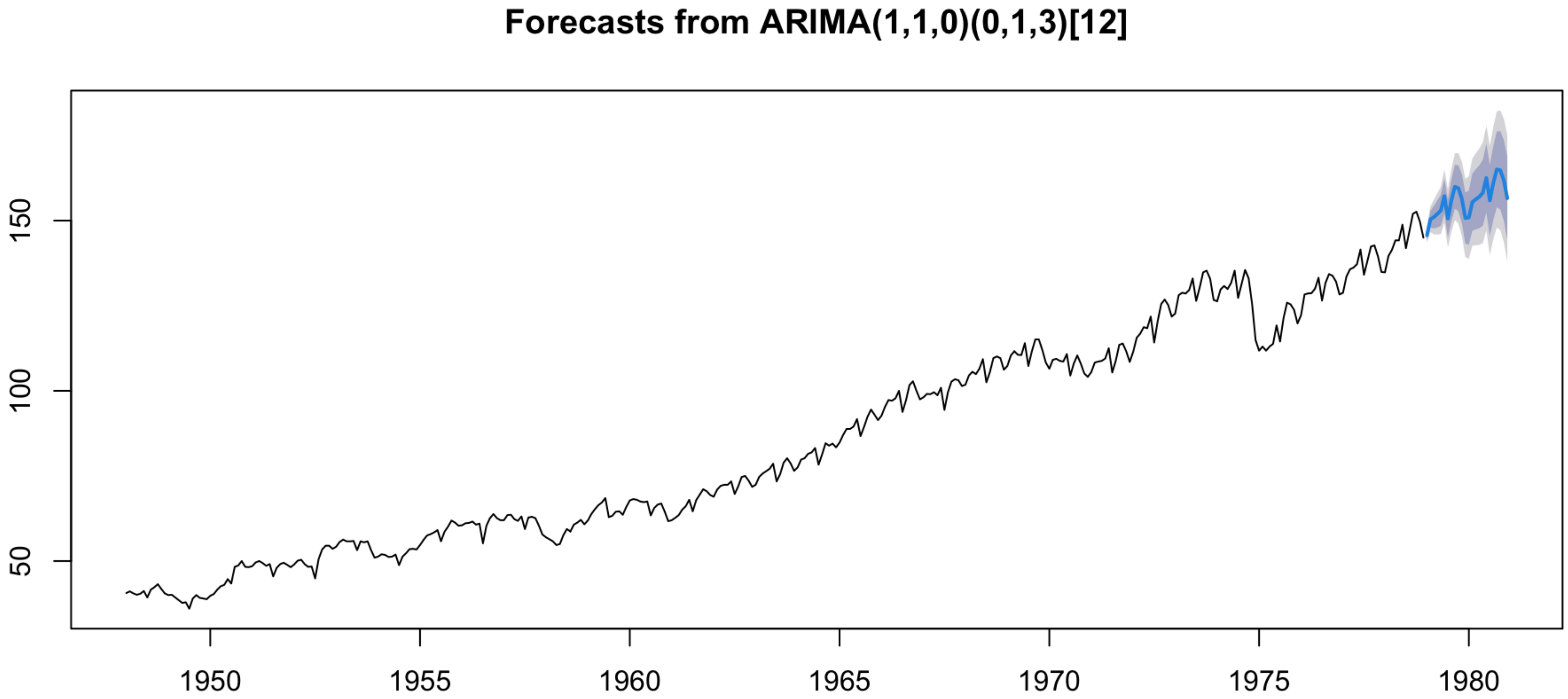
Model Fit



Model 4.1 - forecast::Arima (1,1,0) x (0,1,3)[12]   [RMSE: 1.131]
150
120
90
60
sales
1950
1960
1970
1980
time
prodn
model

