Lagged predictors & AR(1) Models

Lecture 08

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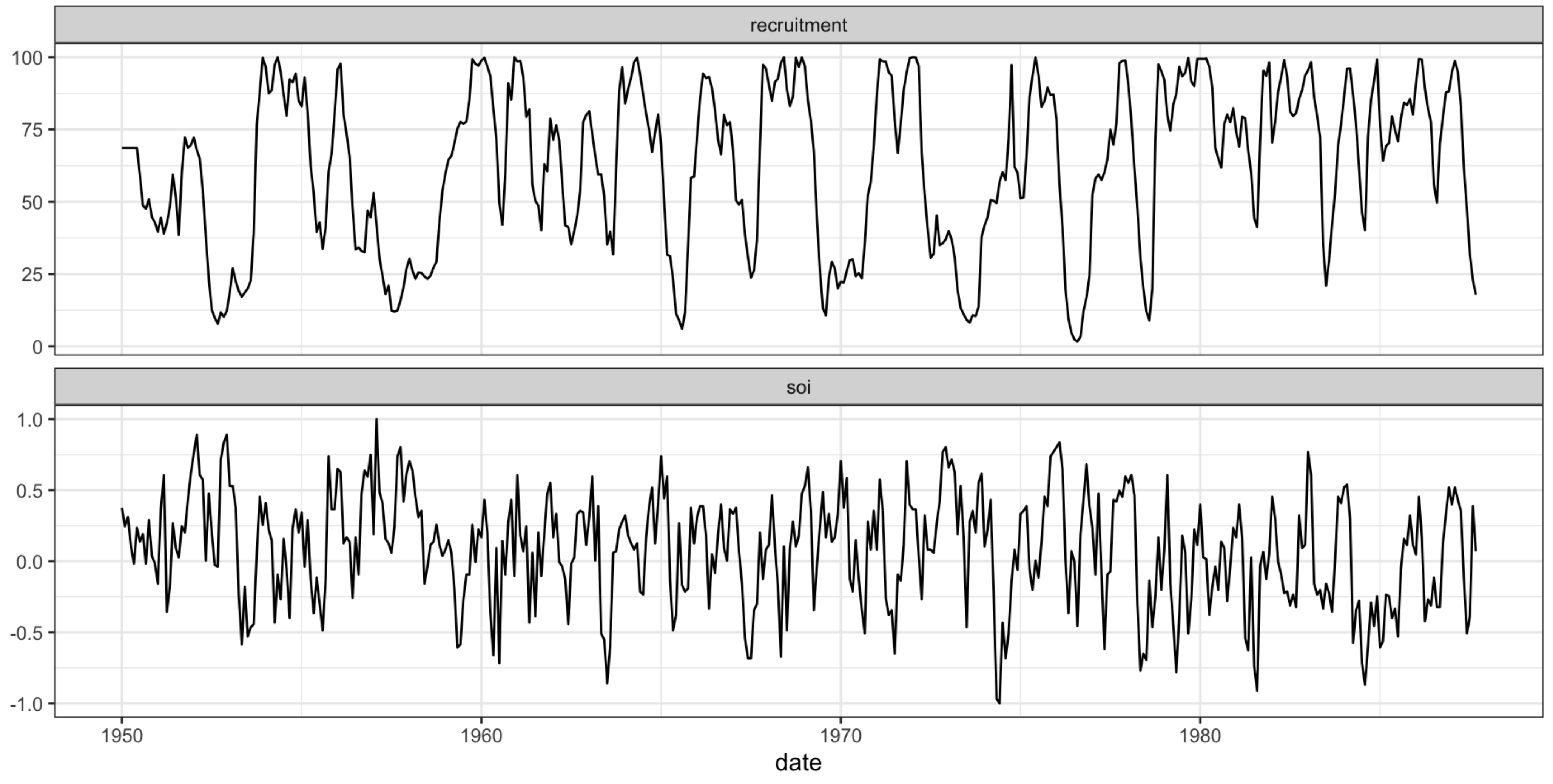
Lagged Predictors and CCFs

Southern Oscillation Index & Recruitment

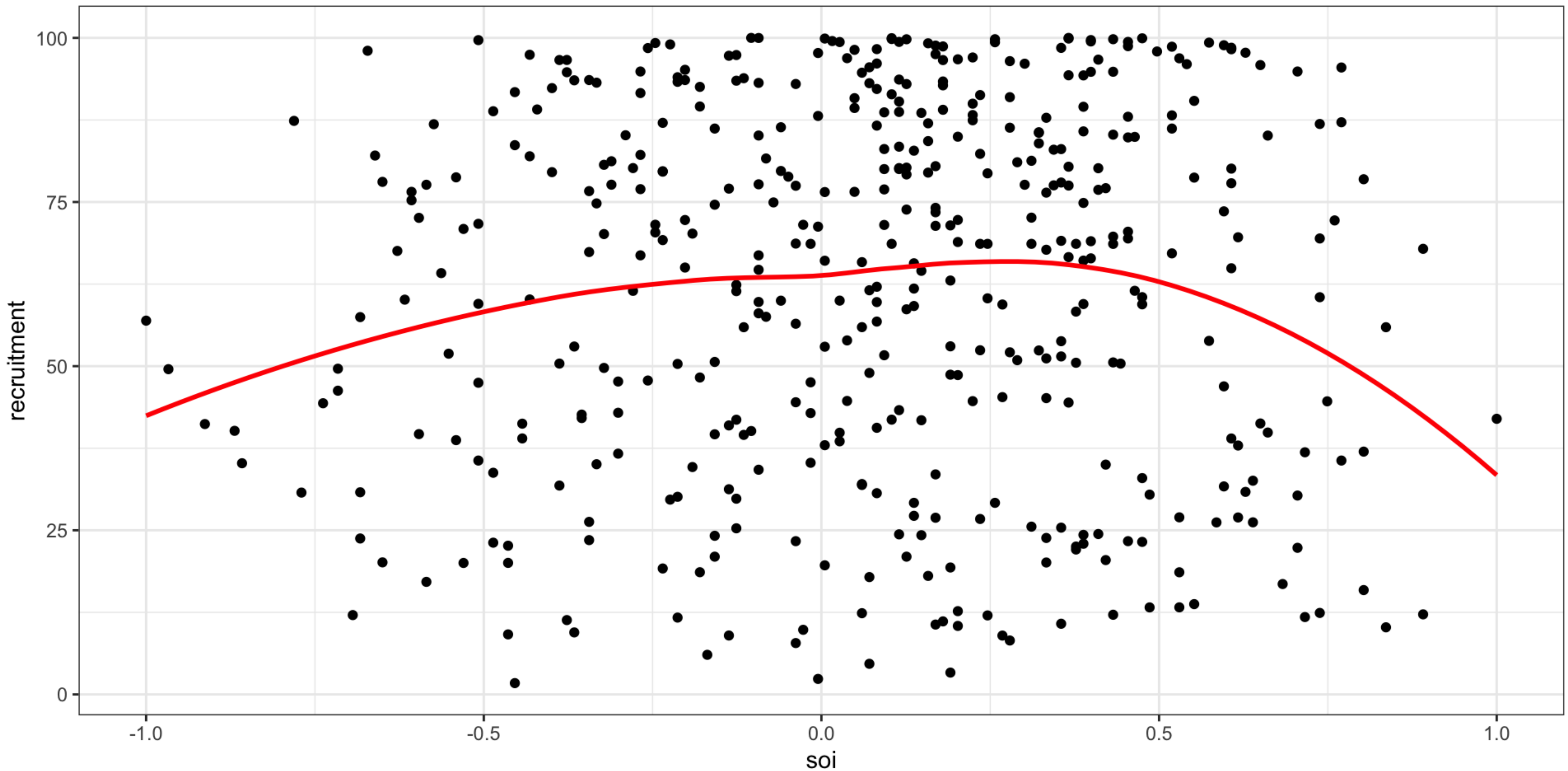
The Southern Oscillation Index (SOI) is an indicator of the development and intensity of El Niño (negative SOI) or La Niña (positive SOI) events in the Pacific Ocean. These data also included the estimate of “recruitment”, which indicate fish population sizes in the southern hemisphere.

```
# A tibble: 453 × 3
    date    soi recruitment
   <dbl>  <dbl>       <dbl>
 1 1950   0.377        68.6
 2 1950.  0.246        68.6
 3 1950.  0.311        68.6
 4 1950.  0.104        68.6
 5 1950. -0.016        68.6
 6 1950.  0.235        68.6
 7 1950.  0.137        59.2
 8 1951.  0.191        48.7
 9 1951. -0.016        47.5
```

