# Lecture 15: Gradient descent – visualization

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# Last time

Gradient descent: $\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} - \alpha_k \nabla f(\mathbf{x}^{(k)})$

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- How do we choose step size $\alpha_k$?

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- Why the gradient? $-\nabla f(\mathbf{x})$ is the **direction of steepest decrease** in $f$

- How do we choose step size $\alpha_k$? **maximize benefit**
  - Would like $\alpha_k$ to minimize $f(\mathbf{x}^{(k)} - \alpha \nabla f(\mathbf{x}^{(k)}))$
  - Exact minimization is expensive and unnecessary; instead, find "good enough" $\alpha_k$ with backtracking line search

# Today: behavior of gradient descent

**Motivating example:** Data on med school admissions for 55 students

- `GPA`: student's undergraduate GPA
- `MCAT`: student's MCAT score

Function to minimize:

$$f(\beta_0, \beta_1) = \sum_{i=1}^{n} (\text{MCAT}_i - \beta_0 - \beta_1 \text{GPA}_i)^2$$

- Compass search beginning at (0, 0): 5282 iterations
- Gradient descent with backtracking linear search beginning at (0, 0): 6517 iterations

**Motivating question:** Why so many iterations?

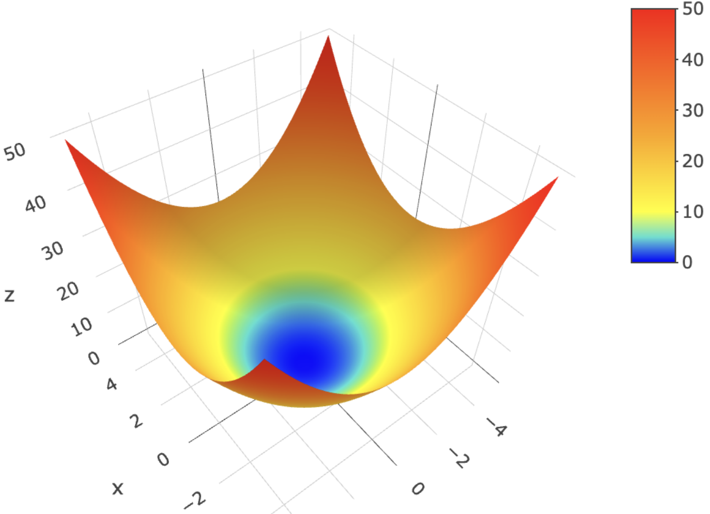
# Preliminaries: plotting a function of two variables

$$f(x, y) = x^2 + y^2$$

- Two inputs: $x$ and $y$
- One output value for each $(x, y)$: $z = f(x, y)$
- In total: function described by 3 coordinates, $(x, y, z)$

# Preliminaries: plotting a function of two variables

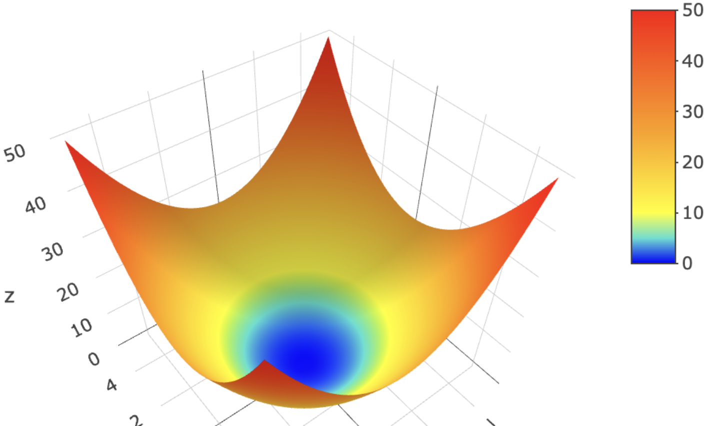
$$f(x, y) = x^2 + y^2$$



# Preliminaries: plotting a function of two variables

$$f(x, y) = x^2 + y^2$$

**Question:** What will happen to the shape if we instead plot $5x^2 + 0.5y^2$?



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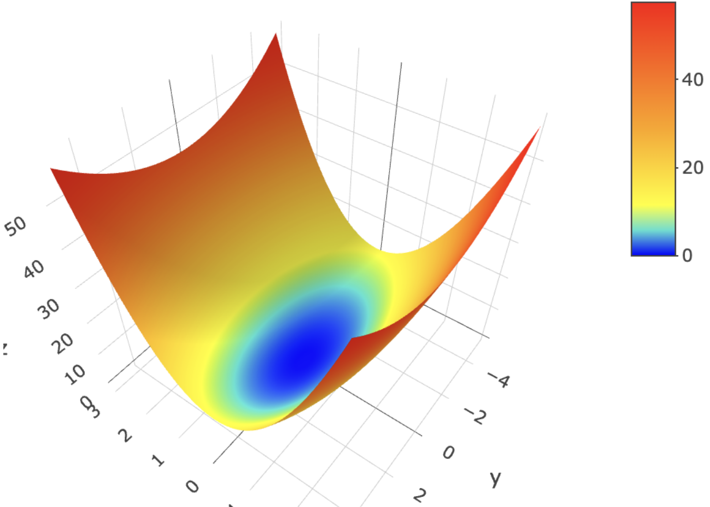
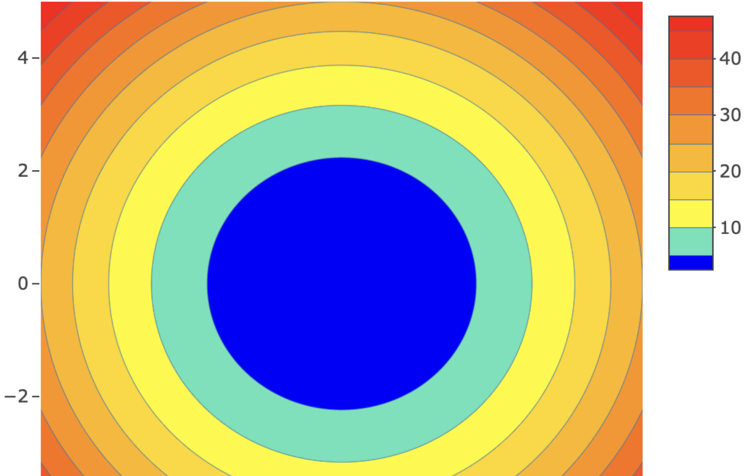


