

tidyverts & prophet

Lecture 12

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Tidy time series

ts objects

In base R, time series are usually encoded using the `ts` S3 class,

```
1 co2
```

| | Jan | Feb | Mar | Apr | May | Jun |
|------|--------|--------|--------|--------|--------|--------|
| 1959 | 315.42 | 316.31 | 316.50 | 317.56 | 318.13 | 318.00 |
| 1960 | 316.27 | 316.81 | 317.42 | 318.87 | 319.87 | 319.43 |
| 1961 | 316.73 | 317.54 | 318.38 | 319.31 | 320.42 | 319.61 |
| 1962 | 317.78 | 318.40 | 319.53 | 320.42 | 320.85 | 320.45 |
| 1963 | 318.58 | 318.92 | 319.70 | 321.22 | 322.08 | 321.31 |
| 1964 | 319.41 | 320.07 | 320.74 | 321.40 | 322.06 | 321.73 |
| 1965 | 319.27 | 320.28 | 320.73 | 321.97 | 322.00 | 321.71 |
| 1966 | 320.46 | 321.43 | 322.23 | 323.54 | 323.91 | 323.59 |
| 1967 | 322.17 | 322.34 | 322.88 | 324.25 | 324.83 | 323.93 |
| 1968 | 322.40 | 322.99 | 323.73 | 324.86 | 325.40 | 325.20 |
| 1969 | 323.83 | 324.26 | 325.47 | 326.50 | 327.21 | 326.54 |
| 1970 | 324.89 | 325.82 | 326.77 | 327.97 | 327.91 | 327.50 |
| 1971 | 326.01 | 326.51 | 327.01 | 327.62 | 328.76 | 328.40 |
| 1972 | 326.60 | 327.47 | 327.58 | 329.56 | 329.90 | 328.92 |
| 1973 | 328.37 | 329.40 | 330.14 | 331.33 | 332.31 | 331.90 |

```
1 typeof(co2)
```

```
[1] "double"
```

```
1 class(co2)
```

```
[1] "ts"
```

```
1 attributes(co2)
```

```
$tsp
```

```
[1] 1959.000 1997.917 12.000
```

```
$class
```

```
[1] "ts"
```

tidyverts

This is an effort headed by Rob Hyndman (of forecast fame) and others to provide a consistent tidy data based framework for working with time series data and models.

Core packages:

- `tsibble` - temporal data frames and related tools
- `fable` - tidy forecasting (modelling)
- `feasts` - feature extraction and statistics
- `tsibbldata` - sample tsibble data sets

tsibble

A tsibble is a tibble with additional infrastructure for encoding temporal data - specifically a tsibble is a tidy data frame with an *index* and *key* where

- the *index* is the variable that describes the inherent ordering of the data (from past to present)
- and the *key* is one or more variables that define the unit of observation over time
- each observation should be uniquely identified by the *index* and *key*

global_economy

```
1 tsibbledata::global_economy
```

```
# A tsibble: 15,150 x 9 [1Y]
# Key:      Country [263]
  Country Code    Year     GDP Growth     CPI Imports
  <fct>   <fct> <dbl>   <dbl> <dbl> <dbl>   <dbl>
1 Afghan... AFG    1960 5.38e8      NA     NA    7.02
2 Afghan... AFG    1961 5.49e8      NA     NA    8.10
3 Afghan... AFG    1962 5.47e8      NA     NA    9.35
4 Afghan... AFG    1963 7.51e8      NA     NA   16.9
5 Afghan... AFG    1964 8.00e8      NA     NA   18.1
6 Afghan... AFG    1965 1.01e9      NA     NA   21.4
7 Afghan... AFG    1966 1.40e9      NA     NA   18.6
8 Afghan... AFG    1967 1.67e9      NA     NA   14.2
```

vic_elec

```
1 tsibbledata::vic_elec
```

```
# A tsibble: 52,608 x 5 [30m]
#   <Australia/Melbourne>
# 
#   Time           Demand Tempera...¹ Date
#   <dttm>        <dbl>    <dbl> <date>
# 1 2012-01-01 00:00:00  4383.    21.4 2012-01-01
# 2 2012-01-01 00:30:00  4263.    21.0 2012-01-01
# 3 2012-01-01 01:00:00  4049.    20.7 2012-01-01
# 4 2012-01-01 01:30:00  3878.    20.6 2012-01-01
# 5 2012-01-01 02:00:00  4036.    20.4 2012-01-01
# 6 2012-01-01 02:30:00  3866.    20.2 2012-01-01
# 7 2012-01-01 03:00:00  3694.    20.1 2012-01-01
# 8 2012-01-01 03:30:00  3562.    19.6 2012-01-01
```

aus_retail

```
1 tsibbledata::aus_retail

# A tsibble: 64,532 x 5 [1M]
# Key:      State, Industry [152]
#       State     Indus...¹ Serie...² Month Turno...³
#       <chr>     <chr>    <chr>    <mth>    <dbl>
# 1 Australian Cafes,... A33498... 1982 Apr      4.4
# 2 Australian Cafes,... A33498... 1982 May      3.4
# 3 Australian Cafes,... A33498... 1982 Jun      3.6
# 4 Australian Cafes,... A33498... 1982 Jul      4
# 5 Australian Cafes,... A33498... 1982 Aug      3.6
# 6 Australian Cafes,... A33498... 1982 Sep      4.2
# 7 Australian Cafes,... A33498... 1982 Oct      4.8
# 8 Australian Cafes,... A33498... 1982 Nov      5.4
```

as_tsibble()

Existing ts objects can be converted to a tsibble easily,

```
1 tsibble::as_tsibble(co2)
```

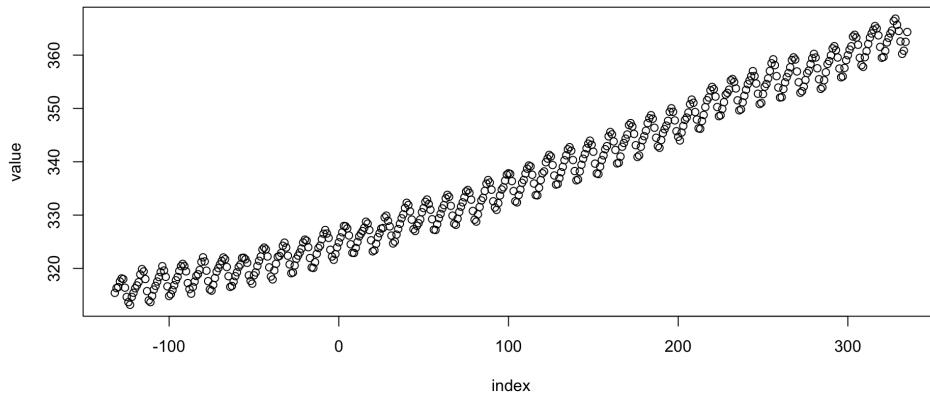
```
# A tsibble: 468 x 2 [1M]
```

| | index | value |
|---|----------|-------|
| | <mth> | <dbl> |
| 1 | 1959 Jan | 315. |
| 2 | 1959 Feb | 316. |
| 3 | 1959 Mar | 316. |
| 4 | 1959 Apr | 318. |
| 5 | 1959 May | 318. |
| 6 | 1959 Jun | 318 |
| 7 | 1959 Jul | 316. |
| 8 | 1959 Aug | 315. |
| 9 | 1959 Sep | 314. |

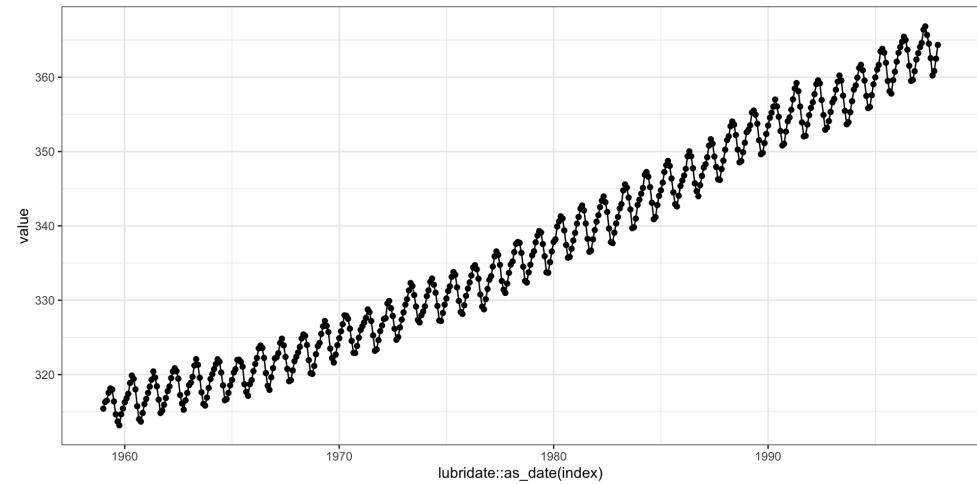
plotting tsibbles

As the tsibble is basically just a tibble which is basically just a data frame both base and ggplot plotting methods will work.

```
1 tsibble::as_tsibble(co2) %>%  
2   plot()
```

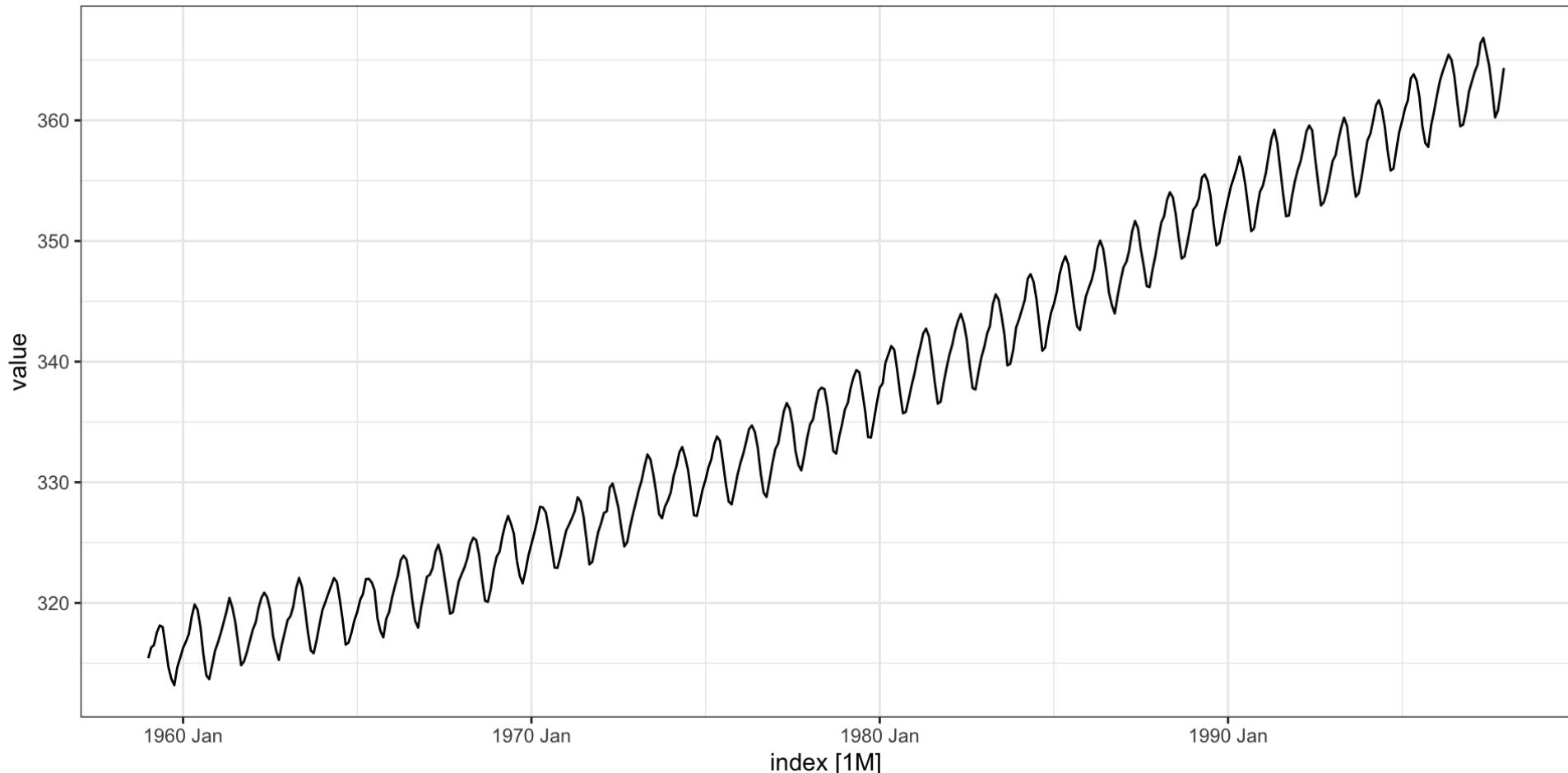


```
1 tsibble::as_tsibble(co2) %>%  
2   ggplot(  
3     aes(x=lubridate::as_date(index), y=value)  
4   ) +  
5     geom_point() +  
6     geom_line()
```



