Warmup: antithetic sampling

Group members:

Monte Carlo integration

Suppose we wish to approximate the integral

$$\theta = \int_0^1 e^x dx = \mathbb{E}[g(U)]$$

where $U \sim Uniform(0,1)$ and $g(x) = e^x$.

Simple Monte Carlo estimate

The simple Monte Carlo estimate does the following:

1. Sample $U_1, ..., U_n \overset{iid}{\sim} Uniform(0,1)$
2. $\widehat{\theta}_{MC} = \frac{1}{n} \sum\limits_{i=1}^{n} g(U_i)$

Here is code to approximate the variance of the simple MC estimate with $n = 1000$:

```
g <- function(x){
  exp(x)
}

n <- 1000
nsim <- 1000

theta_hat_mc <- rep(NA, nsim)
for(i in 1:nsim){
  u <- runif(n)
  theta_hat_mc[i] <- mean(g(u))
}

var(theta_hat_mc)
```

Antithetic sampling

One method for reducing the variance of a Monte Carlo estimate is with **antithetic sampling**:

- Sample $U_1, ..., U_{n/2} \sim Uniform(0,1)$
- $\widehat{\theta}_{AS} = \frac{1}{n} \sum\limits_{i=1}^{n/2} (g(U_i) + g(1 - U_i))$

Questions

1. Using R, approximate the variance of the antithetic estimator $\widehat{\theta}_{AS}$ with $n = 1000$, and compute the percent reduction in variance compared to the simple estimator $\widehat{\theta}_{MC}$.

2. Using R, approximate $\rho = Cor(g(U), g(1 - U))$ and compare to the percent reduction in variance from question 1.