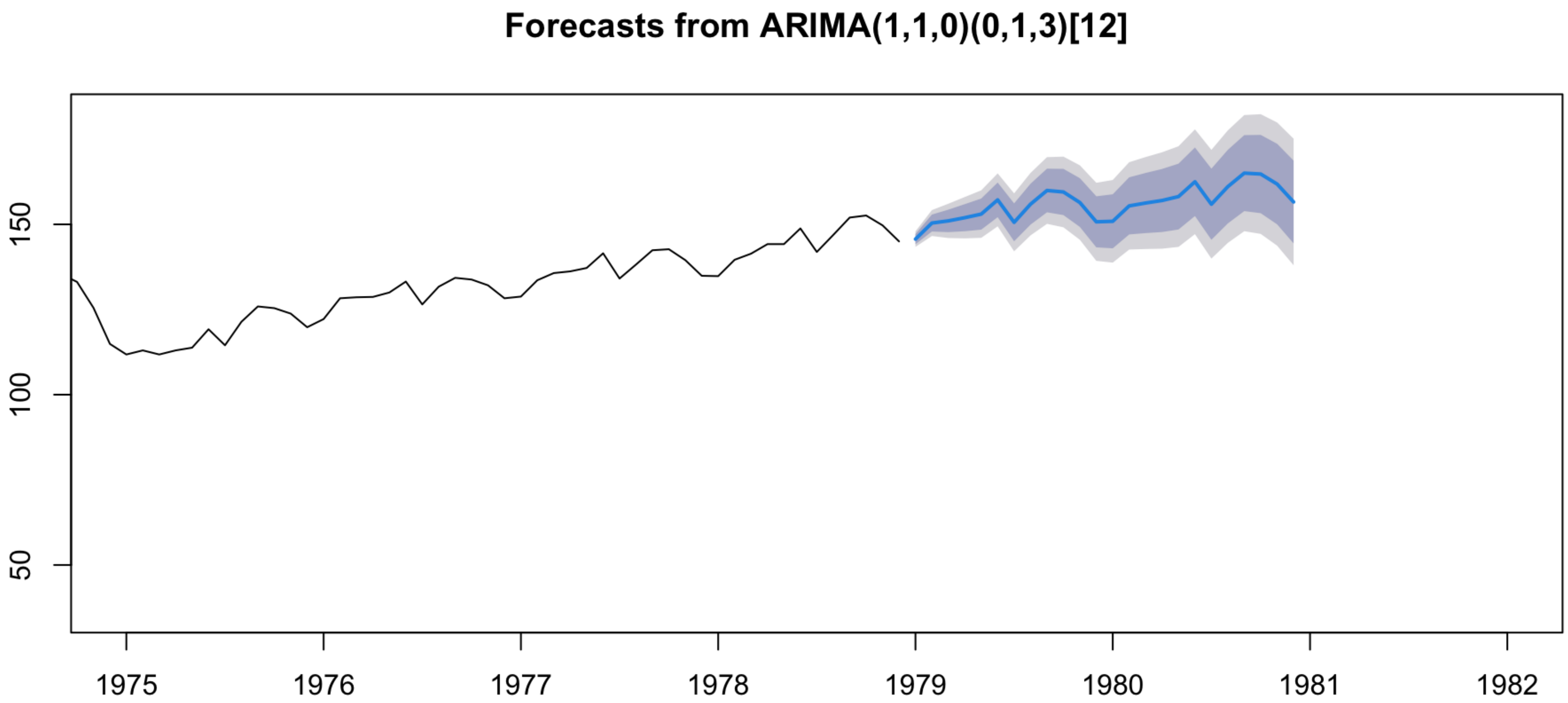
Model Forecast

```
forecast::forecast(fr_m4.1) %>% plot()
```