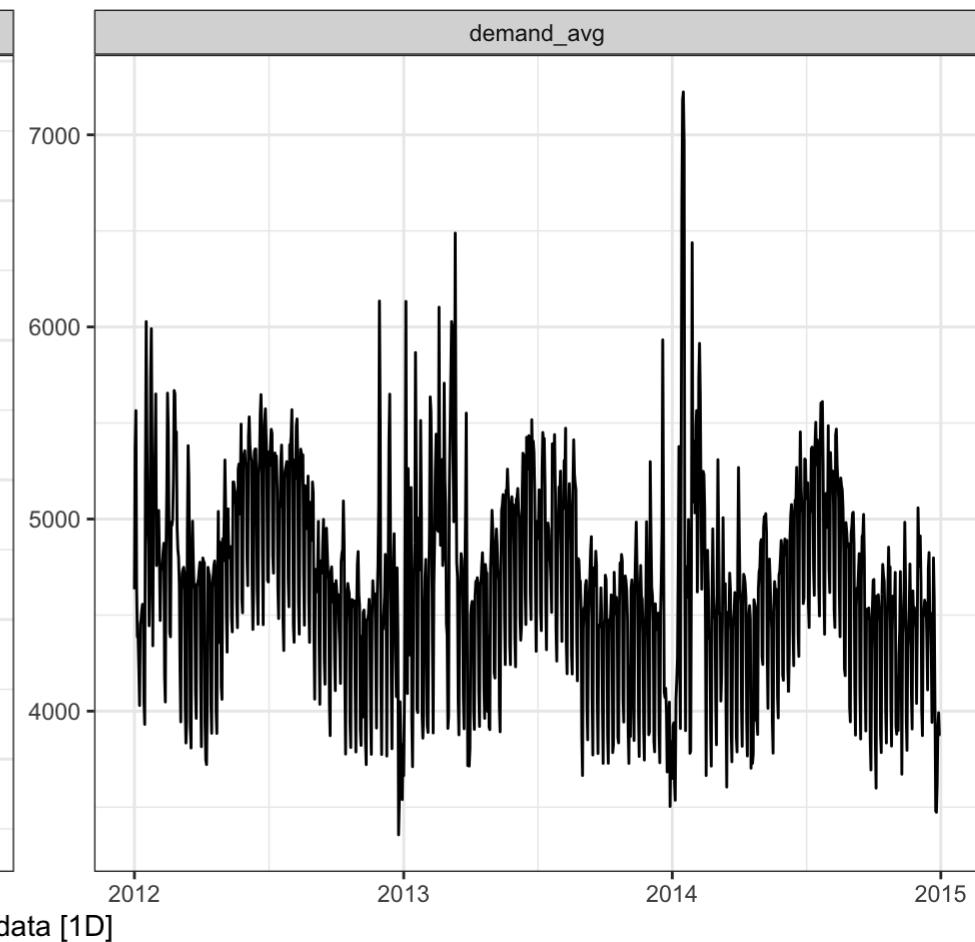
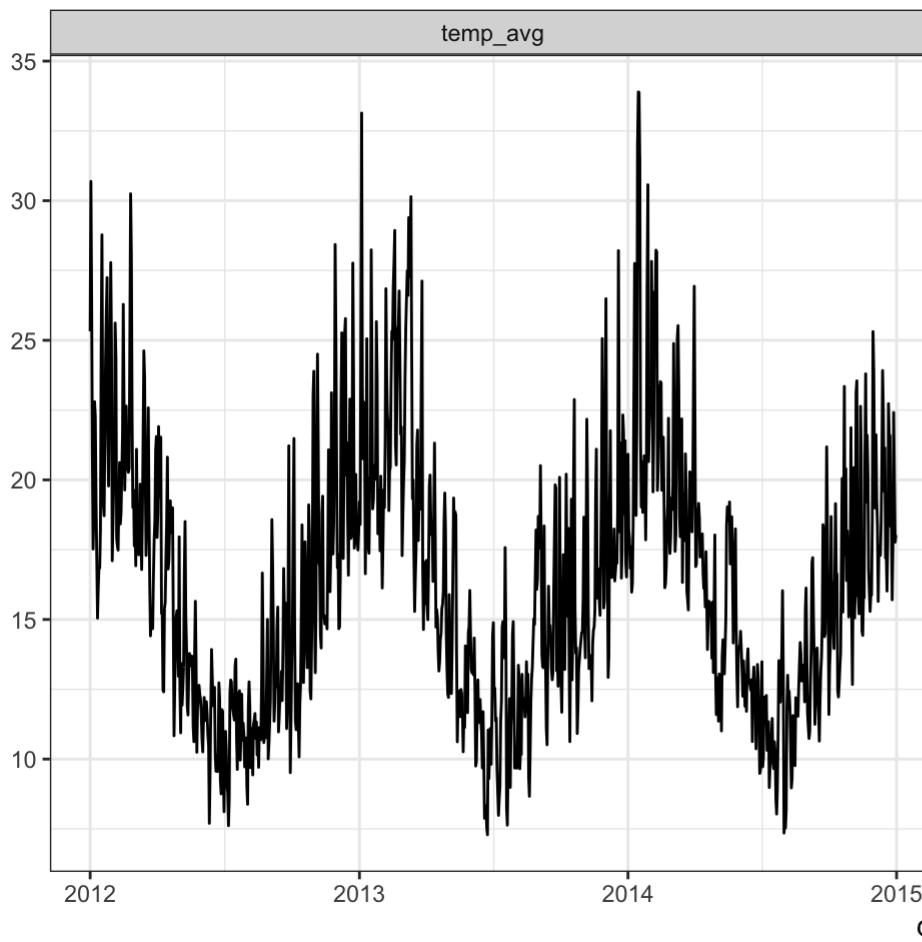
autoplot

```
1 library(fabletools) # needed to support autoplot  
2 tsibble::as_tsibble(co2) %>%  
3   autoplot(.vars = vars(value))
```



Multiple variables

```
1 tsibbledata::vic_elec %>%
2   tsibble::index_by(data = ~ lubridate::as_date(.)) %>%
3   summarize(
4     demand_avg = mean(Demand, na.rm=TRUE),
5     temp_avg = mean(Temperature, na.rm=TRUE)
6   ) %>%
7   autoplot(.vars = vars(temp_avg, demand_avg))
```

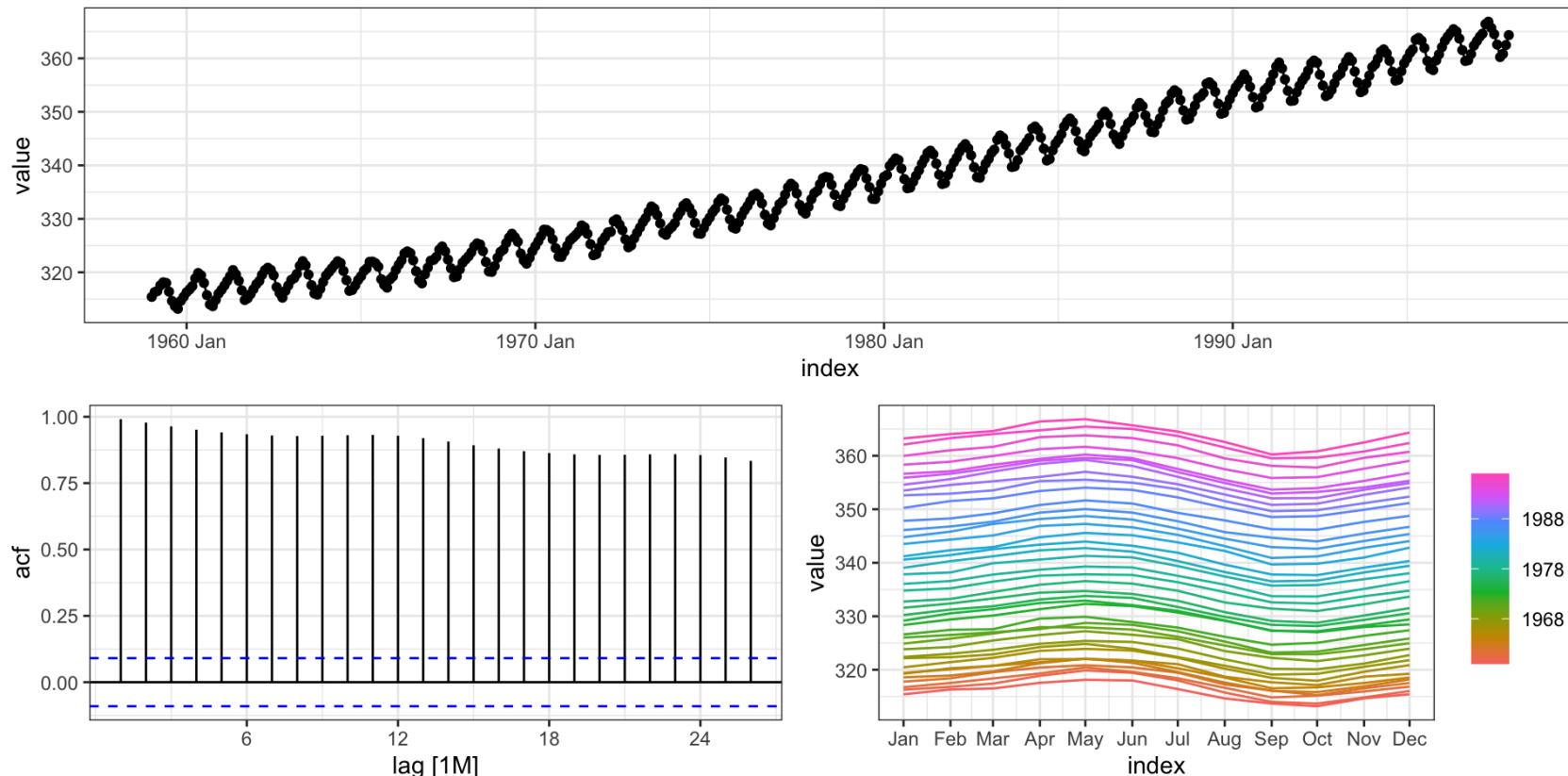


data [1D]

