W-	
7	Today's punchline:
	Assume:
	1, 10,02 ~N(0,02)
(3)	0 102 ~ N(NO, 02/KD)
(3)	$Y, \theta, \sigma^2 \sim N(\theta, \sigma^2)$ $0 \sigma^2 \sim N(\mu_0, \sigma^2/\kappa_0)$ $1/\sigma^2 \sim gamma(\frac{\nu_0}{2}, \frac{\nu_0 \sigma^2}{2})$
	then the posterior
	p(0,02 /y,yn)
	= p(0/02, y1,, yn) p(02/y,,, yn)
	dnorm(0; Mn, In) diny gamma (02; Vn, Vn on2)
	"Gul and the Man, En ding gamma of 2, 2 on
	"full cond'l posterior of 0"
	Today's agenda:
	agenan.
	(i) sketch proof for offely
	(1) sketch proof for p(02/y,, yn) (2) sample from 1 posterior
	(3) sample from posterior predictive p(g) y,,yn)
	e (time permitting)
) '
	Interpretation:
	Mo: prior gress for A
	502: prior guess for 52
	Ko = prive sample size for O
	Vo : prior sample size for 62
Sales in the first of the sales	

(1) Y:10,02 ~N(0,02) Assume (2) 0102 ~N(MO,021KO) 3) 1/02 ~ gamma (vo/2, vooo2) then the joint posterior $p(\theta, \sigma^2 | \vec{y}) = p(\theta | \sigma^2, \vec{y}) p(\sigma^2 | \vec{y})$ = dnorm(B, Mn, Tn). dinvgamma(02, Vn, Vn on2) / Shown 3 Good: to prove 02/9 ~ invgamma Sketch proof: p(02 19) x p(9102) p(02) × p(02) \ p(\forall , \theta | \sigma^2) dt (t)

Sketch
$$\rho^{007}$$
.

 $\rho(\sigma^2 | \vec{y}) \propto \rho(\vec{y} | \sigma^2) \rho(\sigma^2)$
 $\propto \rho(\sigma^2) \int \rho(\vec{y}, \theta | \sigma^2) d\theta$
 $\propto \rho(\sigma^2) \int \rho(\vec{y} | \theta, \sigma^2) \rho(\theta | \sigma^2) d\theta$ (†)

Known by assumption:

(3)

(1)

(2)

The integral (t) above reduces to integrating a normal density.

(1) collect (but do not absorb into normaliting const!) terms

(1) collect (but do not absorb into normalizing const.) Term will will the integral:

who the outside integral:

$$(2\pi\sigma^2)^{-n/2} \cdot (2\pi\sigma^2/\kappa_0)^{-1/2} \cdot \exp\left(\frac{1}{2}\sigma_2(zy_1^2) - \frac{\kappa_0}{2\sigma_2}(\mu_0)^2\right)^{-1/2} \cdot \exp\left(\frac{1}{2}\sigma_2(zy_1^2) - \frac{\kappa_0}{2\sigma_2}(\mu_0)^2\right)^{-1/2} \cdot \exp\left(\frac{1}{2}\sigma_2(-2\mu_0\theta + \theta^2)\right) \cdot \det\left(\frac{1}{2}\sigma_2(-2\mu_0\theta +$$

$$\frac{(2\pi\sigma^{2})^{-1/2} \cdot (2\pi\sigma^{2}/K_{0})}{\int e^{\chi} P_{2}^{2} - \frac{1}{2\sigma^{2}} (-2(zy_{i})\theta + n\theta^{2}) - \frac{K_{0}}{2\sigma^{2}} (-2\mu_{0}\theta + \theta^{2})^{2} d\theta}$$

Compute the integral above using the kernel trick: hint: it's the nevnel of a normal, you must complete the square. (2)

nint: it's the nernel of a normal,
$$\frac{1}{\sqrt{2}}$$
 signal.

$$\int e^{-\frac{1}{2}a\theta^{2} + b\theta} d\theta = \left(2\pi/a\right)^{1/2} \exp\left\{\frac{1}{2}b^{2}/a\right\} = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}b^{2}\right)^{1/2}$$

where $a = \left(\frac{n}{\sqrt{2}} + \frac{\kappa_{0}}{\sqrt{2}}\right)$
 $b = \left(\frac{ny + \kappa_{0}}{\sqrt{2}}M_{0}\right)$

$$b = \left(\frac{ny + K_0 \mu_0}{\sigma^2}\right)$$

(3) Return to (t) and write down:

te down:

$$p(\sigma^2|\vec{y}) \propto p(\sigma^2) - C_1(\sigma^2) - C_2(\sigma^2)$$