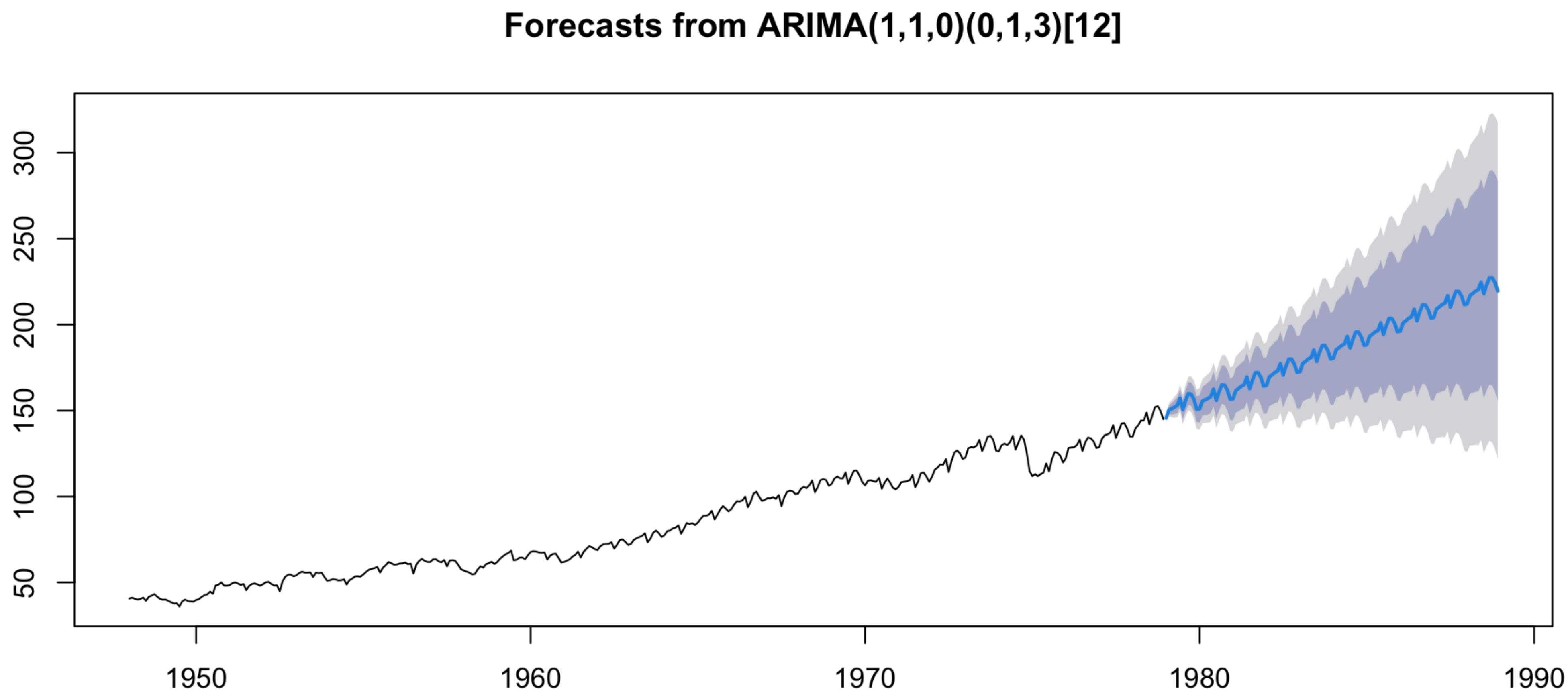
Model Forecast (cont.)

```
forecast::forecast(fr_m4.1) %>% plot(xlim=c(1975,1982))
```



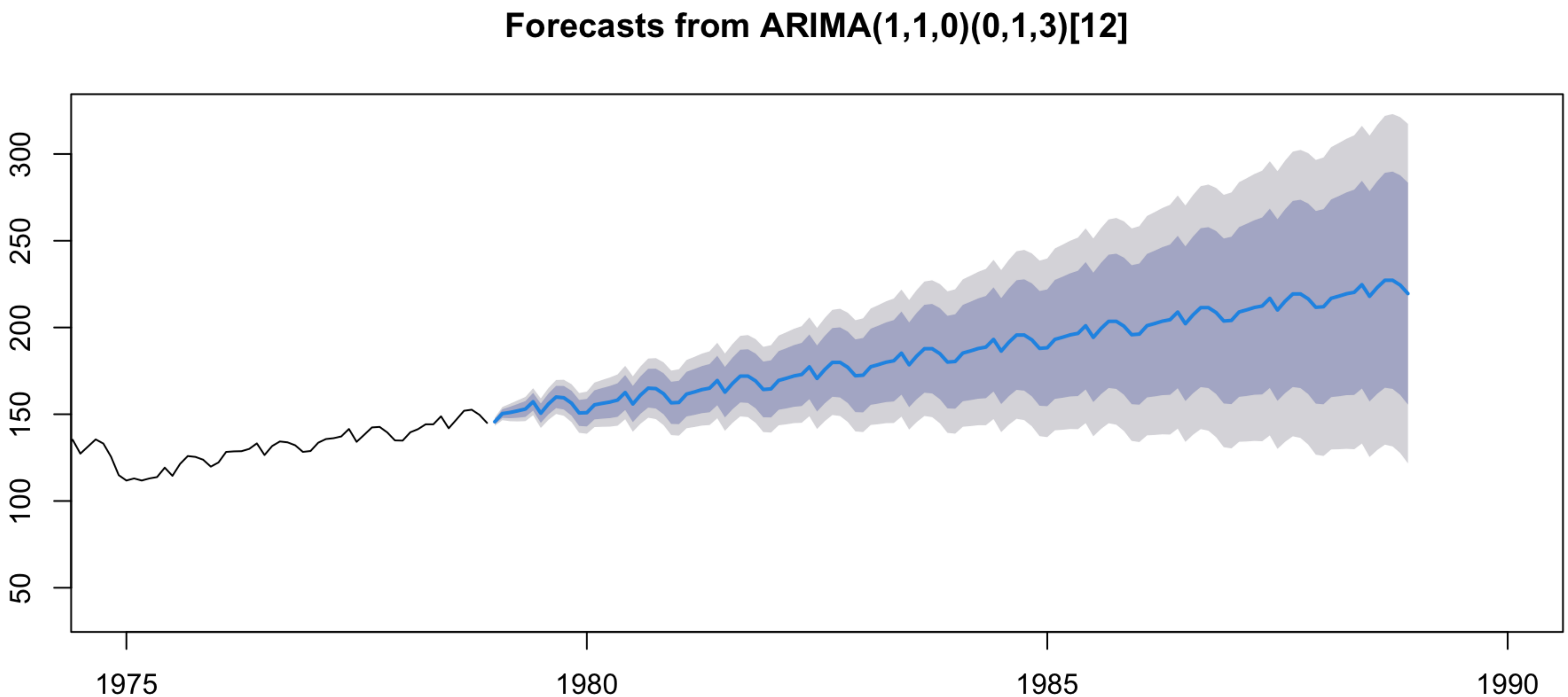
Model Forecast (cont.)

```r
forecast::forecast(fr_m4.1, 120) %>% plot()
```