ggtsdisplay() replacement

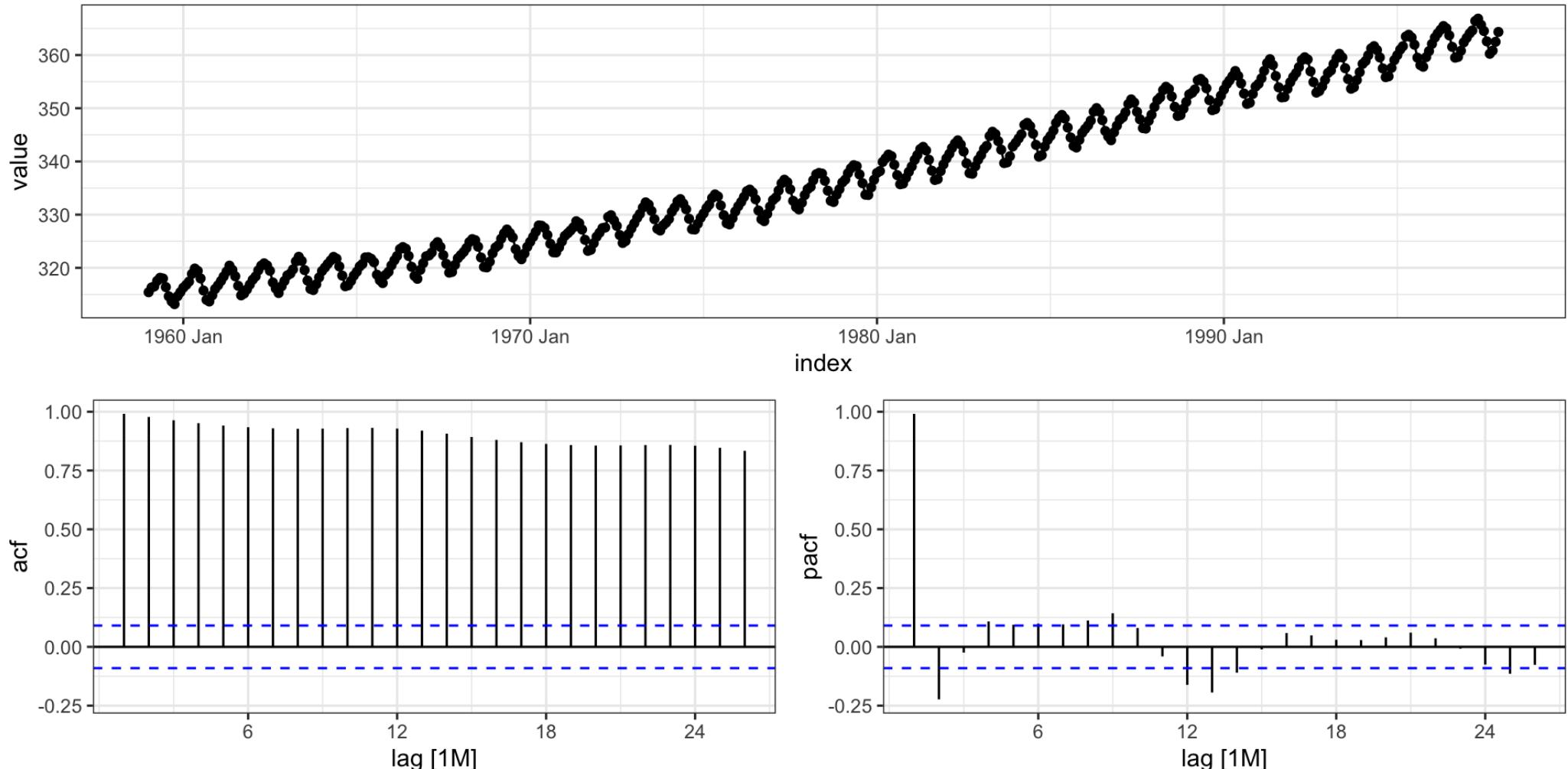
The equivalent to the `ggtsdisplay()` plot is provided by `feasts` with `gg_tsdisplay()`,

```
1 tsibble::as_tsibble(co2) %>%
 2   feasts::gg_tsdisplay(y = value)
```



ggtsdisplay() - pACF

```
1 tsibble::as_tsibble(co2) %>%
2   feasts::gg_tsdisplay(y = value, plot_type = "partial")
```



Modeling with fable

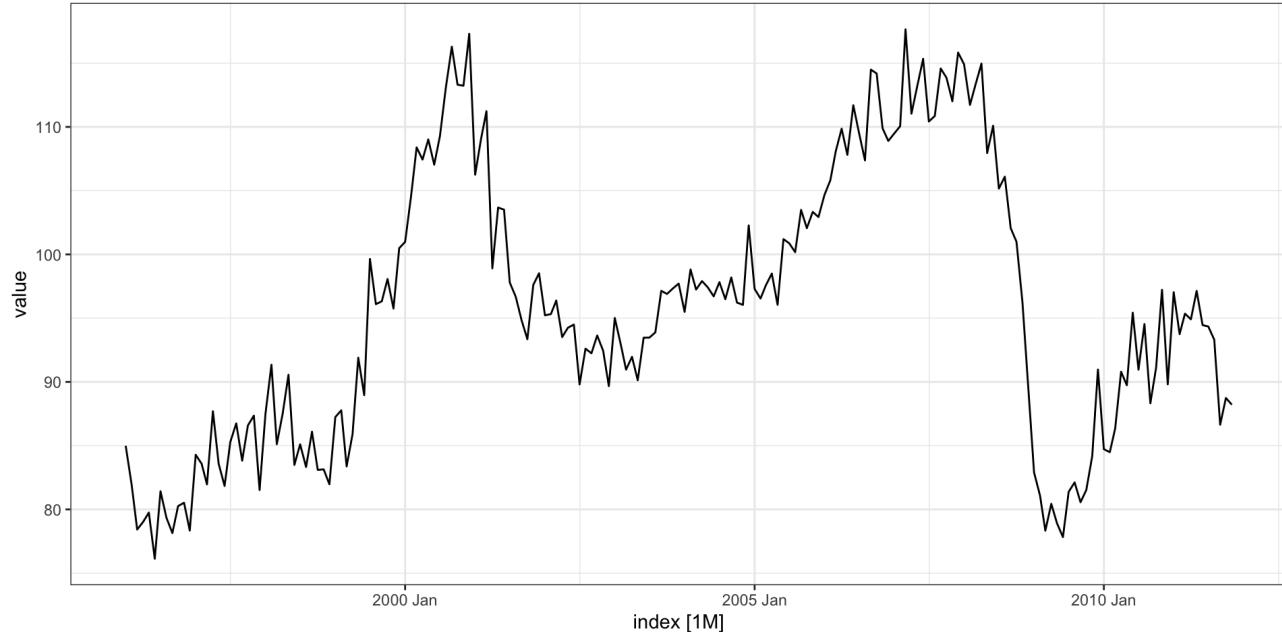
elec_sales

```
1 ( elec_sales = as_tsibble(  
2   elec_sales  
3 ) )
```

```
# A tsibble: 191 x 2 [1M]
```

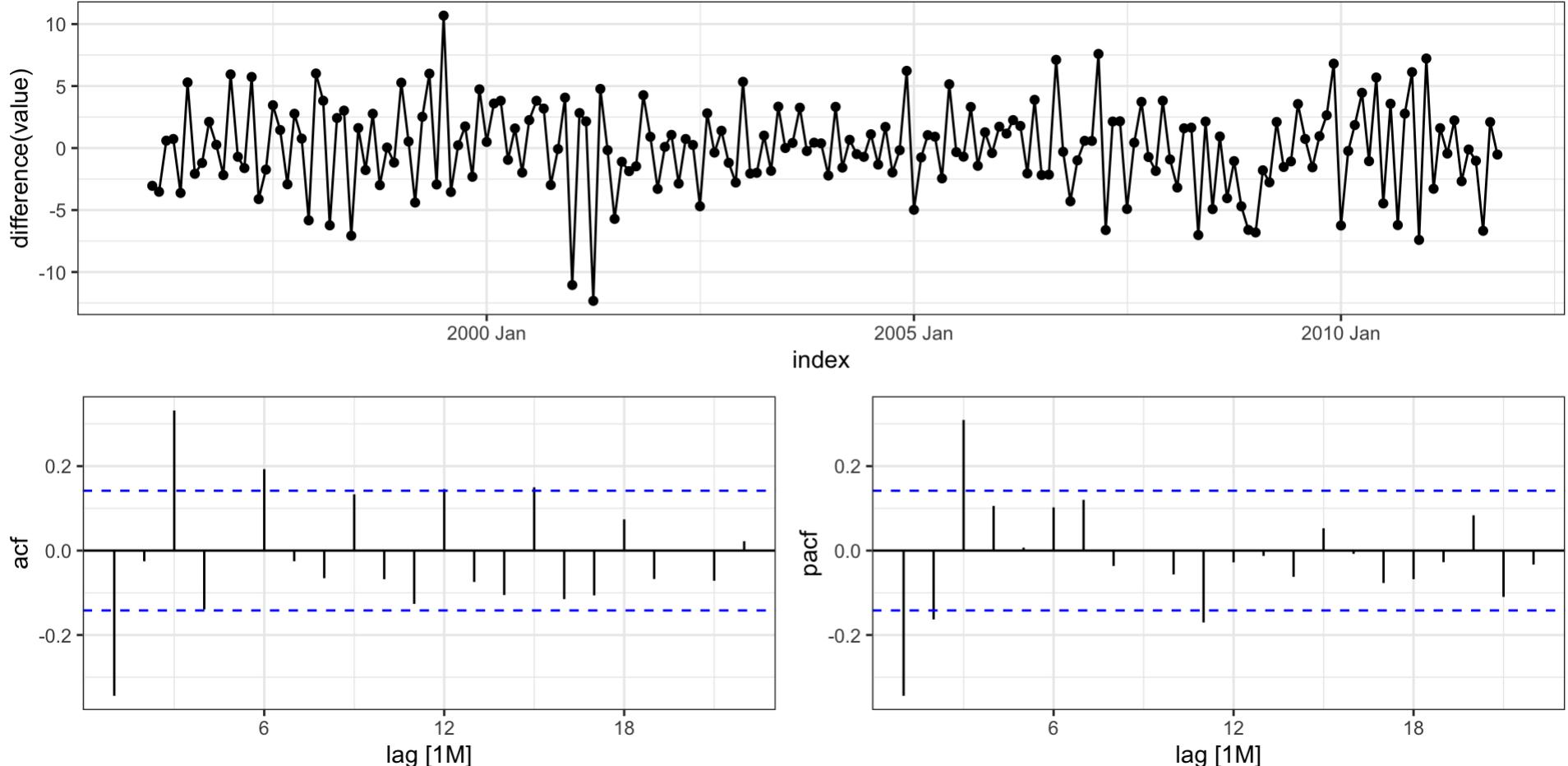
| | index | value |
|--------------------------|----------|-------|
| 1 | 1996 Jan | 85.0 |
| 2 | 1996 Feb | 81.9 |
| 3 | 1996 Mar | 78.4 |
| 4 | 1996 Apr | 79.0 |
| 5 | 1996 May | 79.7 |
| 6 | 1996 Jun | 76.1 |
| 7 | 1996 Jul | 81.4 |
| 8 | 1996 Aug | 79.3 |
| 9 | 1996 Sep | 78.1 |
| 10 | 1996 Oct | 80.3 |
| # ... with 181 more rows | | |

```
1 elec_sales %>%  
2   autoplot(value)
```



Differencing

```
1 elec_sales %>%
2   feasts::gg_tsdisplay(difference(value), plot_type="partial")
```



Modeling

To fit a model with fable we use the `model()` function along with a specific function for the model we are trying to fit (`ARIMA()` here).