Time series

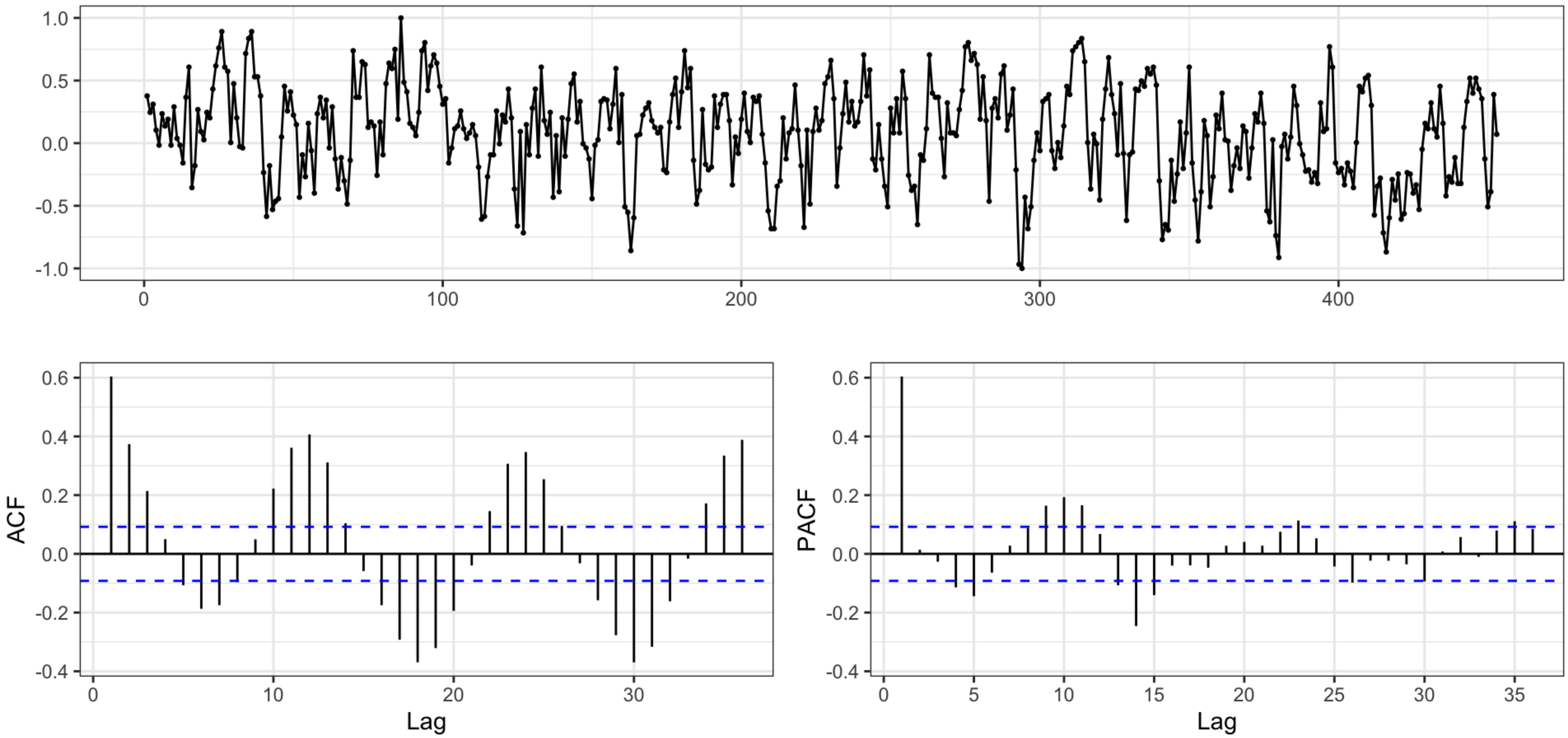

recruitment
100
75
50
25
0
soi
1.0
0.5
0.0
-0.5
-1.0
1950
1960
1970
1980
date

Relationship?



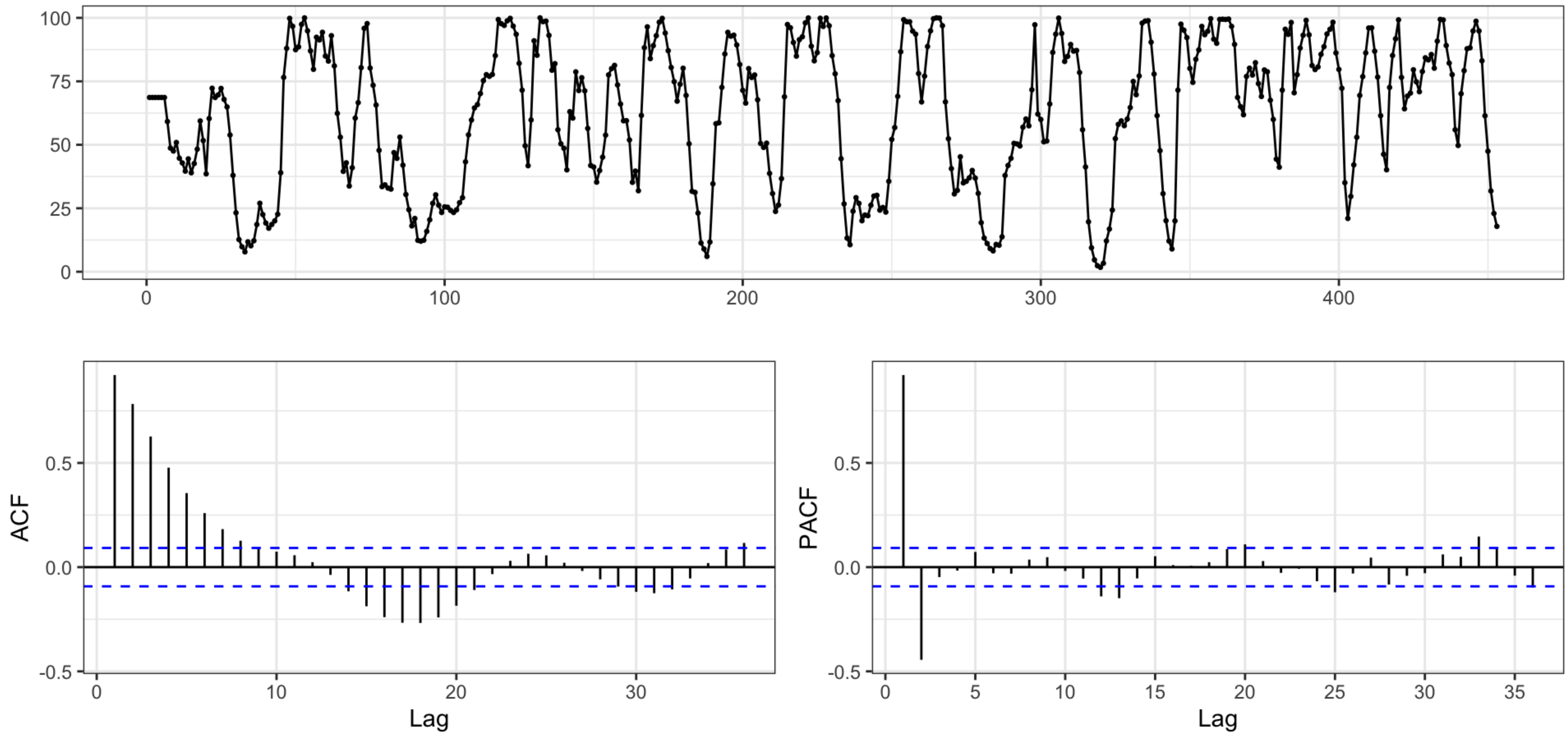
soi - ACF & PACF

```r
forecast::ggtsdisplay(fish$soi, lag.max = 36)
```



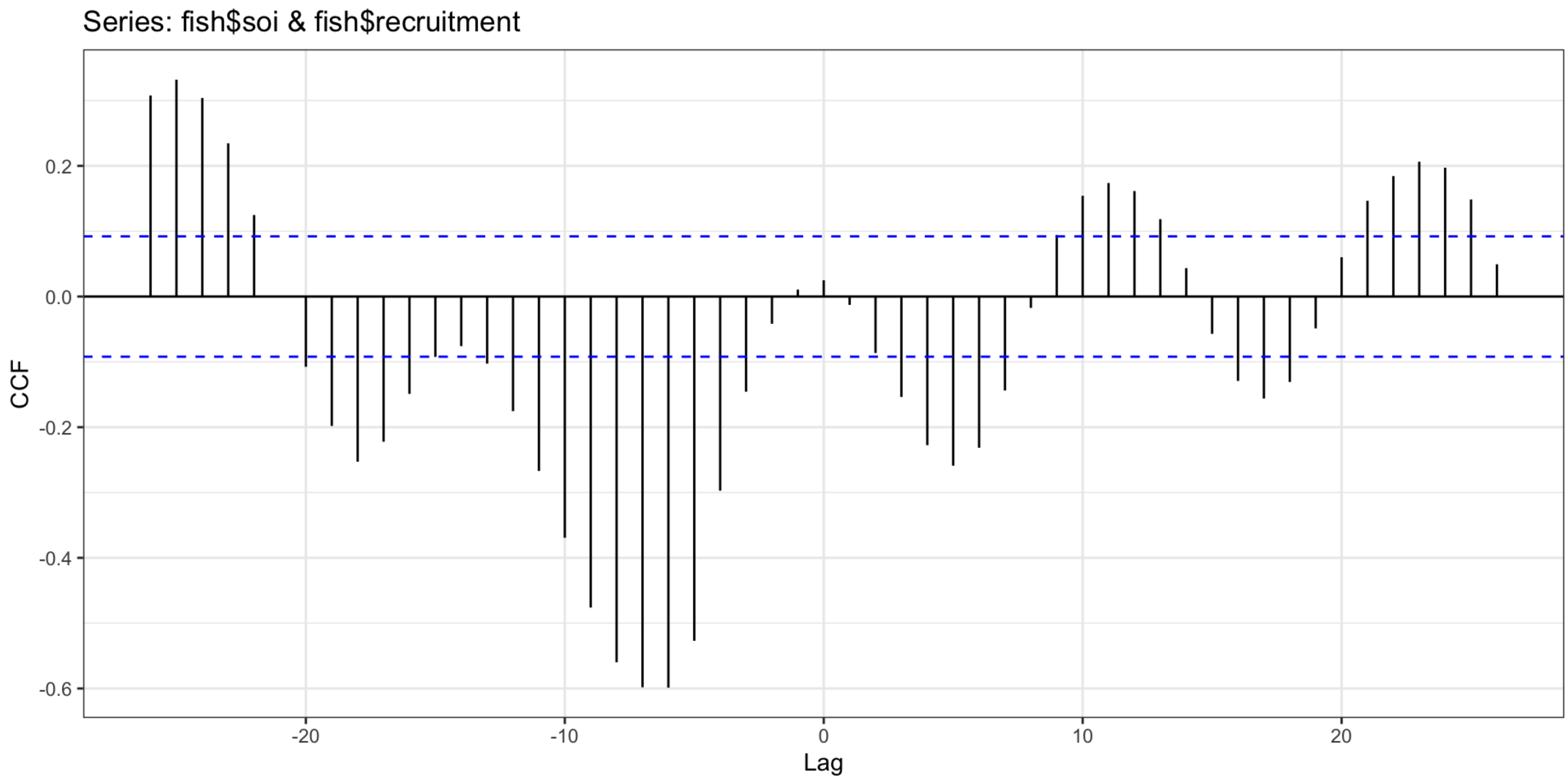
recruitment - ACF & PACF

```r
forecast::ggtsdisplay(fish$recruitment, lag.max = 36)
```

