Lecture 1: Course Overview

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“Statistical computing” vs. “computational statistics”

- **Statistical computing**: programming languages and computing tools for working with statistics (think: storing and accessing data, data transformations and wrangling, some data visualization, etc.)

- **Computational statistics**: the use of computational algorithms to implement statistical methods (think: simulating data from a distribution, fitting a regression model, performing hypothesis testing)

Key points

- Focus on *how* to statistical methods work. E.g., how do we fit a logistic regression model? What is R actually doing "under the hood"?
- Focus on *implementation* – much of your work will be turning statistical methods and algorithms into code
- Focus on efficiency
  - Efficient algorithms (e.g. dynamic programming approaches)
  - Efficient approximations (e.g. integral and Hessian approximations)
  - Efficient languages (particularly the use of C++)
- Focus on iteration

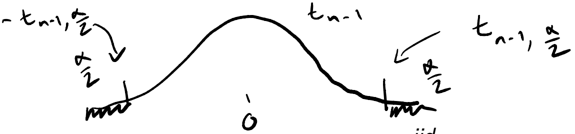
Tentative course outline

- Simulation (simulation studies, generating random numbers, simulating from a distribution)
- Model fitting (linear and generalized linear models, maximum likelihood, Newton and quasi-Newton methods)
- Missing data and EM algorithm (Gaussian mixtures, Hidden Markov Models)
- Integration (numerical integration, Monte Carlo integration, MCMC)
- Bootstrapping

Roadmap for the first few weeks

- Motivation: simulation studies
  - Today: using simulations to answer questions about hypothesis tests
- Simulation: behind the scenes
  - Generating random numbers
  - Transformation methods
  - Acceptance/rejection sampling
  - Computing topics: iteration, functions, C++
  - Simulation for linear models and multivariate normal distributions
- Following unit: how do you actually fit a regression model? (linear, GLMs, GEEs, etc.)

Recap



Suppose we have a sample $X_1, ..., X_n \overset{iid}{\sim} N(\mu, 1)$. Want to test the hypotheses

$$H_0 : \mu = 0 \quad H_A : \mu \neq 0$$

How do I test these hypotheses? (What is my test statistic, and how do I make a decision?)

$z = \dfrac{\bar{x} - 0}{1/\sqrt{n}}$ (assuming we know sd = 1)

$t = \dfrac{\bar{x} - 0}{s/\sqrt{n}}$ (we don't know sd)

$t \sim t_{n-1}$

reject when $|t|$ (or $|z|$) is large

equivalently, calculate p-value, reject if $p < \alpha$

Recap

Suppose we have a sample $X_1, ..., X_n \overset{iid}{\sim} N(\mu, 1)$. Want to test the hypotheses

$$H_0 : \mu = 0 \quad H_A : \mu \neq 0$$

If actually $\mu = 0$, do we want to reject H_0, or fail to reject?

H_0 is true

$\Rightarrow$ want to fail to reject!

Recap

- Either H_0 is true or false (usually we don't know)
- We either reject or fail to reject H_0

Possible outcomes of a hypothesis test:

	H_0 is true	H_0 is false
fail to reject H_0	✓	type II error (false negative)
reject H_0	type I error (false positive)	✓

Recap

	H_0 is true	H_0 is false
fail to reject	correct decision	type II error
reject	type I error	correct decision

Usual goal: Minimize type II error, subject to control of type I error rate (i.e. want $\underbrace{P(\text{reject } H_0 | H_0 \text{ true})}_{\text{type I error rate}} <= \alpha$)

pre-specified significance level

In R

true mean μ

```r
mu_x <- 0
n <- 20
x <- rnorm(n, mean=mu_x, sd=1)
x
```

sample size n

$x_1,\dots,x_n \sim N(\mu,1)$

vector

```
##  [1]  0.6797336 -0.8548404 -1.3210164 -0.4273575 -1.0207
##  [7] -0.4315816 -0.3921952  1.0212251 -0.1325580 -1.0810
## [13]  0.7787329  0.5623875  0.1579840  2.0061821  0.1557
## [19] -0.3435500 -0.2215555
```

```r
t.test(x, alternative="two.sided", mu=0)
```

$H_A: \mu \neq 0$

p-value

```
##
##  One Sample t-test
##
## data:  x
## t = 0.33834, df = 19, p-value = 0.7388
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
```

In R

type I error rate = P(reject H_0 | H_0 is true)

```
t.test(x, alternative="two.sided", mu=0)
```

```
##
##  One Sample t-test
##
## data:  x
## t = 0.33834, df = 19, p-value = 0.7388
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.3297764  0.4569528
## sample estimates:
##  mean of x
## 0.06358819
```

Here we fail to reject H_0. Does this give us our type I error rate?

In R

```r
t.test(x, alternative="two.sided", mu=0)
```

```
##
##  One Sample t-test
##
## data:  x
## t = 0.33834, df = 19, p-value = 0.7388
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.3297764  0.4569528
## sample estimates:
##  mean of x
## 0.06358819
```

Here we fail to reject H_0. Does this give us our type I error rate?

No! Type I error rate $= P(\text{reject } H_0 | H_0 \text{ true})$. Need more than one observation to estimate a probability.

Repeating many times

set a seed for reproducibility

```r
set.seed(379)
n <- 20
mu_x <- 0
nsim <- 1000  # number of times to repeat
test_results <- rep(NA, nsim)  # vector to store the test

for(i in 1:nsim){
  x <- rnorm(n, mean=mu_x, sd=1)
  test_results[i] <- t.test(x, alternative="two.sided", mu=
}

head(test_results)
```

sample size

$\mu = 0$

store results

taking a sample

store in t-test result (p-value)

```
## [1] 0.8213916 0.2525886 0.3818531 0.3760550 0.7342035 0.
```

What is this code doing?

Repeating many times

```
set.seed(379)
n <- 20
mu_x <- 0
nsim <- 1000  # number of times to repeat
test_results <- rep(NA, nsim)  # vector to store the test r

for(i in 1:nsim){
  x <- rnorm(n, mean=mu_x, sd=1)
  test_results[i] <- t.test(x, alternative="two.sided", mu=
}

head(test_results)
```

```
## [1] 0.8213916 0.2525886 0.3818531 0.3760550 0.7342035 0.
```

How do I find the fraction of times we rejected H_0? (use $\alpha = 0.05$)

Repeating many times

```
set.seed(379)
n <- 20
mu_x <- 0
nsim <- 1000  # number of times to repeat
test_results <- rep(NA, nsim)  # vector to store the test r

for(i in 1:nsim){
  x <- rnorm(n, mean=mu_x, sd=1)
  test_results[i] <- t.test(x, alternative="two.sided", mu=
}

mean(test_results < 0.05)
```

```
## [1] 0.045
```

alternative:

$$\frac{\text{sum}(\cdots)}{n}$$

Your turn: Practice questions

Practice questions on the course website:

https://sta379-s25.github.io/practice_questions/pq_1.html

- Experiment with changing μ and n. How does the probability of rejecting change?
- Start in class. You are welcome to work with others
- Practice questions are to help you practice. They are not submitted and not graded
- Solutions are posted on the course website