As with the rest of tidyverts - `fable` using a tidy approach for modeling which means that the model results are stored in a tibble (called a `mable`).

```
1 library(fable)
2 ( m = elec_sales %>%
3   model(
4     ARIMA(value ~ pdq(3,1,0))
5   )
6 )
```

```
# A mable: 1 x 1
`ARIMA(value ~ pdq(3, 1, 0))` 
<model>
1 <ARIMA(3,1,0)>
```

Model summary

```
1 m %>%
2   report()
```

Series: value

Model: ARIMA(3,1,0)

Coefficients:

| | ar1 | ar2 | ar3 |
|------|---------|---------|--------|
| - | -0.3488 | -0.0386 | 0.3139 |
| s.e. | 0.0690 | 0.0736 | 0.0694 |

sigma^2 estimated as 9.853: log likelihood=-485.67

AIC=979.33 AICc=979.55 BIC=992.32

Model details (broom + yardstick)

::: {.small}

```
1 m %>% glance()

# A tibble: 1 × 8
  .model          sigma2  log_lik     AIC    AICc    BIC ar_roots
  <chr>           <dbl>   <dbl>   <dbl>   <dbl>   <dbl> <list>
ma_roots
<list>
1 ARIMA(value ~ pdq(3, 1, 0))    9.85    -486.   979.   980.   992. <cpl [3]>
<cpl [0]>
```

::: {.small}

```
1 m %>% accuracy()

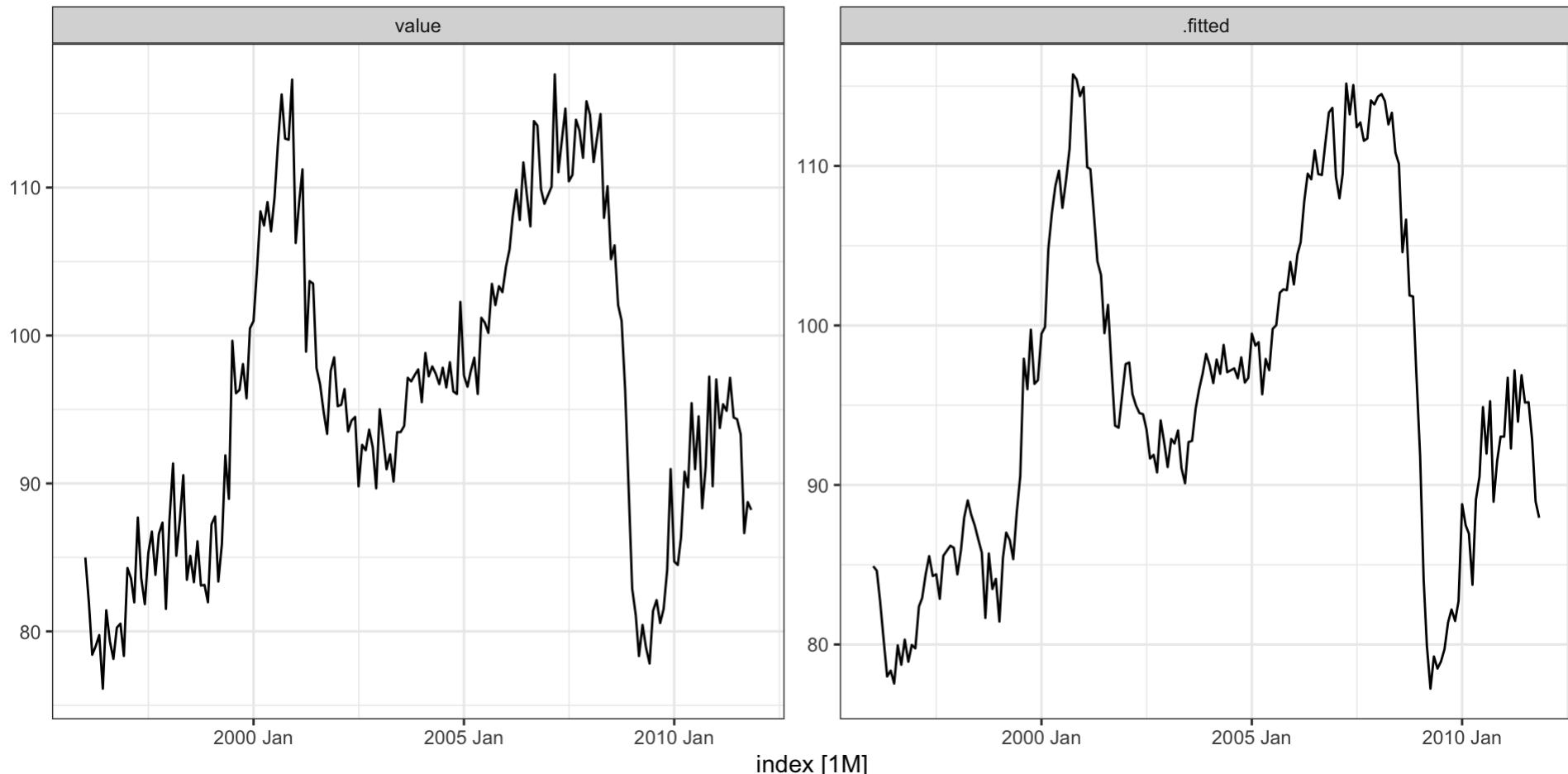
# A tibble: 1 × 10
  .model          .type      ME    RMSE    MAE    MPE    MAPE
  <chr>           <chr>    <dbl>  <dbl>  <dbl>  <dbl>  <dbl>
MASE  RMSSE      ACF1
<dbl> <dbl>      <dbl>
<dbl> <dbl>      <dbl>  <dbl>  <dbl>  <dbl>  <dbl>
```

```
1 ARIMA(value ~ pdq(3, 1, 0)) Training 0.0117 3.11 2.43 -0.0435 2.56  
0.296 0.281 -0.0346
```

```
1 m %>% augment()  
  
# A tsibble: 191 x 6 [1M]  
# Key:   .model [1]  
# ... with 181 more rows  
  
.model          index value .fitted .resid .innov  
<chr>           <mth> <dbl>  <dbl>  <dbl>  <dbl>  
1 ARIMA(value ~ pdq(3, 1, 0)) 1996 Jan  85.0   84.9  0.0850  0.0850  
2 ARIMA(value ~ pdq(3, 1, 0)) 1996 Feb  81.9   84.6 -2.68  -2.68  
3 ARIMA(value ~ pdq(3, 1, 0)) 1996 Mar  78.4   82.7 -4.28  -4.28  
4 ARIMA(value ~ pdq(3, 1, 0)) 1996 Apr  79.0   80.3 -1.25  -1.25  
5 ARIMA(value ~ pdq(3, 1, 0)) 1996 May  79.7   78.0  1.76   1.76  
6 ARIMA(value ~ pdq(3, 1, 0)) 1996 Jun  76.1   78.4 -2.24  -2.24  
7 ARIMA(value ~ pdq(3, 1, 0)) 1996 Jul  81.4   77.5  3.87   3.87  
8 ARIMA(value ~ pdq(3, 1, 0)) 1996 Aug  79.3   79.9 -0.599 -0.599  
9 ARIMA(value ~ pdq(3, 1, 0)) 1996 Sep  78.1   78.7 -0.588 -0.588  
10 ARIMA(value ~ pdq(3, 1, 0)) 1996 Oct  80.3   80.3 -0.0448 -0.0448  
# ... with 181 more rows
```

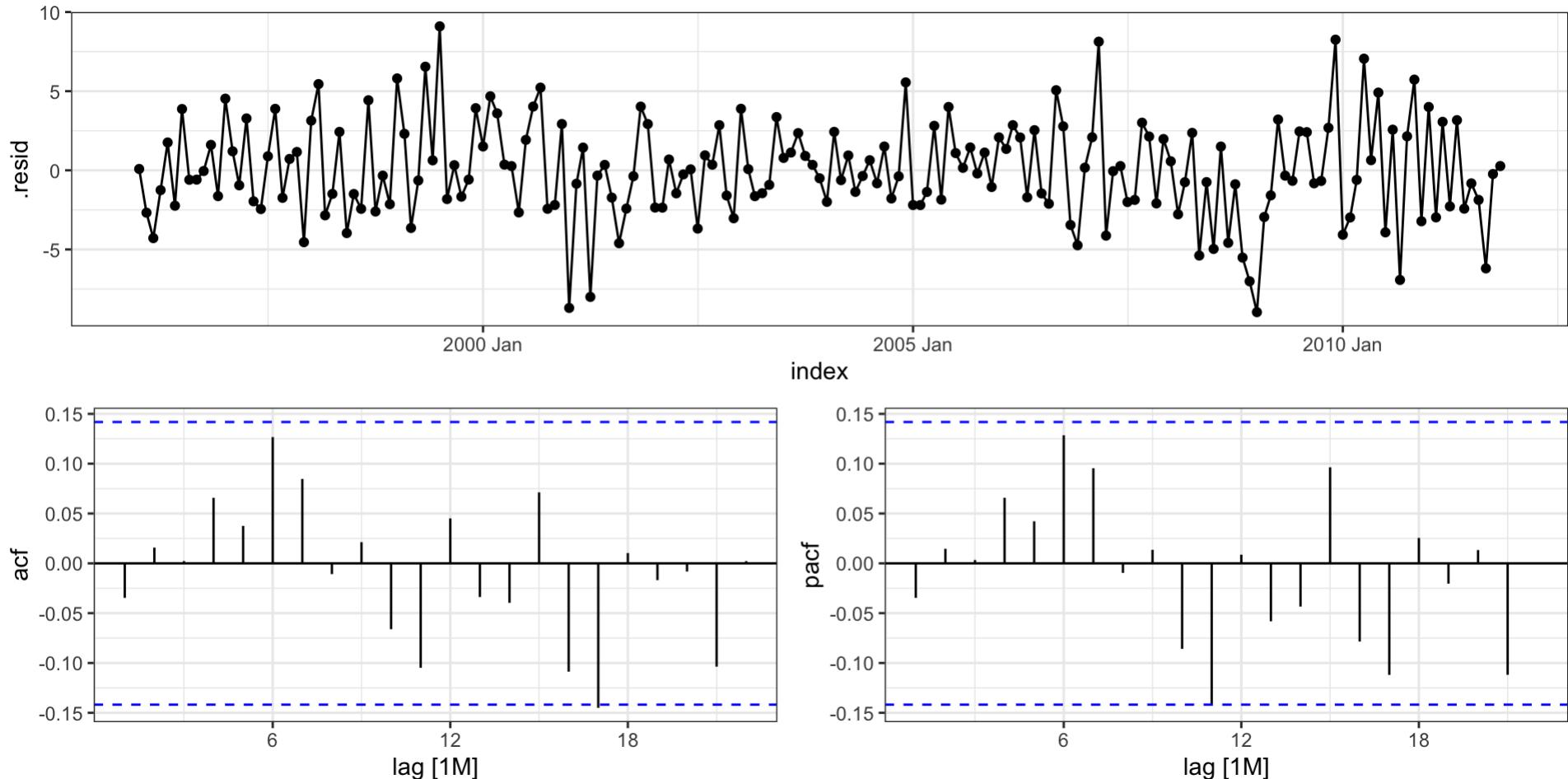
Observed vs predicted

```
1 m %>%
2   augment() %>%
3   autoplot(vars(value, .fitted))
```