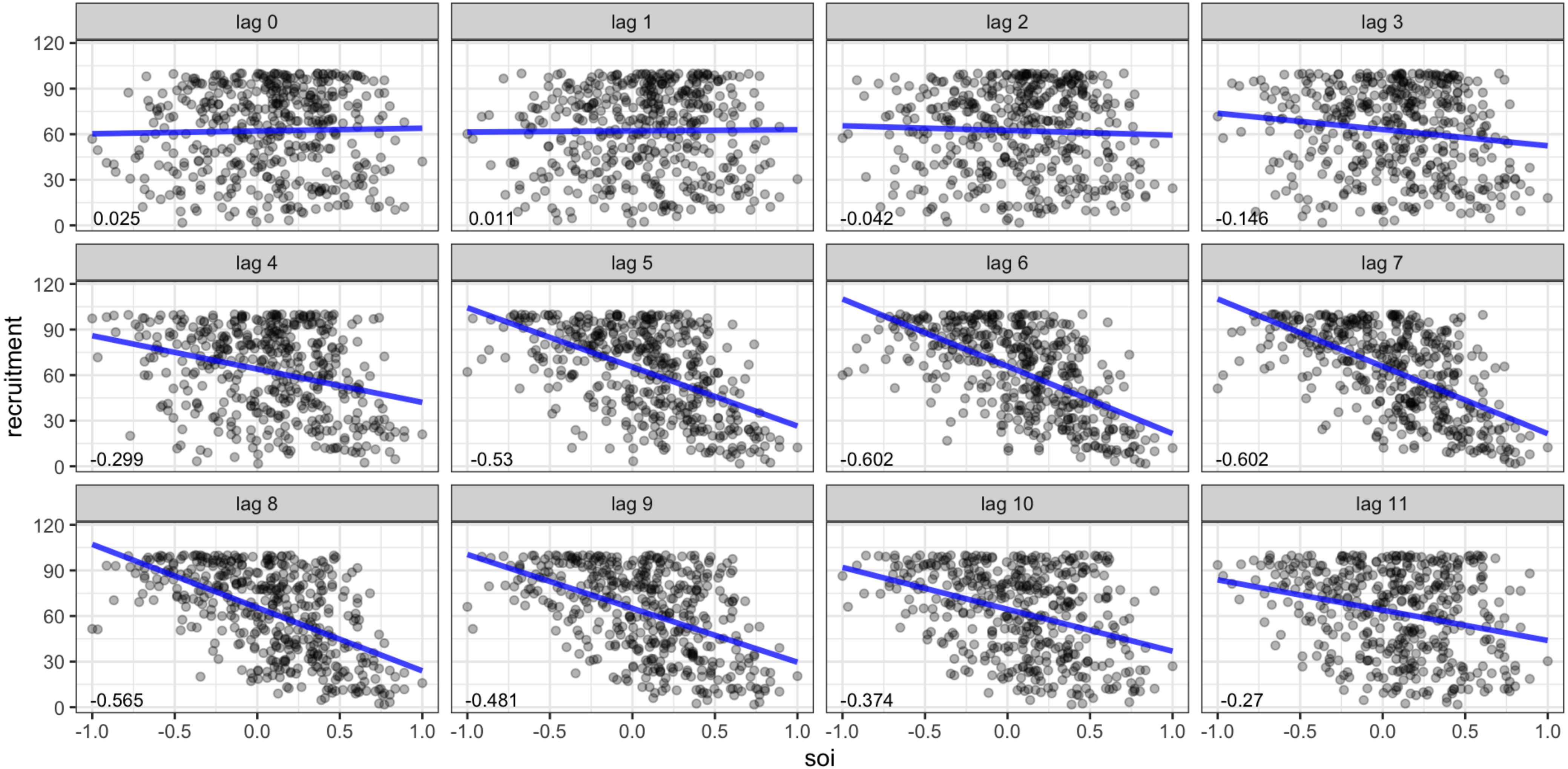


Cross correlation function

```r
forecast::ggCcf(fish$soi, fish$recruitment)
```



Cross correlation function - Scatter plots



The CCF gave us negative lags, why am I not considering them here?

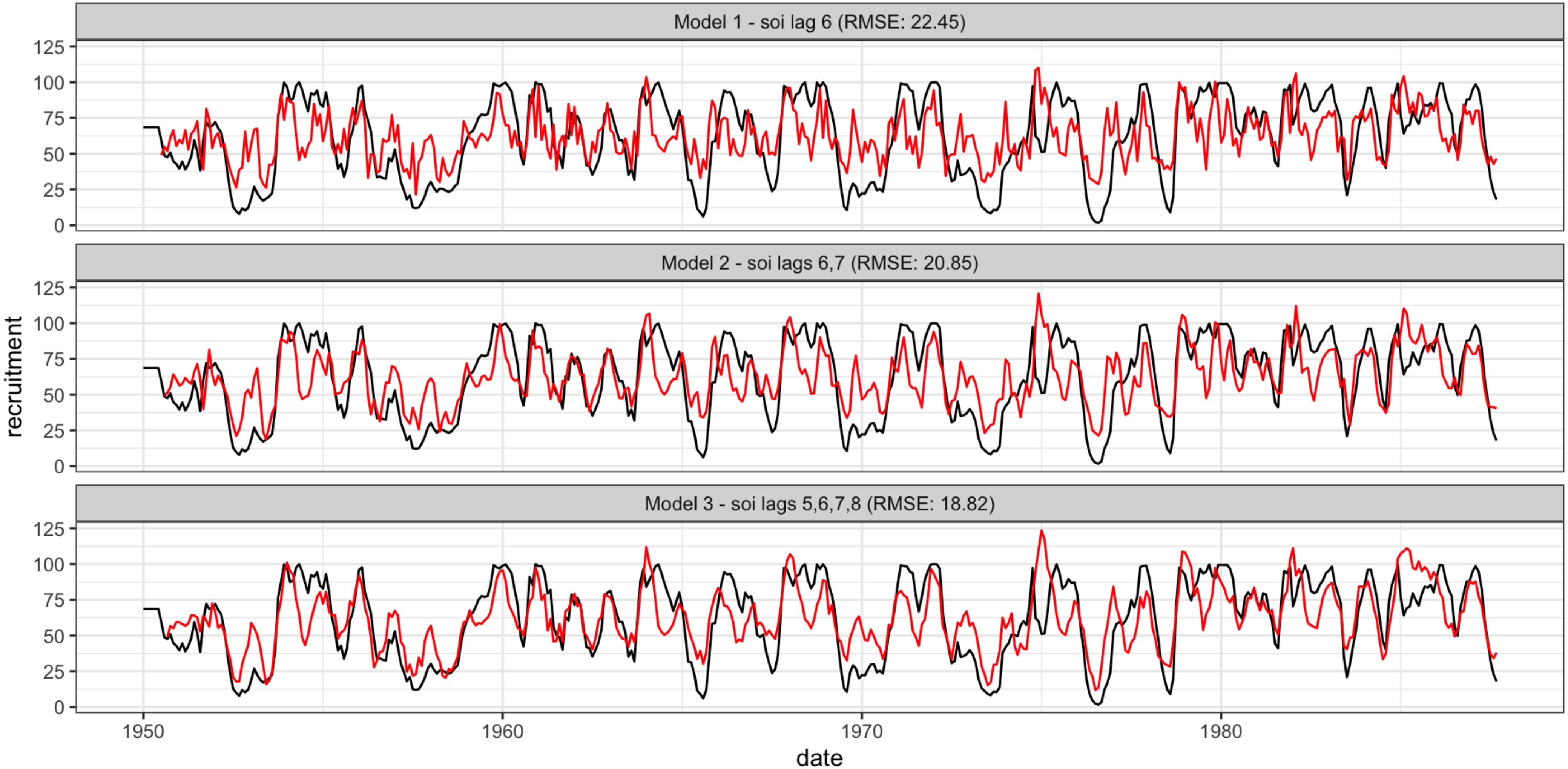
Model

```r
model1 = lm(recruitment~lag(soi,6), data=fish)
model2 = lm(recruitment~lag(soi,6)+lag(soi,7), data=fish)
model3 = lm(recruitment~lag(soi,5)+lag(soi,6)+lag(soi,7)+lag(soi,8), data=fish)
```

```r
summary(model3)
```

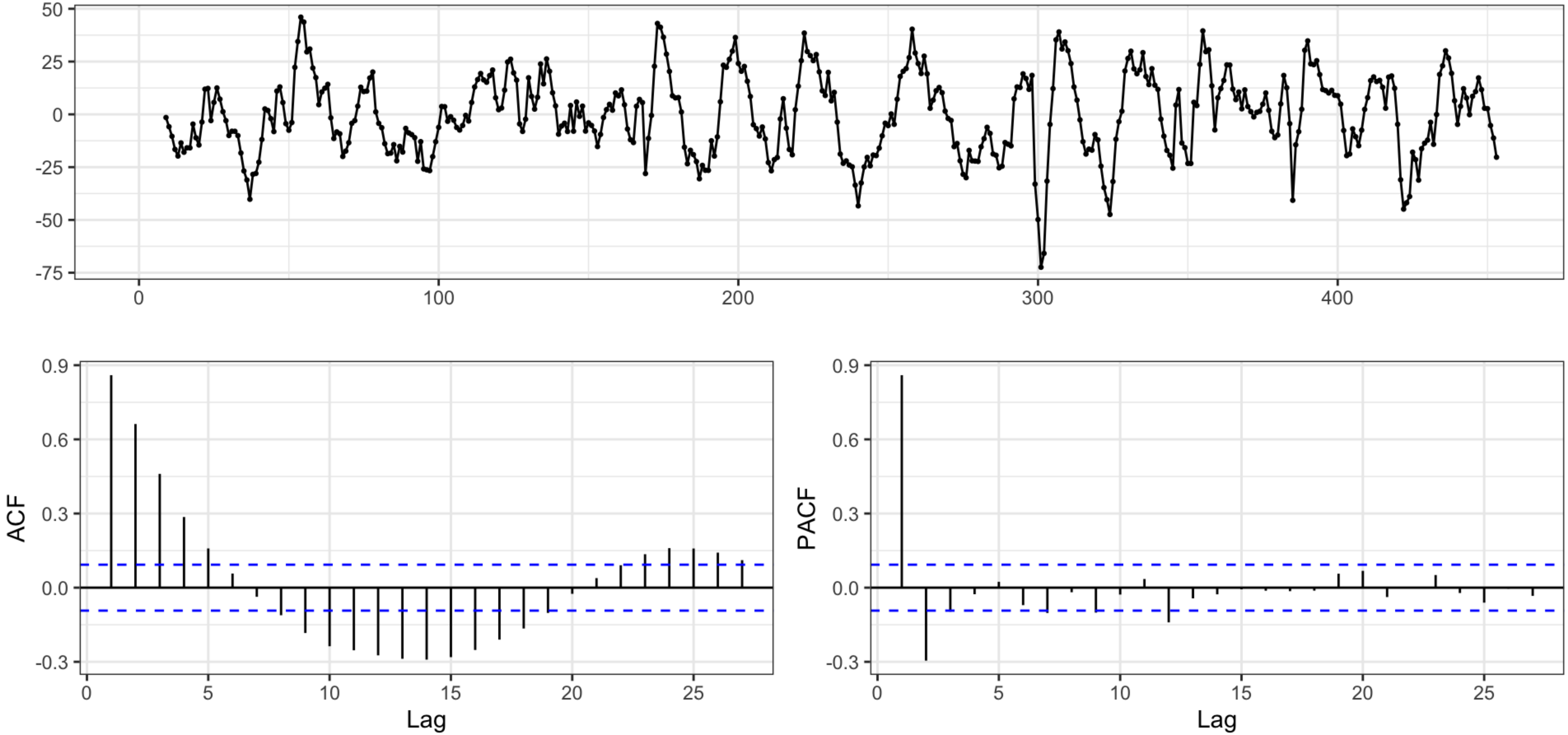
```

Call:
lm(formula = recruitment ~ lag(soi, 5) + lag(soi, 6) + lag(soi,
    7) + lag(soi, 8), data = fish)

Residuals:
    Min      1Q  Median      3Q     Max
-72.409 -13.527   0.191  12.851  46.040

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  67.9438     0.9306  73.007  < 2e-16 ***
lag(soi, 5) -19.1502     2.9508  -6.490 2.32e-10 ***
lag(soi, 6) -15.6894     3.4334  -4.570 6.36e-06 ***
lag(soi, 7) -13.4041     3.4332  -3.904 0.000109 ***
lag(soi, 8) -23.1480     2.9530  -7.839 3.46e-14 ***
```

