Lecture 6: Generating random variables – transformations

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Logistics

- Nice work on HW 1!
- Extra office hours today, 12pm - 1pm and 2pm - 4pm
- Upcoming due dates:
  - HW 2: Friday 10am
  - HW 1 resubmissions: next Monday (February 3)
  - Challenge 1 (command line): next Friday (Feb 7) 10am
- New challenges released:
  - Challenge 2: Mersenne Twister (due Feb 28)
  - Challenge 3: Generating Poisson random variables (due Feb 28)
- Remember: you do not have to do all challenges! Pick and choose ones of interest

Previously

Methods to generate $U \sim \mathit{Uniform}(0, 1)$:

- Linear congruential generator
- Mersenne twister
- lots of other variants and alternatives

Now we want to generate random variables with *other* distributions.

Question: If I have a uniform random variable, how can I get other random variables?

Inverse transform method

Example

Suppose we want to generate $X \sim \mathit{Exponential}(\theta)$

Example

Generating $X \sim \textit{Exponential}(1)$:

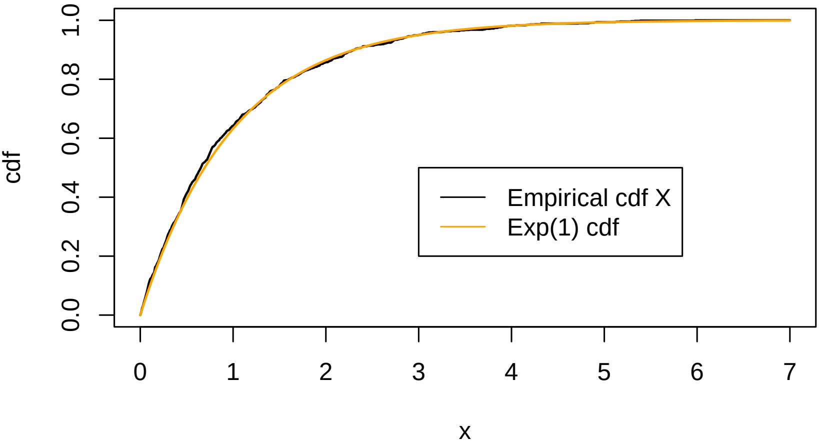
```r
# generate 1000 samples
u <- runif(1000)
x <- -log(u)
```

Question: How can I check that the samples match the desired *Exponential*(1) distribution?

Example

Generating $X \sim \textit{Exponential}(1)$:

```r
# generate 1000 samples
u <- runif(1000)
x <- -log(u)
```



Discrete case

Suppose that we want to generate $X \sim \mathit{Bernoulli}(p)$

Generating a Normal random variable

Suppose we want to simulate $X \sim N(0,1)$

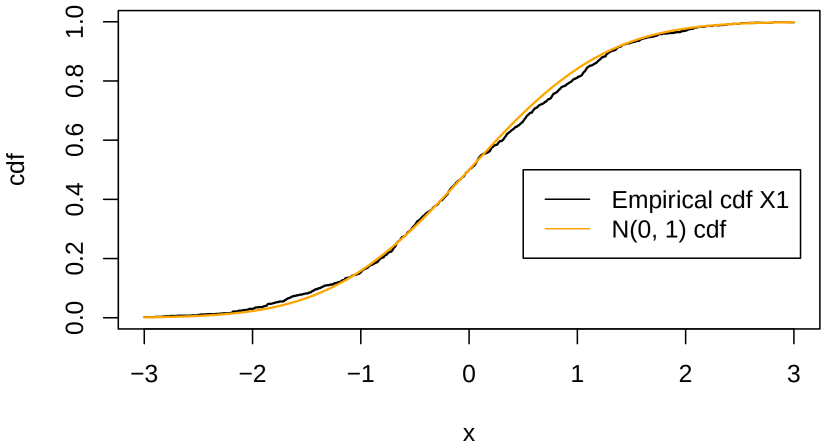
$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}x^2\right\}$$

$$F_X(t) = \, ?$$

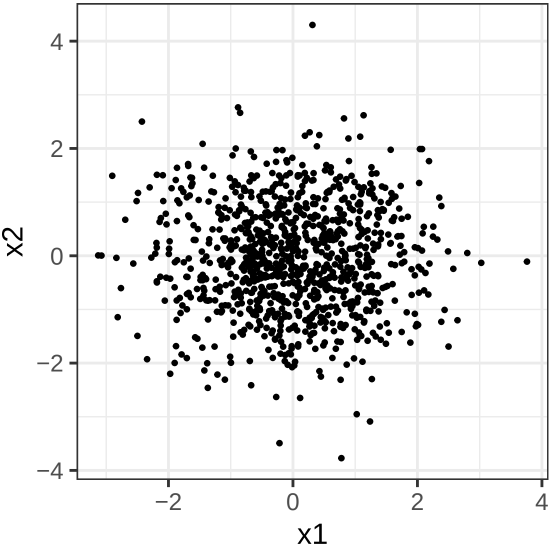
Box-Muller Transformation

Box-Muller in practice

```
# generate 1000 samples
u1 <- runif(1000)
u2 <- runif(1000)
x1 <- sqrt(-2*log(u1)) * cos(2*pi*u2)
x2 <- sqrt(-2*log(u1)) * sin(2*pi*u2)
```



Box-Muller in practice



Other Normals

Suppose that $Z \sim N(0, 1)$. How do I get $X \sim N(\mu, \sigma^2)$?

A few other transformations

- If $X \sim N(\mu, \sigma^2)$, then $e^X \sim Lognormal(\mu, \sigma^2)$
- If $Z_1, ..., Z_k \overset{iid}{\sim} N(0, 1)$, then

$$\sum_{i=1}^{k} Z_i^2 \sim ?$$

- If $V_1 \sim \chi^2_{d_1}$ and $V_2 \sim \chi^2_{d_2}$ are independent, then

$$\frac{V_1/d_1}{V_2/d_2} \sim ?$$

Your turn

Practice questions on the course website:

https://sta379-s25.github.io/practice_questions/pq_6.html

- Practice with inverse transform method
- Start in class. You are welcome to work with others
- Practice questions are to help you practice. They are not submitted and not graded
- Solutions are posted on the course website