Residuals

```
1 m %>%
2   augment() %>%
3   feasts::gg_tsdisplay(.resid, plot_type="partial")
```



Forecasting

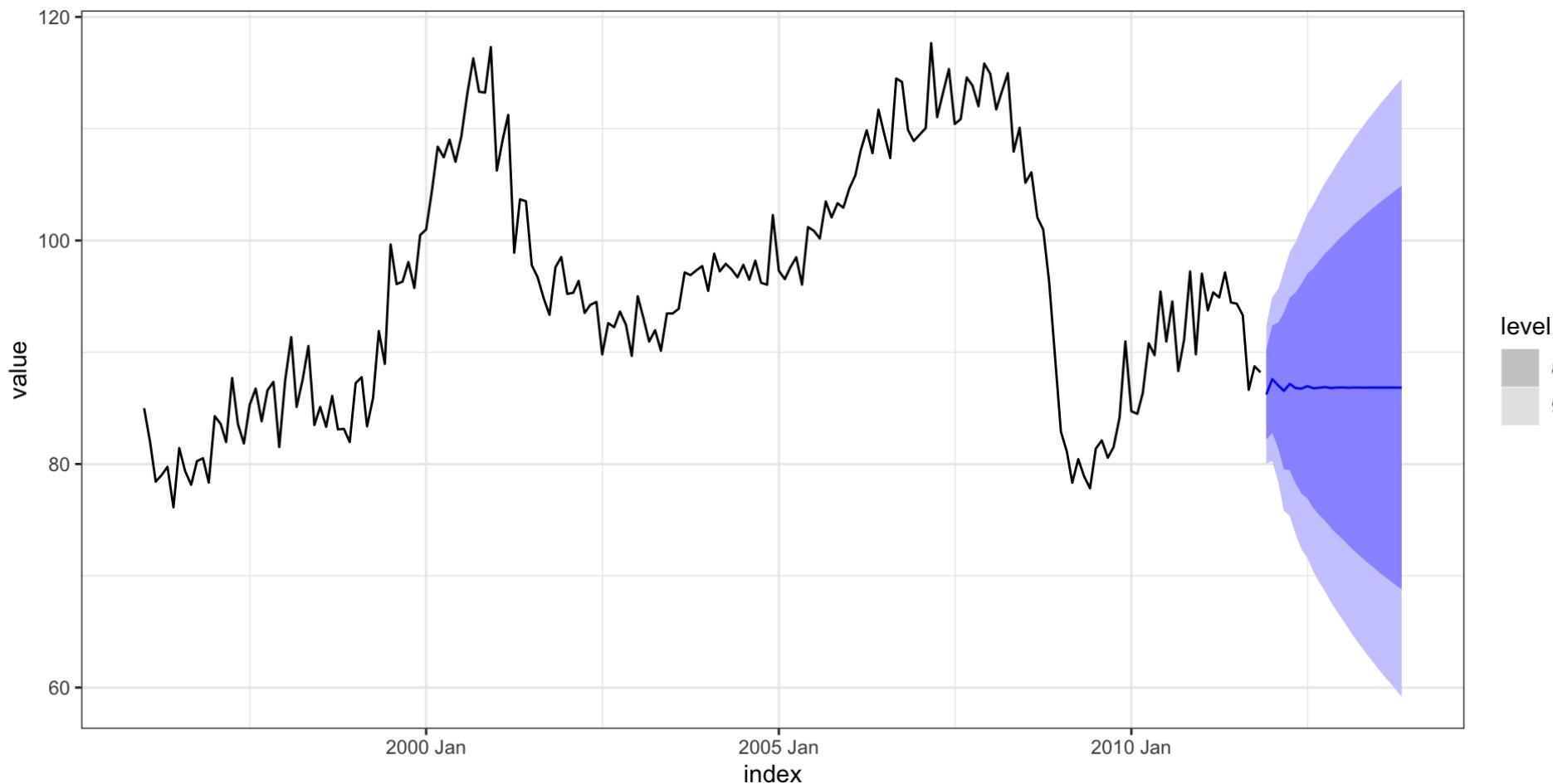
```
1 m %>%
2   forecast()
```

A fable: 24 x 4 [1M]
Key: .model [1]

| | .model | index | value | .mean |
|---|-----------------------------|----------|------------|-------|
| | <chr> | <mth> | <dist> | <dbl> |
| 1 | ARIMA(value ~ pdq(3, 1, 0)) | 2011 Dec | N(86, 9.9) | 86.2 |
| 2 | ARIMA(value ~ pdq(3, 1, 0)) | 2012 Jan | N(88, 14) | 87.6 |
| 3 | ARIMA(value ~ pdq(3, 1, 0)) | 2012 Feb | N(87, 19) | 87.0 |
| 4 | ARIMA(value ~ pdq(3, 1, 0)) | 2012 Mar | N(87, 30) | 86.5 |
| 5 | ARIMA(value ~ pdq(3, 1, 0)) | 2012 Apr | N(87, 36) | 87.2 |
| 6 | ARIMA(value ~ pdq(3, 1, 0)) | 2012 May | N(87, 44) | 86.8 |
| 7 | ARIMA(value ~ pdq(3, 1, 0)) | 2012 Jun | N(87, 54) | 86.7 |
| 8 | ARIMA(value ~ pdq(3, 1, 0)) | 2012 Jul | N(87, 62) | 87.0 |

Forecasting - autoplot

```
1 m %>%
2   forecast() %>%
3   autoplot(elec_sales)
```



Comparing models

The `fable model()` function also has the ability to fit multiple models at the same time which then makes comparison more straight forward.

```
1 ( mm = elec_sales %>%
2   model(
3     arima310 = ARIMA(value ~ pdq(3,1,0)),
4     arima013 = ARIMA(value ~ pdq(0,1,3)),
5     autoarima = ARIMA(value),
6     autoarima_bf = ARIMA(value, stepwise = FALSE)
7   )
8 )
```

```
# A mable: 1 x 4
      arima310      arima013      autoarima    autoarima_bf
      <model>       <model>       <model>        <model>
1 <ARIMA(3,1,0)> <ARIMA(0,1,3)> <ARIMA(3,1,1)> <ARIMA(1,1,5)>
```

broom + multiple models

```
1 mm %>% glance() %>% arrange(AICc)
```

A tibble: 4 × 8

| | .model | sigma2 | log_lik | AIC | AICc | BIC | ar_roots | ma_roots |
|---|--------------|--------|---------|-------|-------|--------|-----------|-----------|
| | <chr> | <dbl> | <dbl> | <dbl> | <dbl> | <list> | <list> | |
| 1 | autoarima | 9.74 | -484. | 978. | 978. | 994. | <cpl [3]> | <cpl [1]> |
| 2 | autoarima_bf | 9.63 | -482. | 978. | 979. | 1001. | <cpl [1]> | <cpl [5]> |
| 3 | arima310 | 9.85 | -486. | 979. | 980. | 992. | <cpl [3]> | <cpl [0]> |
| 4 | arima013 | 10.2 | -489. | 986. | 987. | 999. | <cpl [0]> | <cpl [3]> |

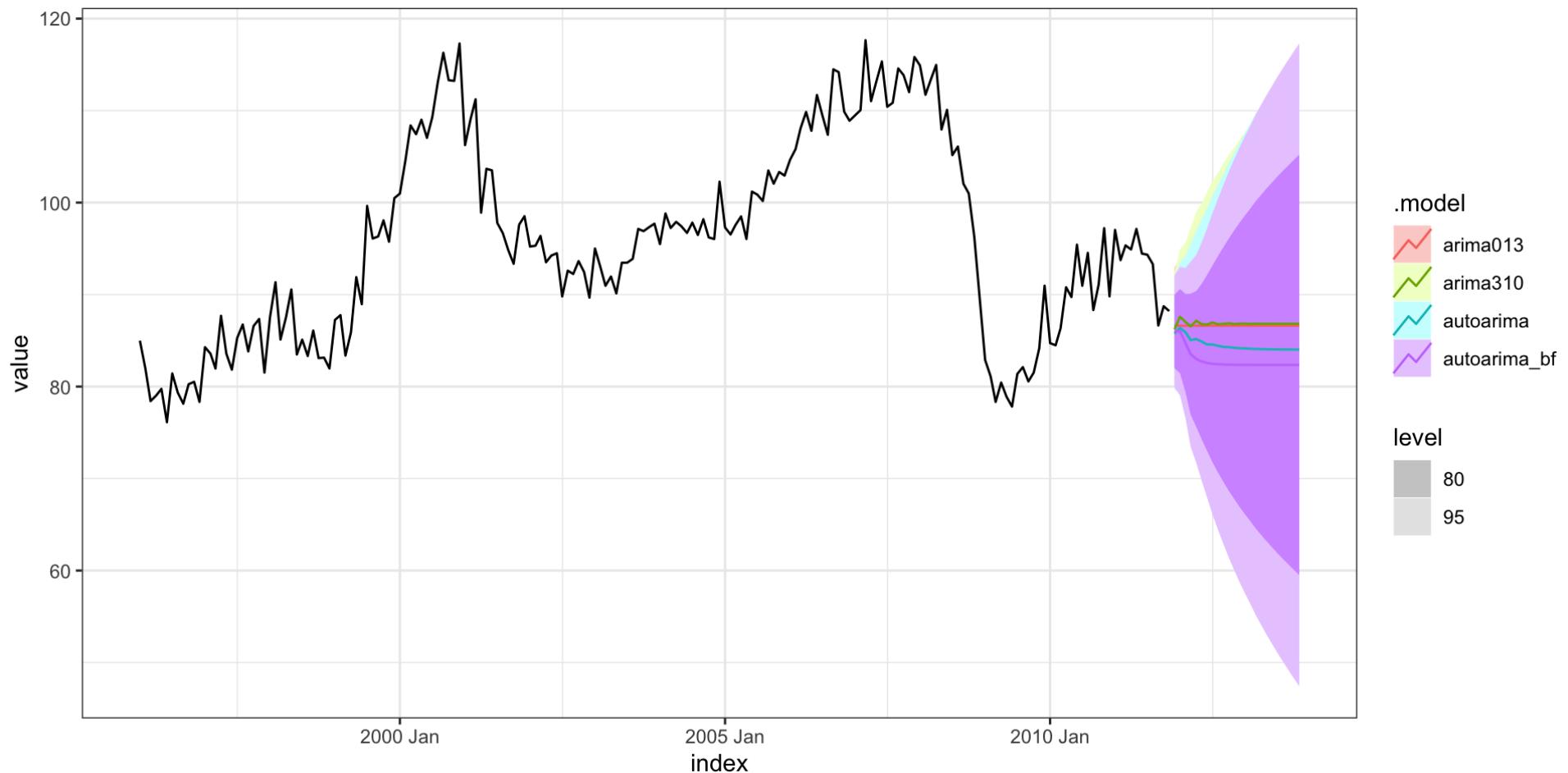
```
1 mm %>% accuracy() %>% arrange(RMSE)
```

