✳ Today's punchline:

Assume:

① $Y_i \mid \theta, \sigma^2 \sim N(\theta, \sigma^2)$

② $\theta \mid \sigma^2 \sim N(\mu_0, \sigma^2/\kappa_0)$

③ $1/\sigma^2 \sim \text{gamma}\left(\frac{\nu_0}{2}, \frac{\nu_0}{2}\sigma_0^2\right)$

then the posterior

$$p(\theta, \sigma^2 \mid y_1, \ldots, y_n)$$

$$= \underbrace{p(\theta \mid \sigma^2, y_1, \ldots, y_n)}_{\downarrow} \; \underbrace{p(\sigma^2 \mid y_1, \ldots, y_n)}_{\downarrow}$$

$$\text{dnorm}\left(\theta; \mu_n, \tau_n\right) \quad \text{dinv gamma}\left(\sigma^2; \frac{\nu_n}{2}, \frac{\nu_n}{2}\sigma_n^2\right)$$

"full cond'l posterior of θ"

Today's agenda:

(1) sketch proof for $p(\sigma^2 \mid y_1, \ldots, y_n)$

(2) sample from joint posterior

(3) sample from posterior predictive $p(\tilde{y} \mid y_1, \ldots, y_n)$
   ↰ (time permitting)

Interpretation:

μ_0 : prior guess for θ

σ_0^2 : prior guess for σ^2

κ_0 : prior sample size for θ

ν_0 : prior sample size for ν_0