Prediction



Model 1 - soi lag 6 (RMSE: 22.45)
Model 2 - soi lags 6,7 (RMSE: 20.85)
Model 3 - soi lags 5,6,7,8 (RMSE: 18.82)
recruitment
125
100
75
50
25
0
1950
1960
1970
1980
date

Residual ACF - Model 3



Autoregessive model 4

```r
model4 = lm(
  recruitment~lag(recruitment,1) + lag(recruitment,2) +
              lag(soi,5)+lag(soi,6)+lag(soi,7)+lag(soi,8),
  data=fish
)
summary(model4)
```

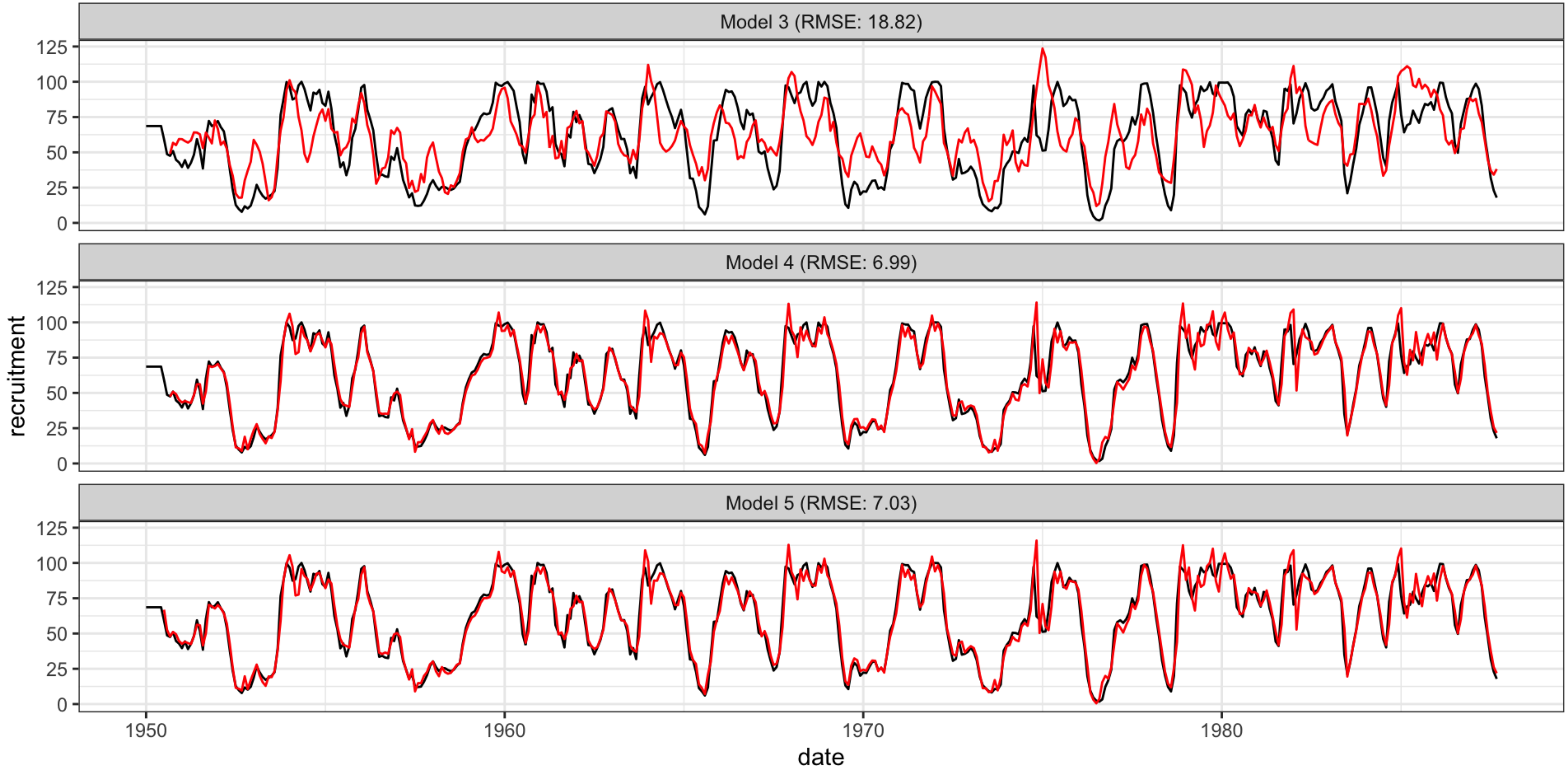
```
Call:
lm(formula = recruitment ~ lag(recruitment, 1) + lag(recruitment, 
    2) + lag(soi, 5) + lag(soi, 6) + lag(soi, 7) + lag(soi, 8), 
    data = fish)

Residuals:
    Min      1Q  Median      3Q     Max 
-51.996  -2.892   0.103   3.117  28.579 

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)    
(Intercept)          10.25007    1.17081   8.755  < 2e-16 ***
lag(recruitment, 1)   1.25301    0.04312  29.061  < 2e-16 ***
lag(recruitment, 2)  -0.39961    0.03998  -9.995  < 2e-16 ***
lag(soi, 5)         -20.76309    1.09906 -18.892  < 2e-16 ***
```

Autoregessive model 5

```r
model5 = lm(
  recruitment~lag(recruitment,1) + lag(recruitment,2) +
              lag(soi,5) + lag(soi,6),
  data=fish
)
summary(model5)
```

```
Call:
lm(formula = recruitment ~ lag(recruitment, 1) + lag(recruitment,
    2) + lag(soi, 5) + lag(soi, 6), data = fish)

Residuals:
    Min      1Q  Median      3Q     Max
-53.786  -2.999  -0.035   3.031  27.669

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)           8.78498    1.00171   8.770  < 2e-16 ***
lag(recruitment, 1)   1.24575    0.04314  28.879  < 2e-16 ***
lag(recruitment, 2)  -0.37193    0.03846  -9.670  < 2e-16 ***
lag(soi, 5)         -20.83776    1.10208 -18.908  < 2e-16 ***
lag(soi, 6)           8.55600    1.43146   5.977 4.68e-09 ***
```

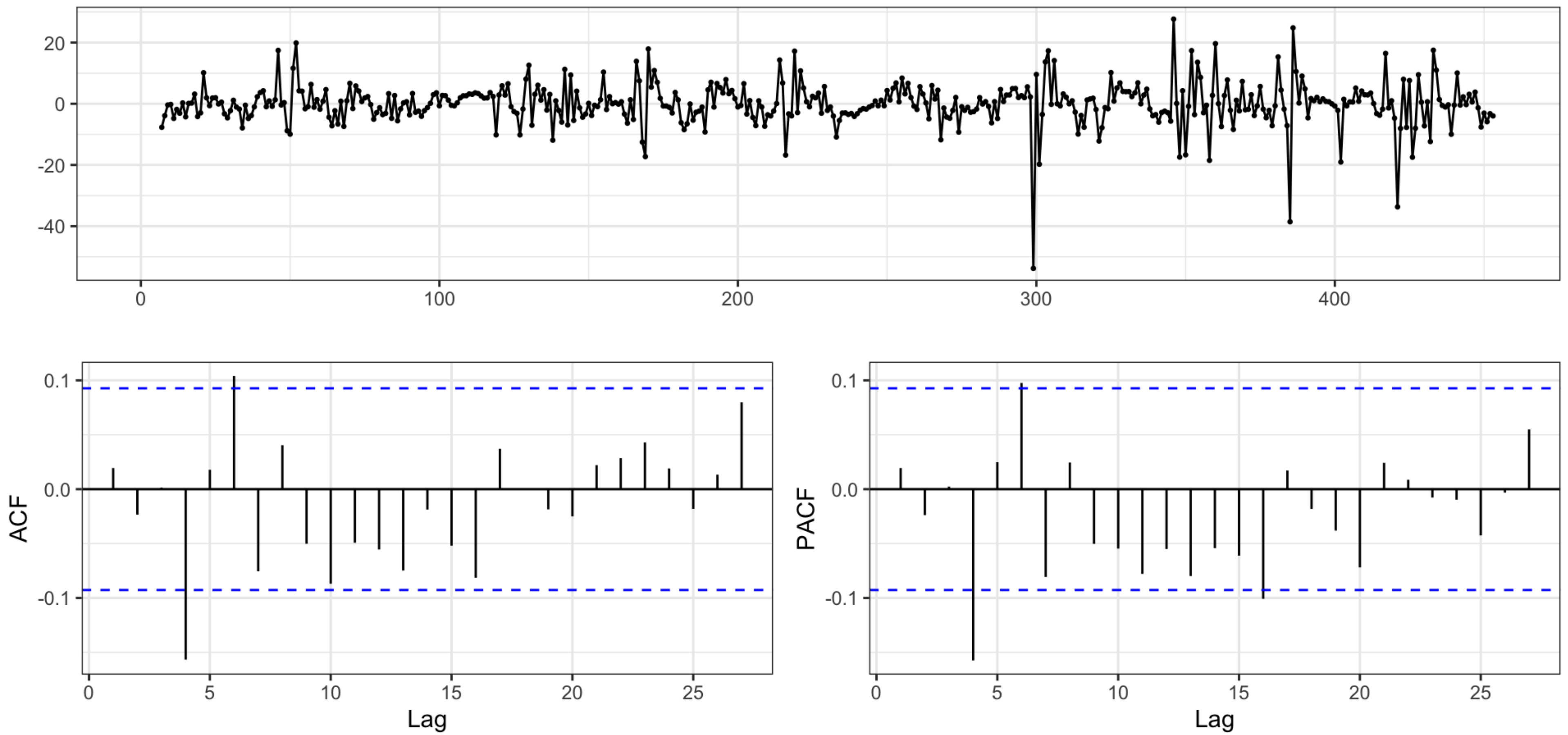
Prediction



Model 3 (RMSE: 18.82)
Model 4 (RMSE: 6.99)
Model 5 (RMSE: 7.03)
recruitment
125
100
75
50
25
0
1950
1960
1970
1980
date