A tibble: 4 × 10

| | .model | .type | ME | RMSE | MAE | MPE | MAPE | MASE | RMSSE | ACF1 |
|---|--------------|----------|----------|-------|-------|---------|-------|-------|-------|---------|
| | <chr> | <chr> | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> |
| 1 | autoarima_bf | Training | -0.00719 | 3.05 | 2.41 | -0.0458 | 2.55 | 0.294 | 0.275 | 0.00916 |
| 2 | autoarima | Training | -0.00123 | 3.08 | 2.39 | -0.0429 | 2.52 | 0.291 | 0.278 | 0.00893 |
| 3 | arima310 | Training | 0.0117 | 3.11 | 2.43 | -0.0435 | 2.56 | 0.296 | 0.281 | -0.0346 |
| 4 | arima013 | Training | 0.0105 | 3.17 | 2.40 | -0.0486 | 2.53 | 0.292 | 0.286 | -0.0210 |

Forecasting - autoplot

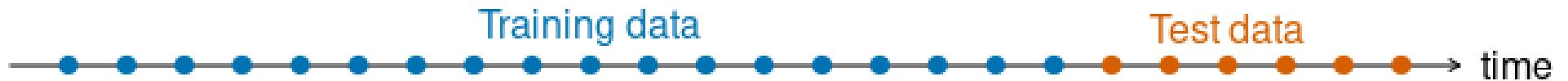
```
1 mm %>%
2   forecast() %>%
3   autoplot(elec_sales)
```



Cross validation

Test train split

The general approach is to keep the data ordered and split the first `prop%` into the training data and the remainder as testing data.



```
1 elec_sales_split = rsample::initial_time_split(elec_sales, prop=0.9)
```

```
1 rsample:::training(elec_sales_split)
```

```
# A tsibble: 171 x 2 [1M]
  index value
  <mth> <dbl>
1 1996 Jan  85.0
2 1996 Feb  81.9
3 1996 Mar  78.4
4 1996 Apr  79.0
5 1996 May  79.7
6 1996 Jun  76.1
7 1996 Jul  81.4
8 1996 Aug  79.3
9 1996 Sep  78.1
10 1996 Oct  80.3
# ... with 161 more rows
```

```
1 rsample:::testing(elec_sales_split)
```

```
# A tsibble: 20 x 2 [1M]
  index value
  <mth> <dbl>
1 2010 Apr  90.8
2 2010 May  89.7
3 2010 Jun  95.4
4 2010 Jul  91.0
5 2010 Aug  94.5
6 2010 Sep  88.3
7 2010 Oct  91.1
8 2010 Nov  97.2
9 2010 Dec  89.8
10 2011 Jan  97.0
11 2011 Feb  93.7
12 2011 Mar  95.4
13 2011 Apr  94.9
```

Model fit (training)

```
1 ( mm = rsample::training(elec_sales_split) %>%
2   model(
3     arima310 = ARIMA(value ~ pdq(3,1,0)),
4     arima013 = ARIMA(value ~ pdq(0,1,3)),
5     autoarima = ARIMA(value),
6     autoarima_bf = ARIMA(value, stepwise = FALSE)
7   )
8 )
```

```
# A mable: 1 x 4
      arima310      arima013      autoarima    autoarima_bf
      <model>       <model>       <model>        <model>
1 <ARIMA(3,1,0)> <ARIMA(0,1,3)> <ARIMA(3,1,0)> <ARIMA(3,1,0)>
```

Forecasting

```
1 mm %>%
2   forecast() %>%
3   autoplot(
4     elec_sales
5   )
```



Accuracy

Out-of-sample:

```
1 mm %>%
2   forecast() %>%
3   accuracy(elec_sales)
```

A tibble: 4 × 10

| | .model | .type | ME | RMSE | MAE | MPE | MAPE | MASE | RMSSE | ACF1 |
|---|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | <chr> | <chr> | <dbl> |
| 1 | arima013 | Test | 7.73 | 8.39 | 7.73 | 8.24 | 8.24 | 0.935 | 0.742 | 0.148 |
| 2 | arima310 | Test | 8.60 | 9.17 | 8.60 | 9.18 | 9.18 | 1.04 | 0.811 | 0.162 |
| 3 | autoarima | Test | 8.60 | 9.17 | 8.60 | 9.18 | 9.18 | 1.04 | 0.811 | 0.162 |
| 4 | autoarima_bf | Test | 8.60 | 9.17 | 8.60 | 9.18 | 9.18 | 1.04 | 0.811 | 0.162 |

Within-sample:

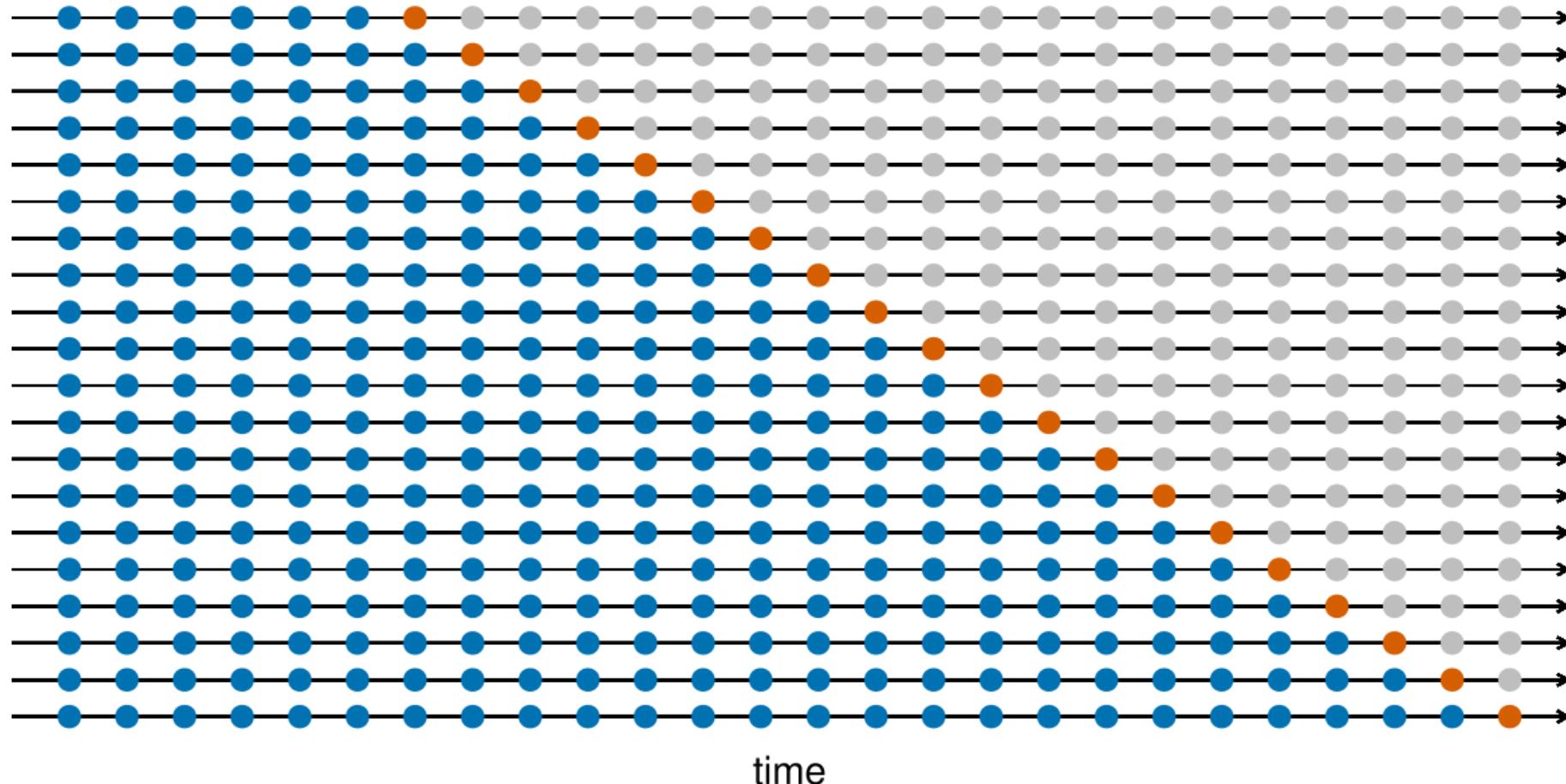
```
1 mm %>%
2   accuracy()
```

A tibble: 4 × 10

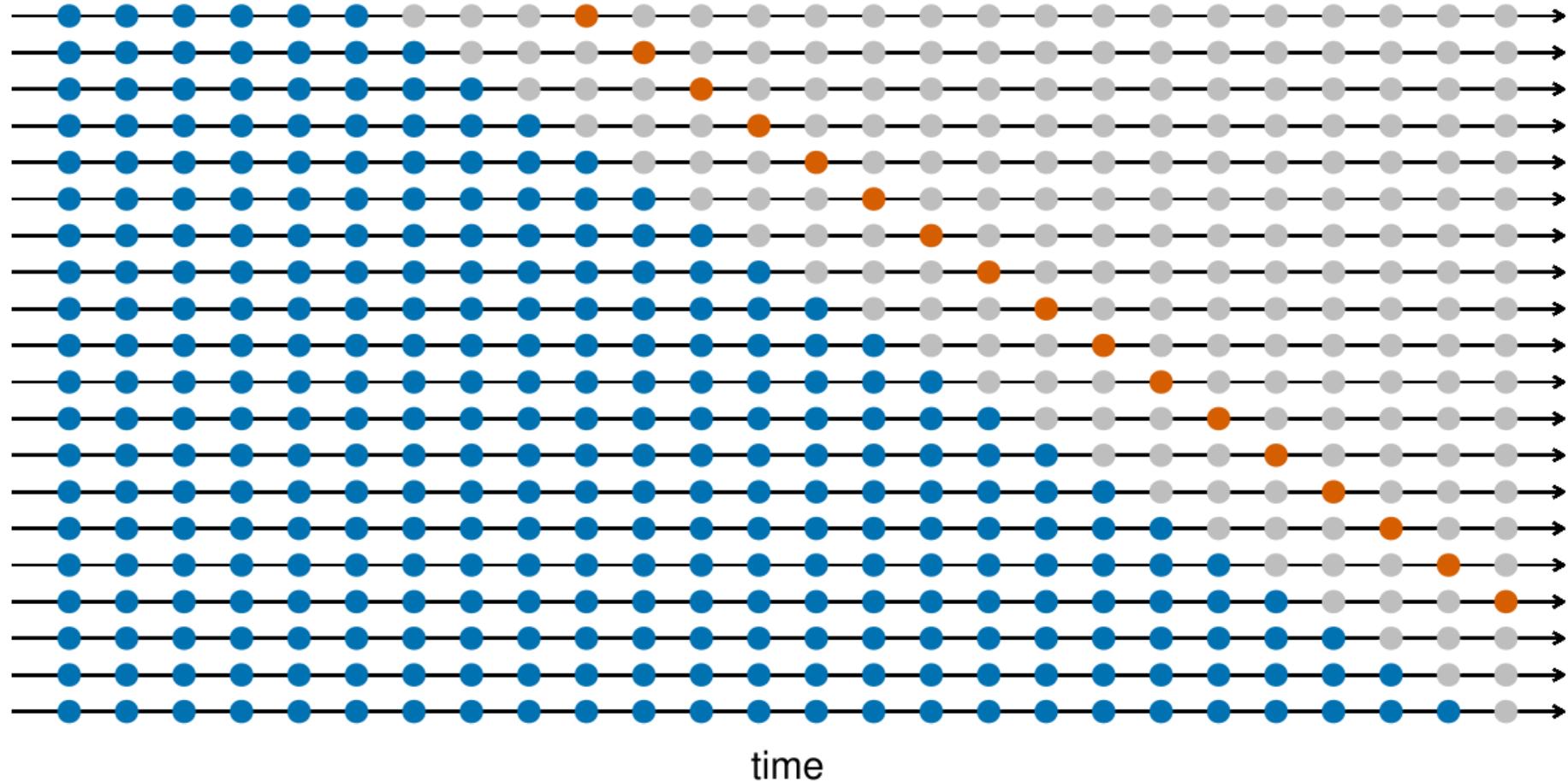
| | .model | .type | ME | RMSE | MAE | MPE | MAPE | MASE | RMSSE | ACF1 |
|---|--------------|----------|-----------|-------|-------|---------|-------|-------|-------|---------|
| | <chr> | <chr> | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> | <dbl> |
| 1 | arima310 | Training | -0.00435 | 3.00 | 2.31 | -0.0535 | 2.42 | 0.279 | 0.265 | -0.0317 |
| 2 | arima013 | Training | -0.000151 | 3.08 | 2.32 | -0.0541 | 2.44 | 0.281 | 0.272 | -0.0318 |
| 3 | autoarima | Training | -0.00435 | 3.00 | 2.31 | -0.0535 | 2.42 | 0.279 | 0.265 | -0.0317 |
| 4 | autoarima_bf | Training | -0.00435 | 3.00 | 2.31 | -0.0535 | 2.42 | 0.279 | 0.265 | -0.0317 |

Rolling forecasting origin

One-step ahead predictive performance



Four-step ahead predictive performance



Prophet

Prophet model

Prophet uses a modeling framework that looks a lot like traditional GAM approaches. Specifically the time series is modeled as

$$y(t) = g(t) + s(t) + h(t) + \epsilon_t$$

where

- $g(t)$ is a piecewise linear trend component
- $s(t)$ is a seasonal component based on Fourier terms with specified period(s) and order
- $h(t)$ are holiday effects (specific dummy coded variables for important dates / times)
- ϵ_t white noise error

Implementation

Prophet is implemented in its own R package and Python packages ([prophet](#)) which provide all of the basic functionality.

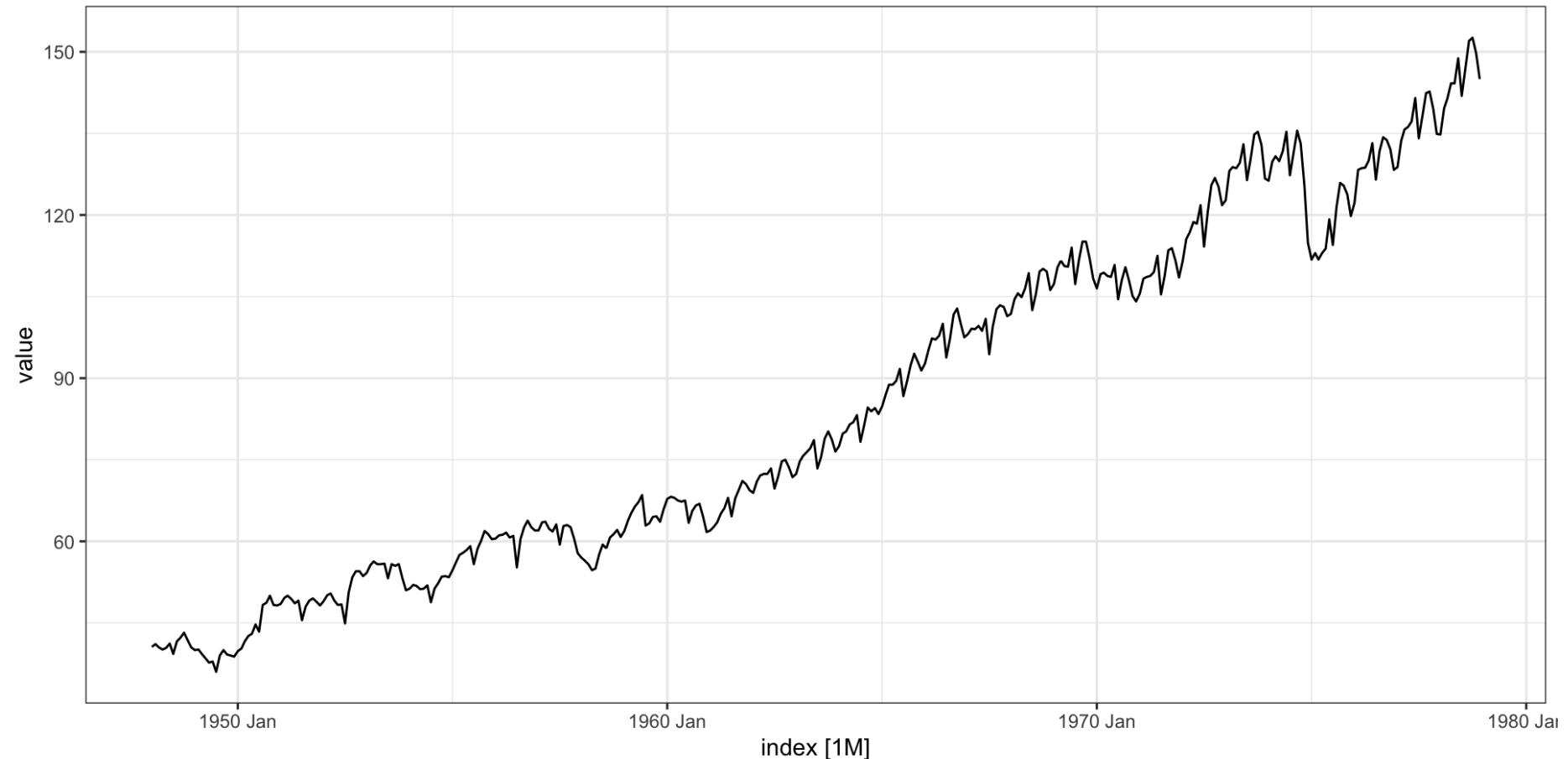
The model fitting is done using a Bayesian approach with the specific implementation using Stan.

Other frameworks like [fable](#) (or [modeltime](#)) provide higher level interfaces to this package.

prodn from astsa

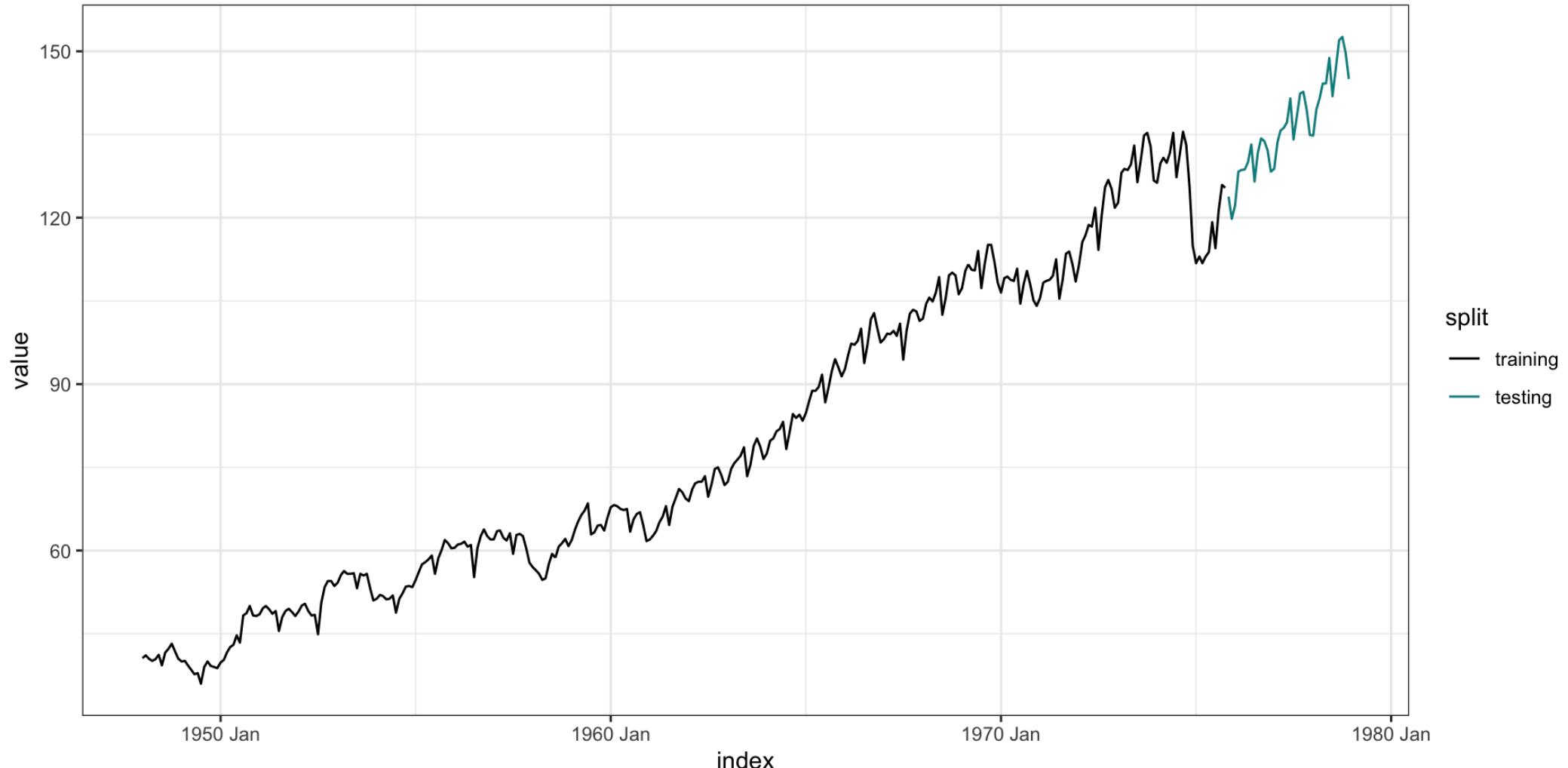
Monthly Federal Reserve Board Production Index (1948-1978)

```
1 prodn = tsibble::as_tsibble(astsa::prodn)
2 autoplot(prodn, value)
```



