Lecture 6: Generating random variables – transformations
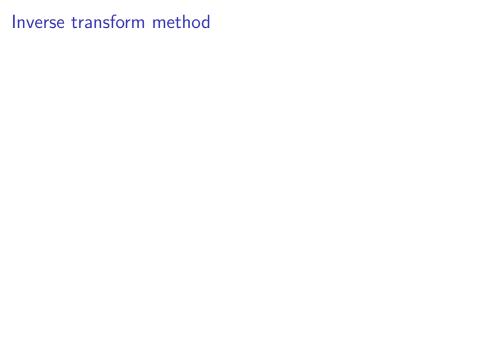
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Previously

Methods to generate $U \sim Uniform(0, 1)$:

- ▶ Linear congruential generator
- ▶ Mersenne twister
- ▶ lots of other variants and alternatives

Now we want to generate random variables with *other* distributions.

Question: If I have a uniform random variable, how can I get other random variables?

Inverse transform method

Example

Suppose we want to generate $X \sim Exponential(\theta)$

Discrete case

Suppose that we want to generate $X \sim Bernoulli(p)$

Generating a Normal random variable

Suppose we want to simulate $X \sim N(0, 1)$

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}x^2\right\}$$

$$F_X(t) = ?$$

Box-Muller Transformation

Other Normals

Suppose that $Z \sim N(0, 1)$. How do I get $X \sim N(\mu, \sigma^2)$?

A few other transformations

- If $X \sim N(\mu, \sigma^2)$, then $e^X \sim Lognormal(\mu, \sigma^2)$
- If $Z_1, ..., Z_k \overset{iid}{\sim} N(0, 1)$, then

$$\sum_{i=1}^{k} Z_i^2 \sim ?$$

- If $V_1 \sim \chi_{d_1}^2$ and $V_2 \sim \chi_{d_2}^2$ are independent, then

$$\frac{V_1/d_1}{V_2/d_2} \sim ?$$

Your turn

Practice questions on the course website:

https://sta379-s25.github.io/practice_questions/pq_6.html

- ▶ Practice generating random variables
- ▶ Start in class. You are welcome to work with others
- ▶ Practice questions are to help you practice. They are not submitted and not graded
- ▶ Solutions are posted on the course website