Residual ACF - Model 5

```r
broom::augment(model5, newdata=fish) %>%
  pull(.resid) %>%
  forecast::ggtsdisplay()
```



Non-stationarity

Non-stationary models

> All happy families are alike; each unhappy family is unhappy in its own way.
>
> - Tolstoy, Anna Karenina

This applies to time series models as well, just replace happy family with stationary model.

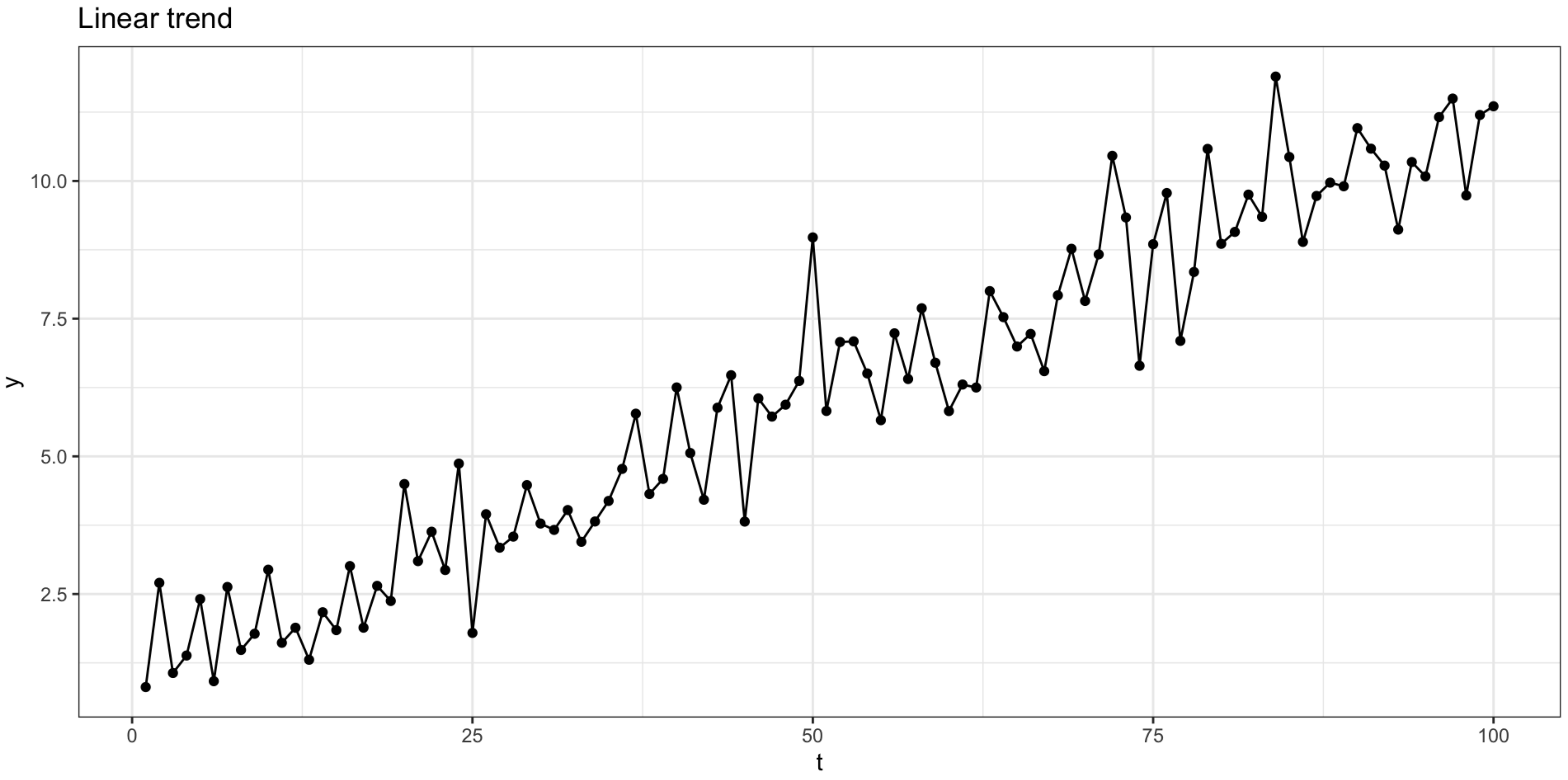
A simple example of a non-stationary time series is a trend stationary model

$$y_t = \mu(t) + w_t$$

where $\mu(t)$ denotes a time dependent trend and w_t is a white noise (stationary) process.

Linear trend model

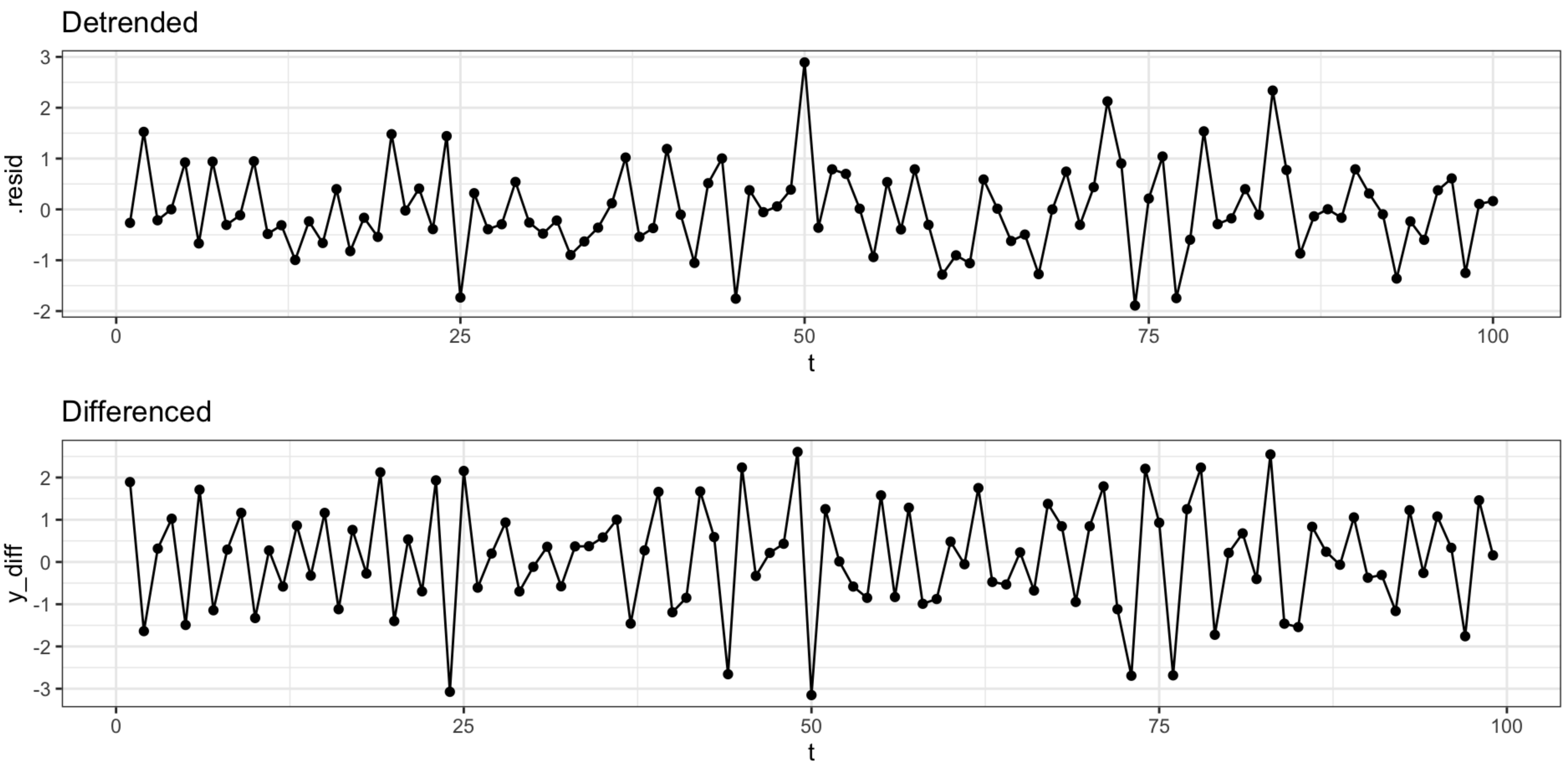
Lets imagine a simple model where $y_t = \delta + \beta t + x_t$ where δ and β are constants and x_t is a stationary process.