Test train split

```
1 prodn_split = rsample::initial_time_split(prodn, prop=0.90)
```



Models

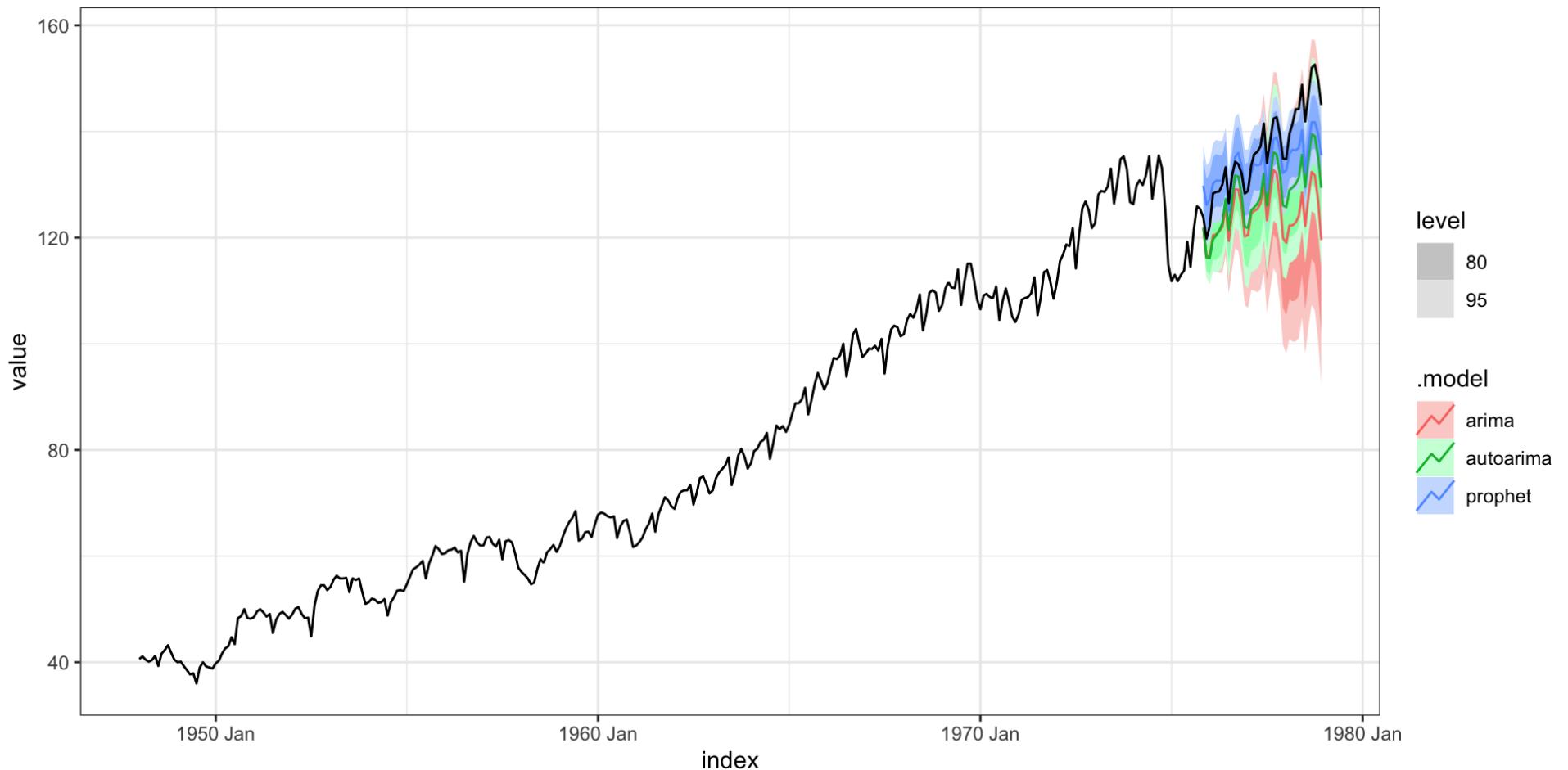
```
1 library(fable.prophet)
2
3 ( prodn_fit = rsample:::training(prodn_split) %>%
4   model(
5     arima = ARIMA(value ~ pdq(1,1,0) + PDQ(0,1,3)),
6     autoarima = ARIMA(value),
7     prophet = prophet(value ~ season(period = 12,
8                               type = "multiplicative")))
9   )
10 )
```

A mable: 1 x 3

| | arima | autoarima | prophet |
|---|---------------------------|--------------------------|-----------|
| | <model> | <model> | <model> |
| 1 | <ARIMA(1,1,0)(0,1,3)[12]> | <ARIMA(2,0,1)(0,1,1)[12] | w/ drift |
| | | | <prophet> |

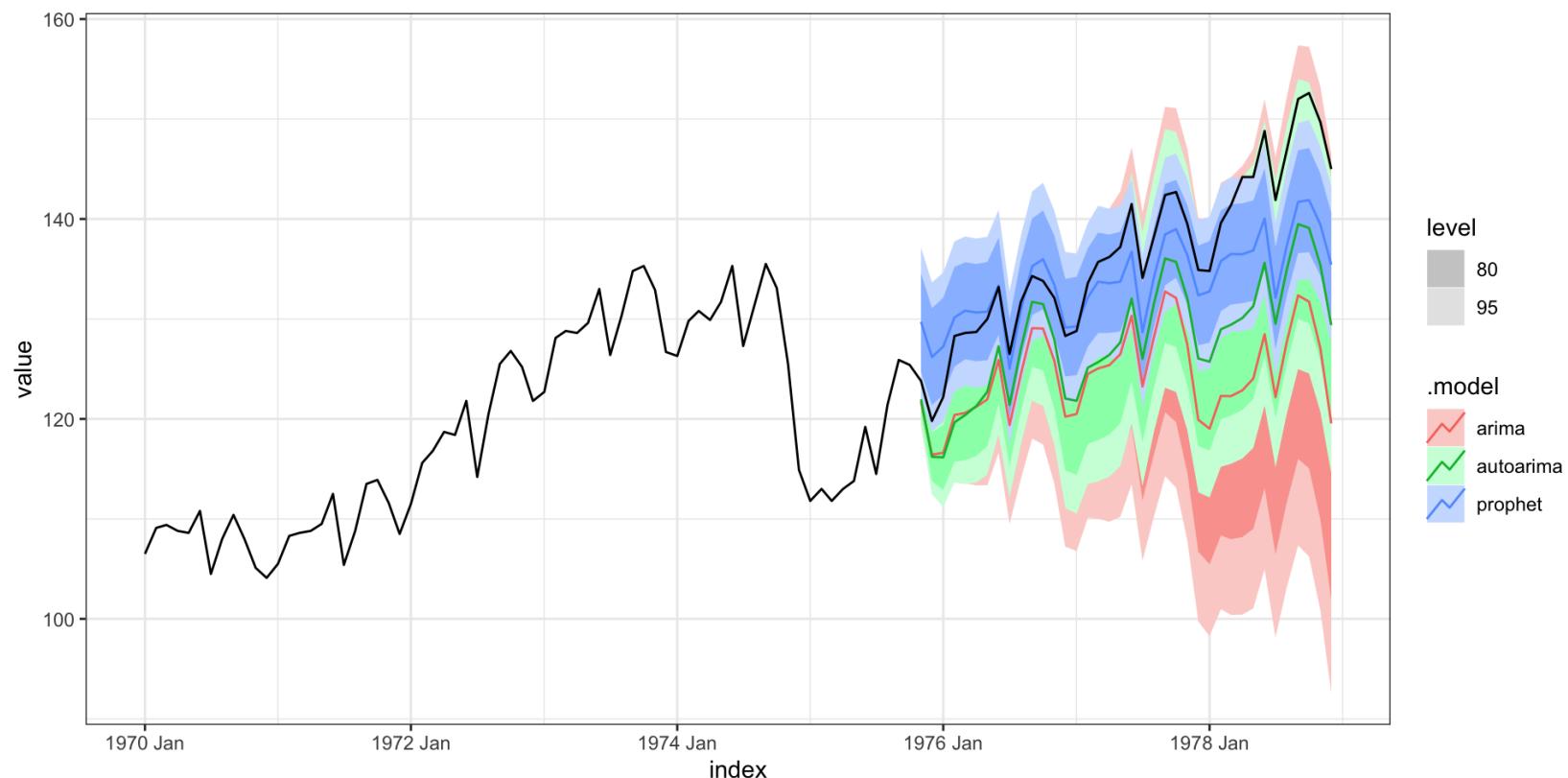
Forecast

```
1 prodn_fc = forecast(prodn_fit, h=38)
2 prodn_fc %>%
3   autoplot(prodn)
```



Forecast - zoom

```
1 prodn_fc = forecast(prodn_fit, h=38)
2 prodn_fc %>%
3   autoplot(
4     prodn %>% filter(index >= make_yearmonth(1970L, 1L))
5   )
```



Accuracy

Out-of-sample:

```
1 prodn_fit %>%
2   forecast() %>%
3   accuracy(prodn)

# A tibble: 3 × 10
  .model    .type     ME   RMSE    MAE     MPE    MAPE    MASE   RMSSE   ACF1
  <chr>    <chr>  <dbl> <dbl>  <dbl>  <dbl>  <dbl>  <dbl>  <dbl>  <dbl>
1 arima     Test    7.93  8.29  7.93  5.95  5.95  1.56  1.34  0.755
2 autoarima Test    6.52  6.93  6.52  4.91  4.91  1.28  1.12  0.759
3 prophet   Test    0.145 3.22  2.65  0.0162 2.01  0.522 0.521  0.833
```

Within-sample:

```
1 prodn_fit %>%
2   accuracy()

# A tibble: 3 × 10
  .model    .type      ME   RMSE    MAE     MPE    MAPE    MASE   RMSSE   ACF1
  <chr>    <chr>  <dbl> <dbl>  <dbl>  <dbl>  <dbl>  <dbl>  <dbl>  <dbl>
1 arima    Training -0.00242  1.14  0.821  0.0129  1.11  0.162  0.185 -0.0348
2 autoarima Training  0.000504  1.17  0.831 -0.0402  1.11  0.164  0.189  0.00432
3 prophet   Training  0.000883  3.83  2.87  -0.234   3.70  0.565  0.620  0.950
```

Components

```
1 prodn_fit %>%
2   select(prophet) %>%
3   components() %>%
4   autoplot()
```

Prophet decomposition

value = trend * (1 + multiplicative_terms) + additive_terms + .resid

