

Prediction

Let $\vec{y} = \{y_1, \dots, y_n\}$ be n outcomes from a pop'n.

Let \tilde{y} be an additional outcome from the same pop'n.

Predict \tilde{y} : "posterior predictive distr."

$$p(\tilde{y} | \vec{y}) = \int p(\tilde{y}, \theta | \vec{y}) d\theta$$

"marginalization"
"law of total prob"

$$= \int \underbrace{p(\tilde{y} | \theta, \vec{y})}_{\text{if conditionally independent}} p(\theta | \vec{y}) d\theta$$

$$p(\tilde{y} | \vec{y}) = \int p(\tilde{y} | \theta) p(\theta | \vec{y}) d\theta$$

Remember : \tilde{y} is just a #. We view $p(\tilde{y} | \theta)$ as a function of θ .

If I say $p(\tilde{y} | \theta) = g(\theta)$ then I have:

$$\int g(\theta) p(\theta | \vec{y}) d\theta$$

posterior expectation!

$$\mathbb{E} g(\theta) | y$$