Differencing

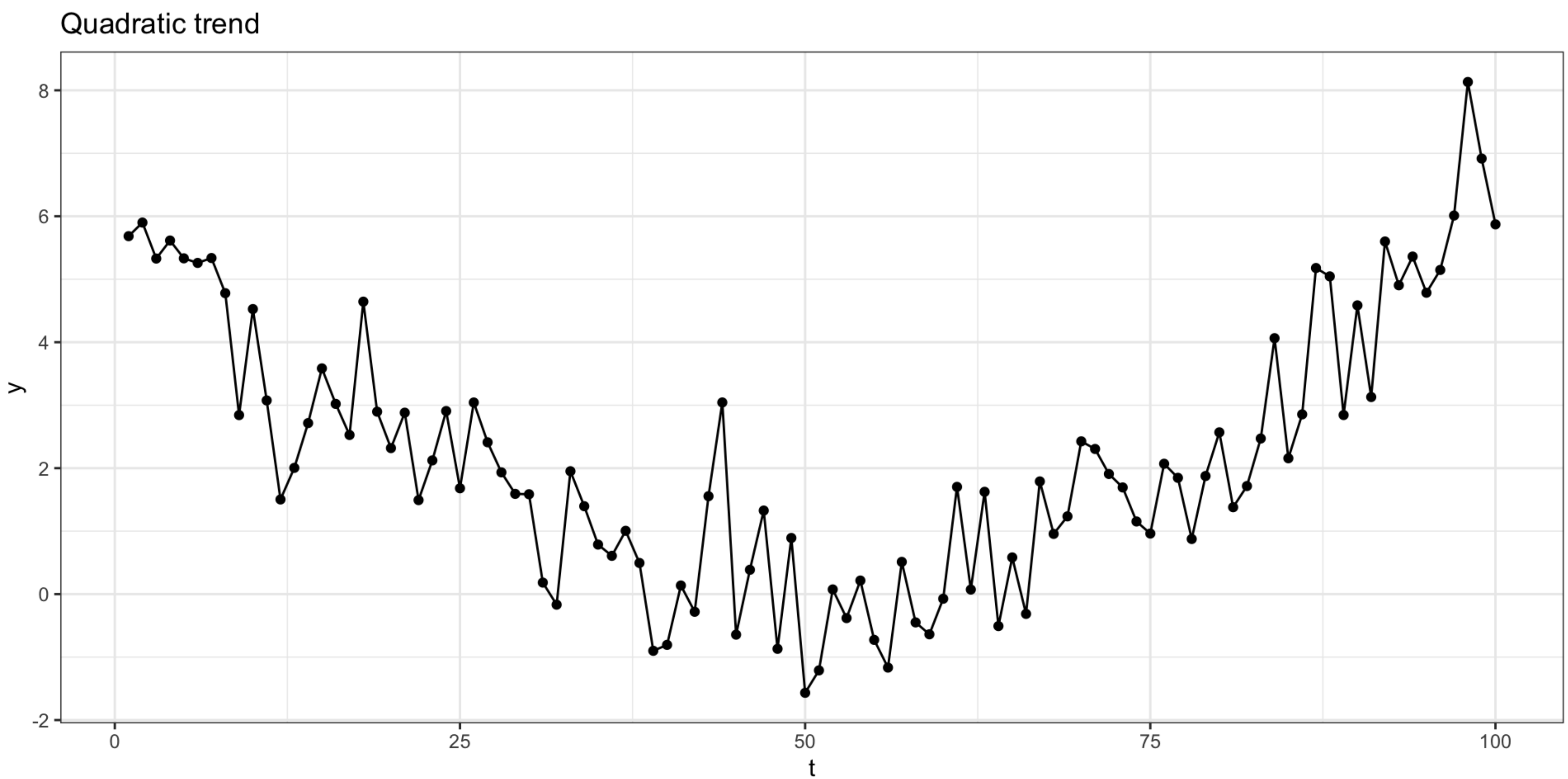
An simple approach to remove trend is to difference your response variable, specifically examine $d_t = y_t - y_{t-1}$ instead of y_t.

Detrending vs Differencing

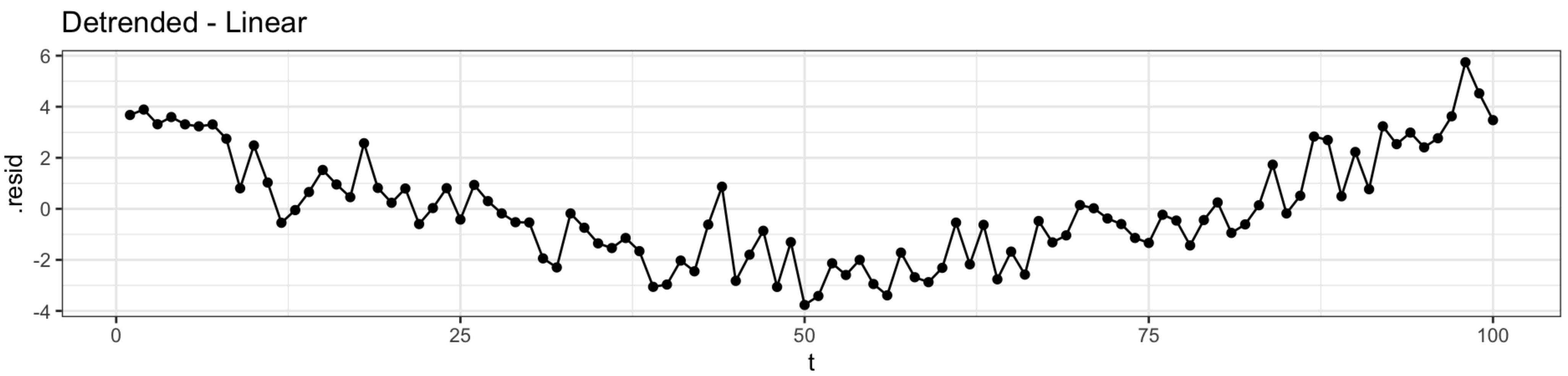


Quadratic trend model

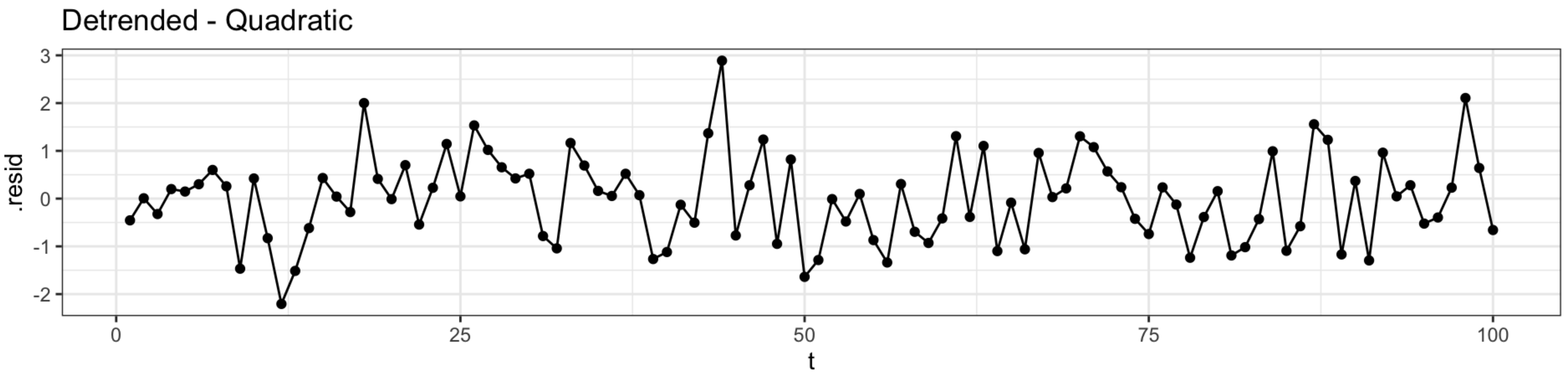
Lets imagine another simple model where $y_t = \delta + \beta t + \gamma t^2 + x_t$ where δ, β, and γ are constants and x_t is a stationary process.



Detrending



Detrended - Linear
6
4
2
0
-2
-4
.resid
0
25
50
75
100
t



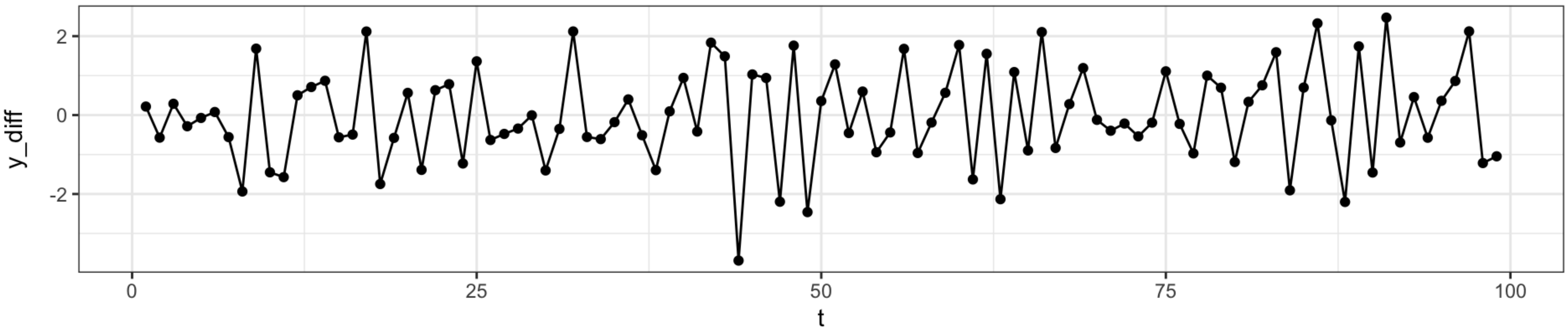
Detrended - Quadratic
3
2
1
0
-1
-2
.resid
0
25
50
75
100
t

2nd order differencing

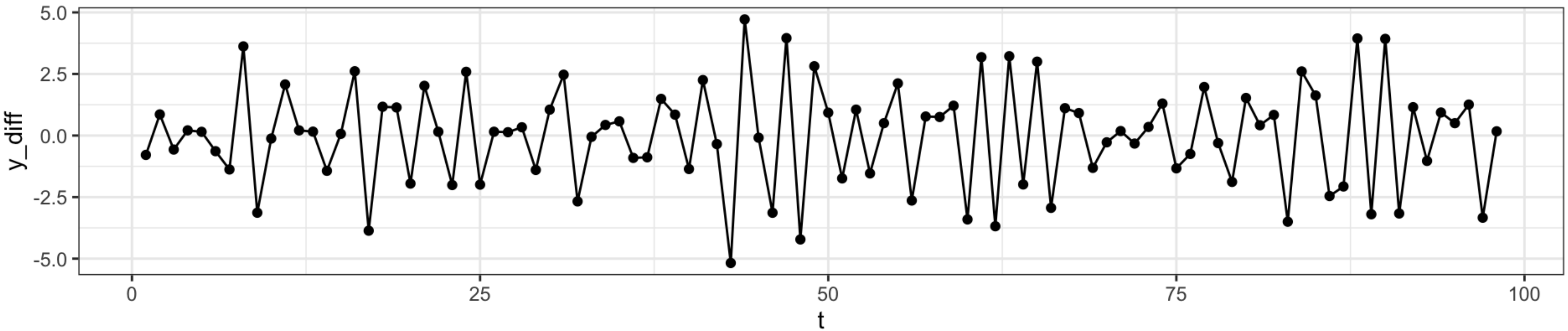
Let $d_t = y_t - y_{t-1}$ be a first order difference then $d_t - d_{t-1}$ is a 2nd order difference.

