Lecture 8: Acceptance-rejection sampling continued

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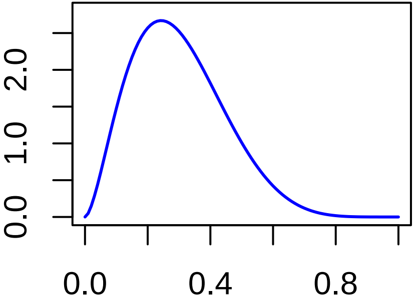
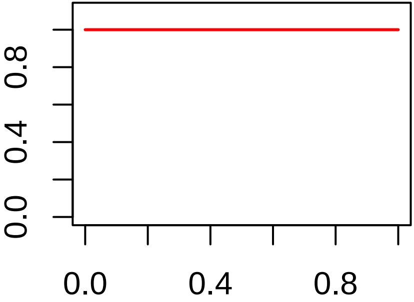
Acceptance-rejection sampling

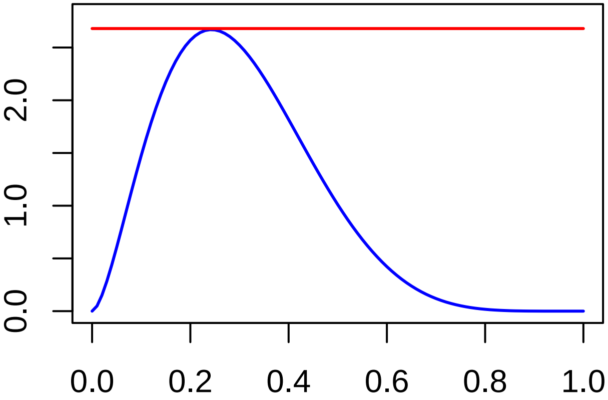
Suppose we want to simulate from this Beta distribution:



We can *get* samples from this uniform distribution:



Acceptance-rejection sampling



Acceptance-rejection sampling

- Want to sample continuous r.v. $X \sim f$
- Can easily sample from a different density: $Y \sim g$, such that

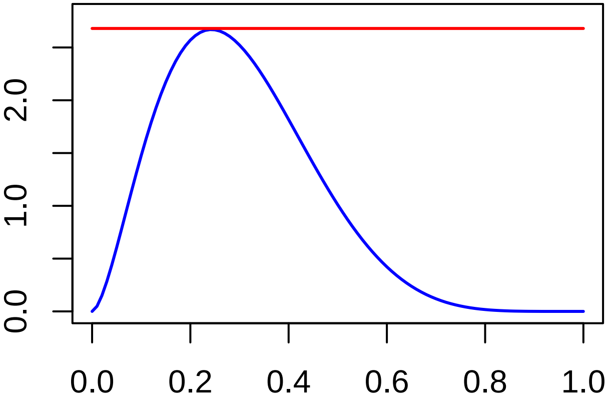
$$\frac{f(t)}{g(t)} \leq c \qquad \text{for all } t \text{ where } f(t) > 0$$

Do the following:

1. Sample $Y \sim g$
2. Sample $U \sim Uniform(0, 1)$
3. If $U \leq \frac{f(Y)}{cg(Y)}$, set $X = Y$. Otherwise, return to step 1.

Illustration

- $Y \sim g$ and $U \sim Uniform(0, 1)$
- *Accept* Y if $U \leq \frac{f(Y)}{cg(Y)}$



Finding c

Acceptance-rejection sampling requires that

$$\frac{f(t)}{g(t)} \leq c \qquad \text{for all } t \text{ where } f(t) > 0$$

So,

$$c = \max_{t : f(t) > 0} \frac{f(t)}{g(t)}$$

Finding c: example

Example from last time:

- $f(t) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} t^{\alpha-1}(1-t)^{\beta-1}$
- $g(t) = 1$

$$c = \max_{t:f(t)>0} \frac{f(t)}{g(t)}$$

Why does acceptance-rejection sampling work?

Your turn

Practice questions on the course website:

https://sta379-s25.github.io/practice_questions/pq_8.html

- Implement acceptance-rejection sampling for the beta example
- Start in class. You are welcome to work with others
- Practice questions are to help you practice. They are not submitted and not graded
- Solutions are posted on the course website
- If done early, start on HW 3

Next time:

- Challenges with acceptance-rejection sampling
- Other transformation methods to generate random variables