

Lecture 6: Generating random variables – transformations

Ciaran Evans

Previously

Methods to generate $U \sim \text{Uniform}(0, 1)$:

- ▶ Linear congruential generator
- ▶ Mersenne twister
- ▶ lots of other variants and alternatives

Now we want to generate random variables with *other* distributions.

Question: If I have a uniform random variable, how can I get other random variables?

Inverse transform method

Example

Suppose we want to generate $X \sim \text{Exponential}(\theta)$

Example

Generating $X \sim \text{Exponential}(1)$:

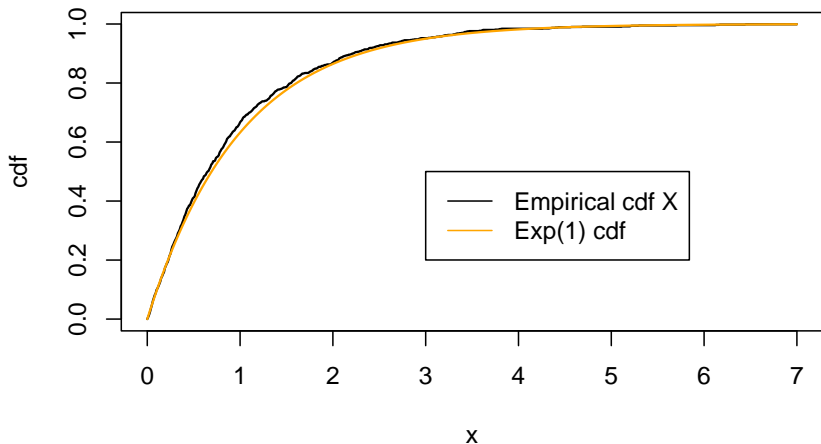
```
# generate 1000 samples  
u <- runif(1000)  
x <- -log(u)
```

Question: How can I check that the samples match the desired $\text{Exponential}(1)$ distribution?

Example

Generating $X \sim \text{Exponential}(1)$:

```
# generate 1000 samples  
u <- runif(1000)  
x <- -log(u)
```



Discrete case

Suppose that we want to generate $X \sim \text{Bernoulli}(p)$

Generating a Normal random variable

Suppose we want to simulate $X \sim N(0, 1)$

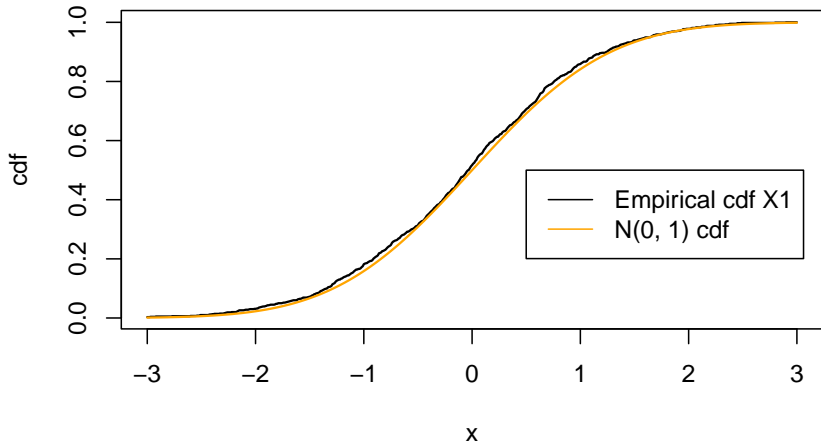
$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{1}{2}x^2 \right\}$$

$$F_X(t) = ?$$

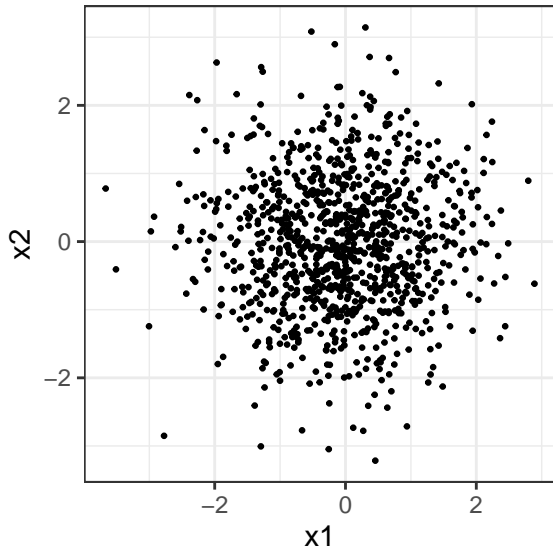
Box-Muller Transformation

Box-Muller in practice

```
# generate 1000 samples  
u1 <- runif(1000)  
u2 <- runif(1000)  
x1 <- sqrt(-2*log(u1)) * cos(2*pi*u2)  
x2 <- sqrt(-2*log(u1)) * sin(2*pi*u2)
```



Box-Muller in practice



Other Normals

Suppose that $Z \sim N(0, 1)$. How do I get $X \sim N(\mu, \sigma^2)$?

A few other transformations

- ▶ If $X \sim N(\mu, \sigma^2)$, then $e^X \sim \text{Lognormal}(\mu, \sigma^2)$
- ▶ If $Z_1, \dots, Z_k \stackrel{iid}{\sim} N(0, 1)$, then

$$\sum_{i=1}^k Z_i^2 \sim ?$$

- ▶ If $V_1 \sim \chi_{d_1}^2$ and $V_2 \sim \chi_{d_2}^2$ are independent, then

$$\frac{V_1/d_1}{V_2/d_2} \sim ?$$

Your turn

Practice questions on the course website:

https://sta379-s25.github.io/practice_questions/pq_6.html

- ▶ Practice with inverse transform method
- ▶ Start in class. You are welcome to work with others
- ▶ Practice questions are to help you practice. They are not submitted and not graded
- ▶ Solutions are posted on the course website