Differencing

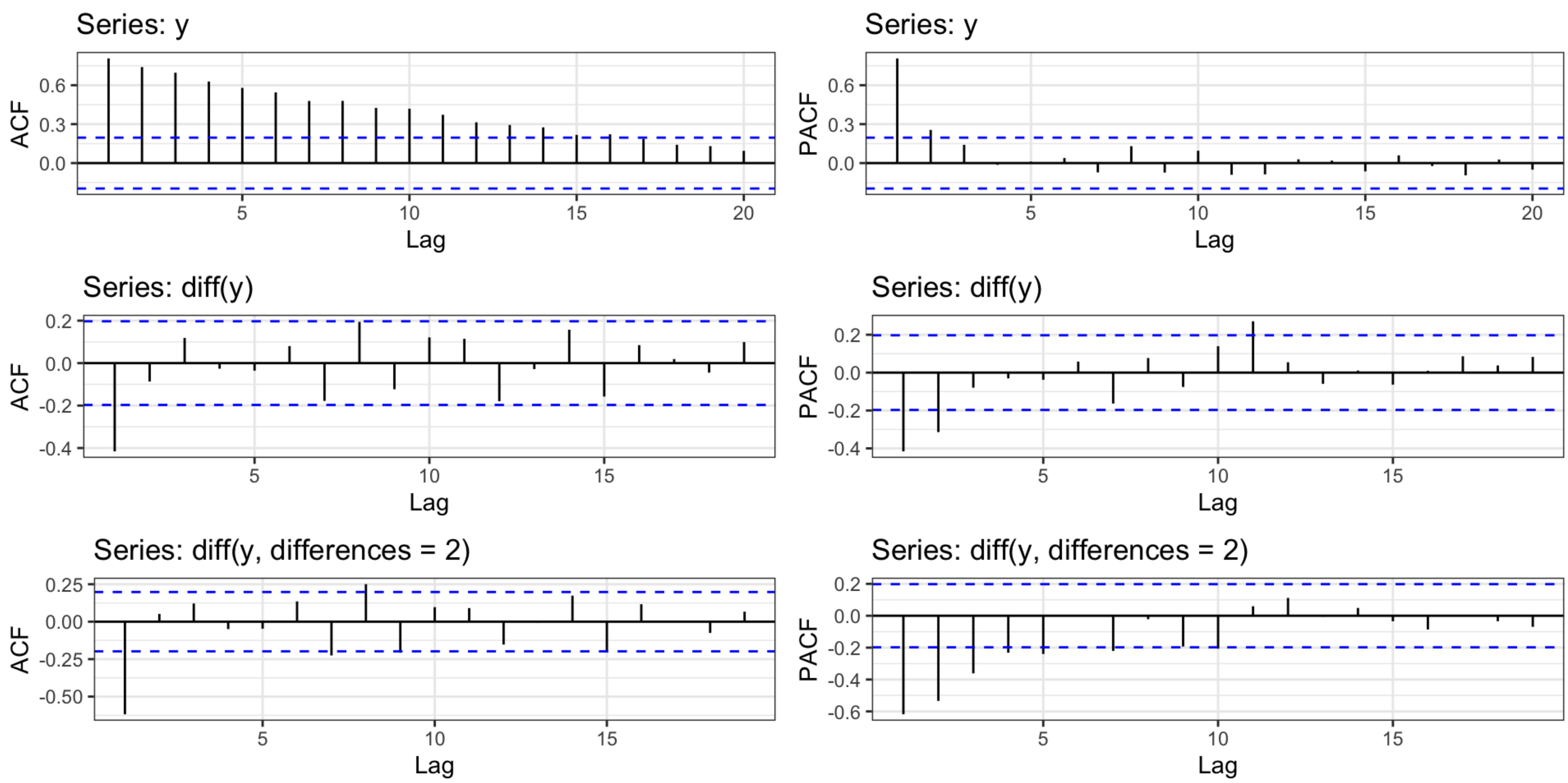
1st Difference



2nd Difference



Differencing - ACF



AR Models

AR(1)

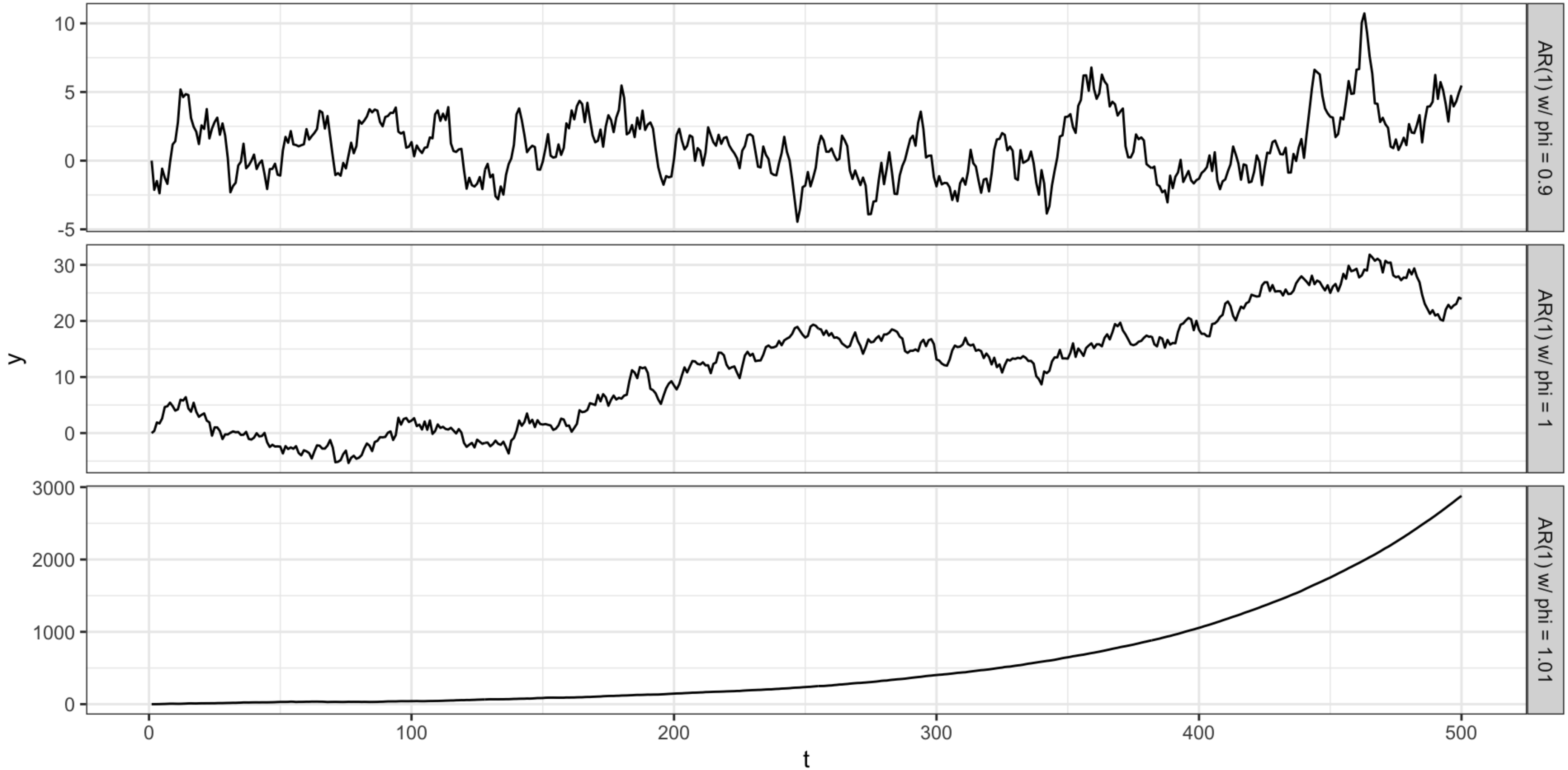
Last time we mentioned a random walk with trend process where $y_t = \delta + y_{t-1} + w_t$.

The AR(1) process is a generalization of this where we include a coefficient in front of the y_{t-1} term.