# Preliminaries: contour plots

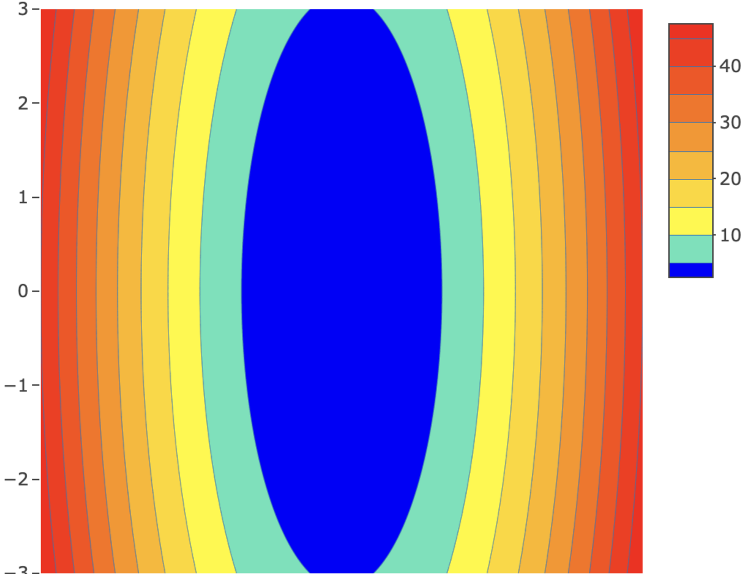
**Definition:** The *contour* of $f(x, y)$ at $c$ is the set of all points $(x, y)$ for which $f(x, y) = c$

Example: $f(x, y) = x^2 + y^2$



# Preliminaries: contour plots

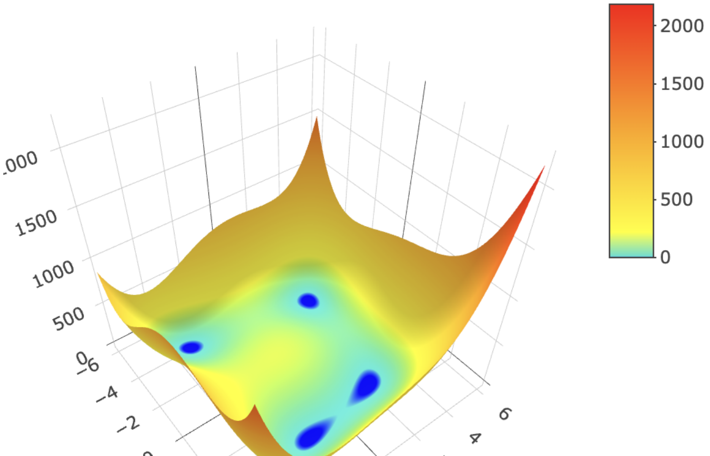
Example: $f(x, y) = 5x^2 + 0.5y^2$



# Another example

$$f(x, y) = (x^2 + y - 11)^2 + (x + y^2 - 7)^2$$

**Question:** What do you notice about this function?



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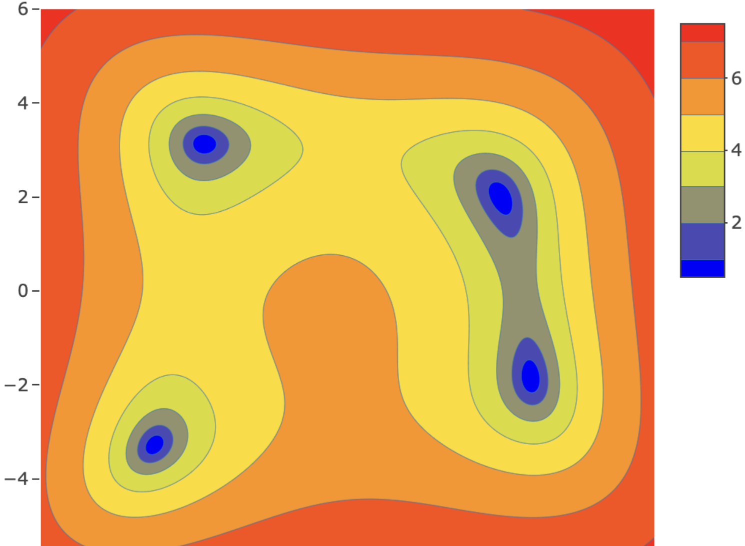

# Activity

Activity on the course website:

https://sta379-s25.github.io/practice_questions/pq_15.html

- Visualize shape of loss function for regression problem
- Explore how shape of loss function impacts performance of gradient descent