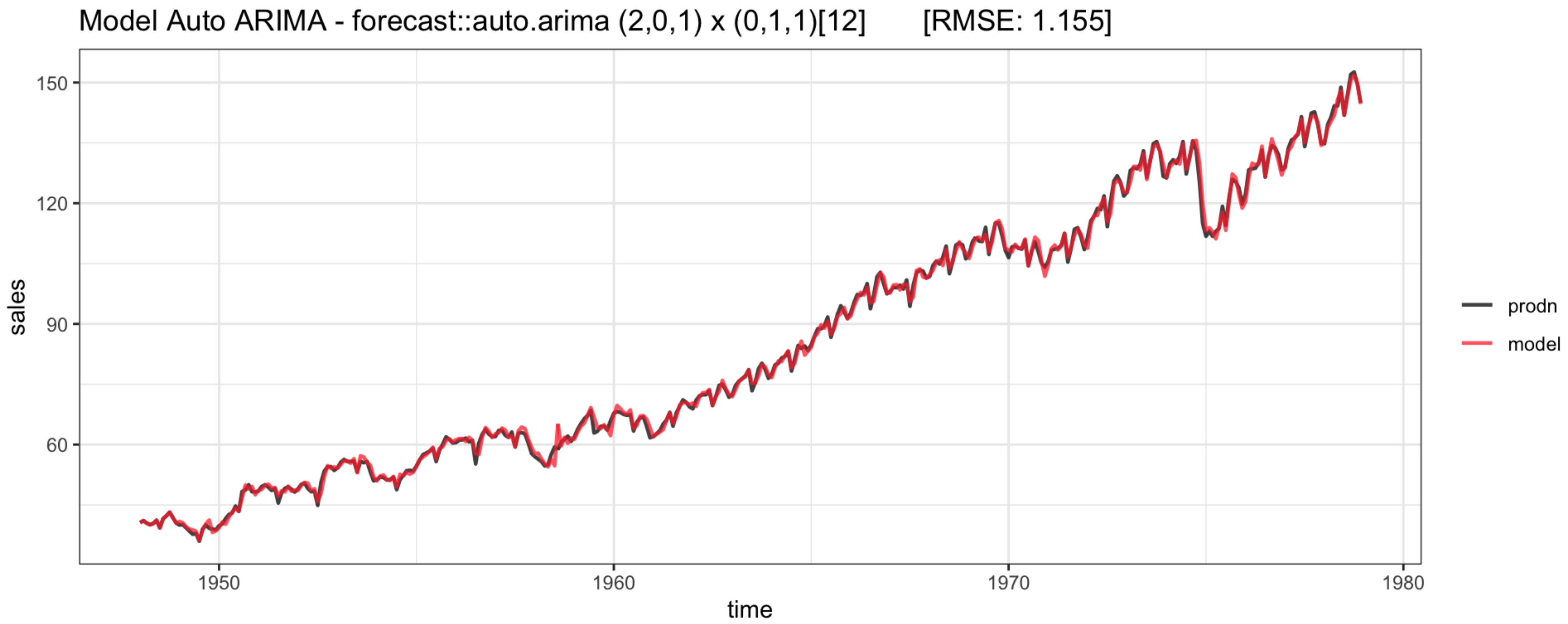


Model Forecast (cont.)

```r
forecast::forecast(fr_m4.1, 120) %>% plot(xlim=c(1975,1990))
```



Auto ARIMA - Model Fit



Exercise - Cortecosteroid Drug Sales

Monthly cortecosteroid drug sales in Australia from 1992 to 2008.

```r
data(h02, package="fpp")
forecast::ggtsdisplay(h02, points=FALSE)
```