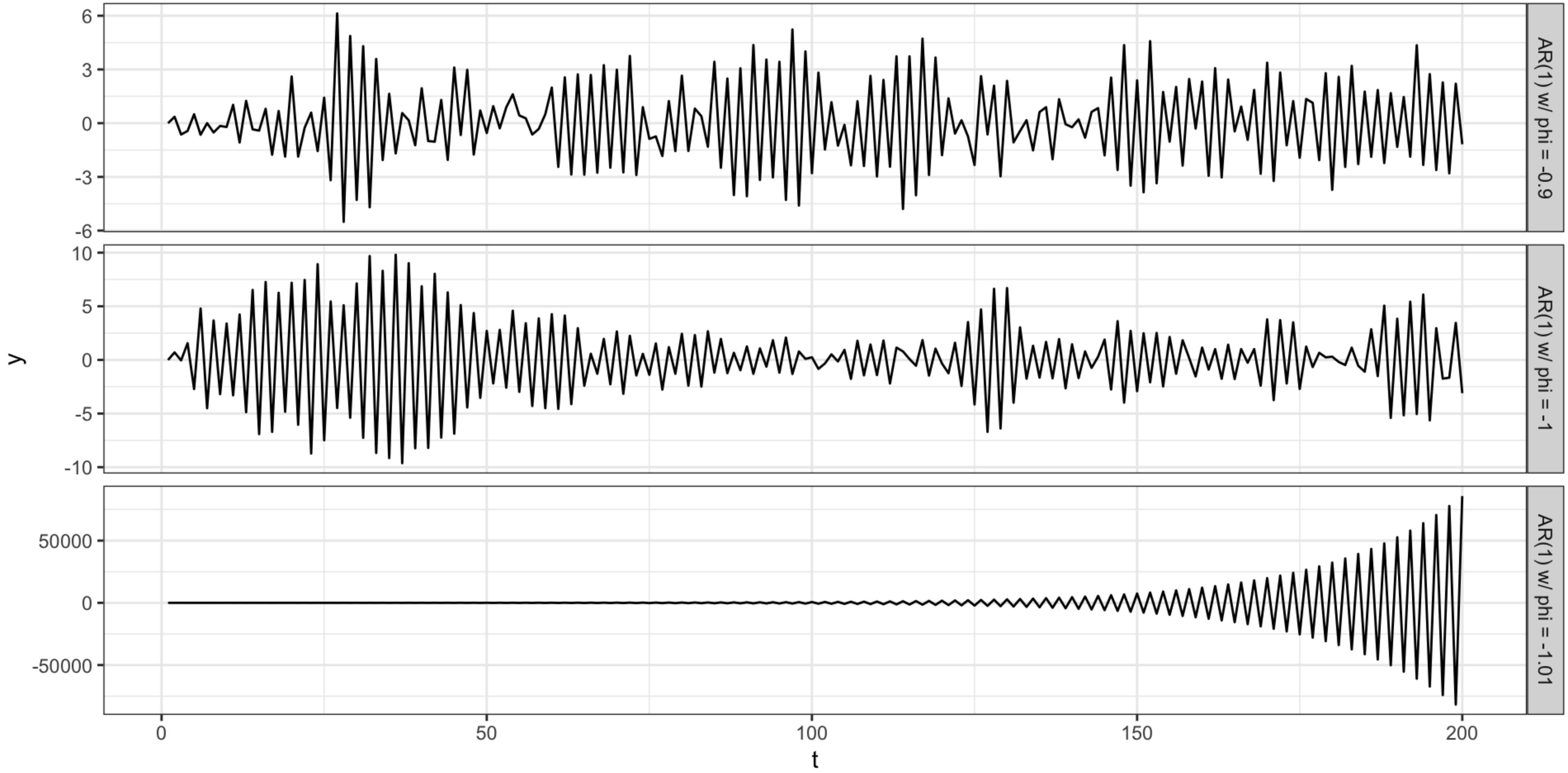
$$\text{AR}(1): \quad y_t = \delta + \phi\, y_{t-1} + w_t$$

$$w_t \sim N(0, \sigma^2_w)$$

AR(1) - Positive ϕ



AR(1) - Negative ϕ



Stationarity of $\text{AR}(1)$ processes

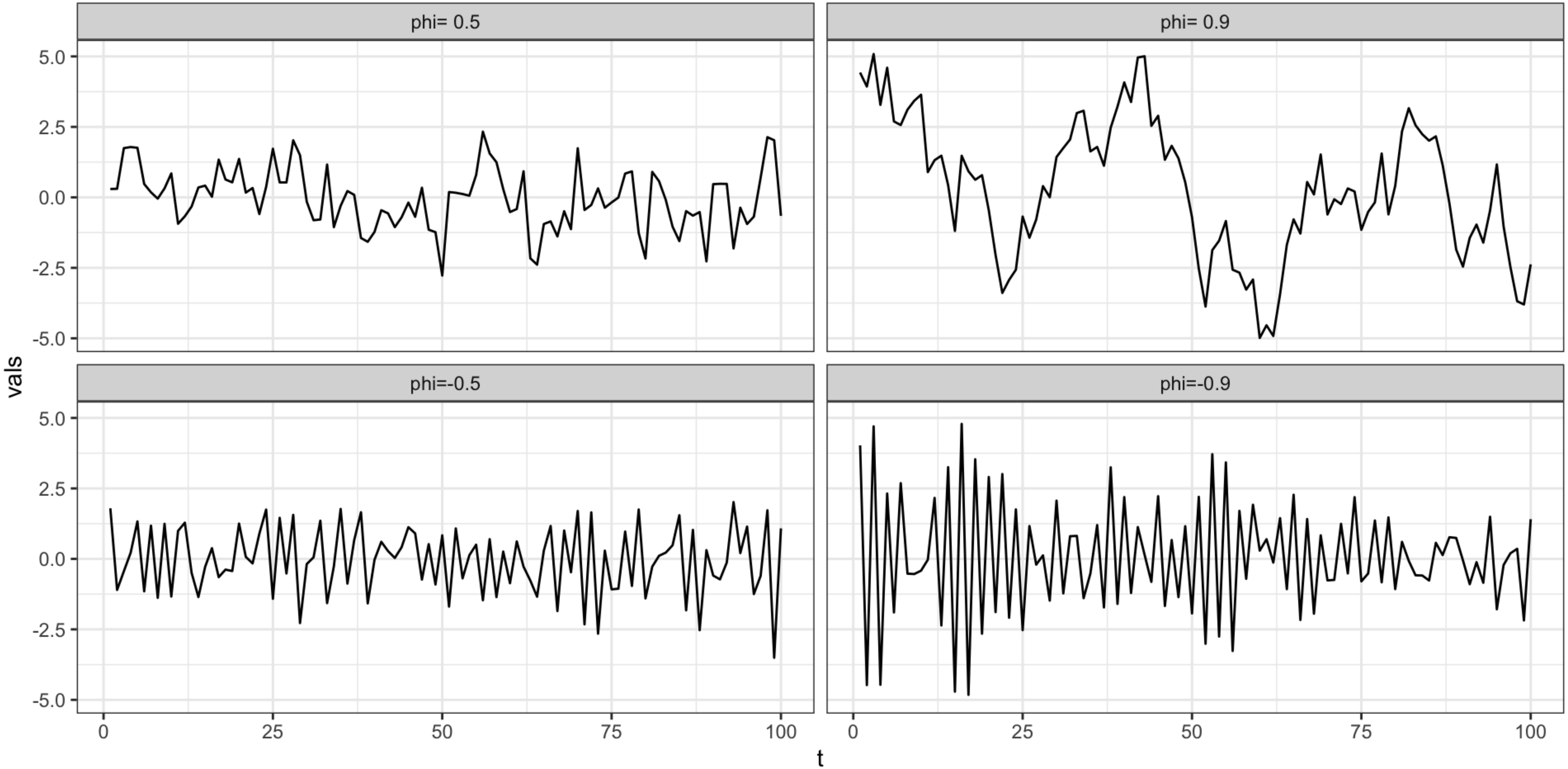
Lets rewrite the AR(1) without any autoregressive terms

Stationarity of $\text{AR}(1)$ processes

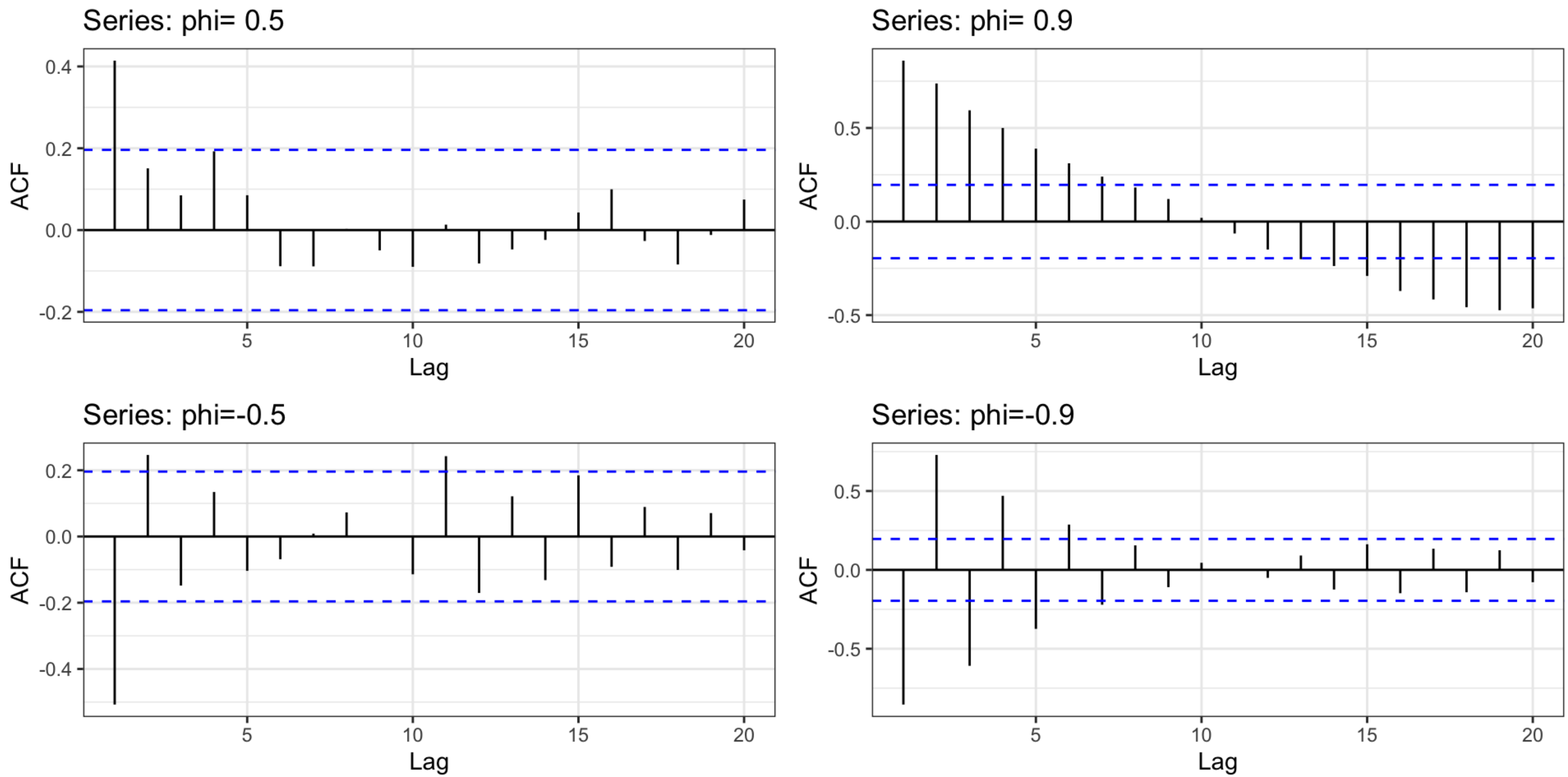
Under what conditions will an AR(1) process be stationary?

Properties of a stationary $\mathrm{AR}(1)$ process

Identifying AR(1) Processes



Identifying AR(1) Processes - ACFs



Identifying AR(1) Processes - PACFs