patsy + statsmodels

Lecture 18

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patsy

patsy

> `patsy` is a Python package for describing statistical models (especially linear models, or models that have a linear component) and building design matrices. It is closely inspired by and compatible with the formula mini-language used in R and S.
> …
> Patsy's goal is to become the standard high-level interface to describing statistical models in Python, regardless of what particular model or library is being used underneath.

Formulas

```python
from patsy import ModelDesc
```

```python
ModelDesc.from_formula("y ~ a + a:b + np.log(x)")
```

```
ModelDesc(lhs_termlist=[Term([EvalFactor('y')])],
          rhs_termlist=[Term([]),
                        Term([EvalFactor('a')]),
                        Term([EvalFactor('a'), EvalFactor('b')]),
                        Term([EvalFactor('np.log(x)')])])
```

```python
ModelDesc.from_formula("y ~ a*b + np.log(x) - 1")
```

```
ModelDesc(lhs_termlist=[Term([EvalFactor('y')])],
          rhs_termlist=[Term([EvalFactor('a')]),
                        Term([EvalFactor('b')]),
                        Term([EvalFactor('a'), EvalFactor('b')]),
                        Term([EvalFactor('np.log(x)')])])
```

Model matrix

```
from patsy import demo_data, dmatrix, dmatrices
```

```
data = demo_data("y", "a", "b", "x1", "x2")
data
```

```
{'a': ['a1', 'a1', 'a2', 'a2', 'a1', 'a1', 'a2', 'a2
```

```
pd.DataFrame(data)
```

```
    a   b        x1        x2         y
0  a1  b1  1.764052 -0.103219  1.494079
1  a1  b2  0.400157  0.410599 -0.205158
2  a2  b1  0.978738  0.144044  0.313068
3  a2  b2  2.240893  1.454274 -0.854096
4  a1  b1  1.867558  0.761038 -2.552990
5  a1  b2 -0.977278  0.121675  0.653619
6  a2  b1  0.950088  0.443863  0.864436
7  a2  b2 -0.151357  0.333674 -0.742165
```

```
dmatrix("a + a:b + np.exp(x1)", data)
```

```
DesignMatrix with shape (8, 5)
  Intercept  a[T.a2]  a[a1]:b[T.b2]  a[a2]:b[T.b2]  r
          1        0              0              0
          1        0              1              0
          1        1              0              0
          1        1              0              1
          1        0              0              0
          1        0              1              0
          1        1              0              0
          1        1              0              1
  Terms:
    'Intercept' (column 0)
    'a' (column 1)
    'a:b' (columns 2:4)
    'np.exp(x1)' (column 4)
```


Note the `T.` in `a[T.a2]` is there to indicate treatment coding (i.e. typical dummy coding)

Model matrices

```python
y, x  = dmatrices("y ~ a + a:b + np.exp(x1)", data)
```

```python
y
```

```
DesignMatrix with shape (8, 1)
         y
   1.49408
  -0.20516
   0.31307
  -0.85410
  -2.55299
   0.65362
   0.86444
  -0.74217
  Terms:
    'y' (column 0)
```

```python
x
```

```
DesignMatrix with shape (8, 5)
  Intercept  a[T.a2]  a[a1]:b[T.b2]  a[a2]:b[T.b2]  r
          1        0              0              0
          1        0              1              0
          1        1              0              0
          1        1              0              1
          1        0              0              0
          1        0              1              0
          1        1              0              0
          1        1              0              1
  Terms:
    'Intercept' (column 0)
    'a' (column 1)
    'a:b' (columns 2:4)
    'np.exp(x1)' (column 4)
```

as DataFrames

```
dmatrix("a + a:b + np.exp(x1)", data, return_type='dataframe')
```

	Intercept	a[T.a2]	a[a1]:b[T.b2]	a[a2]:b[T.b2]	np.exp(x1)
0	1.0	0.0	0.0	0.0	5.836039
1	1.0	0.0	1.0	0.0	1.492059
2	1.0	1.0	0.0	0.0	2.661096
3	1.0	1.0	0.0	1.0	9.401725
4	1.0	0.0	0.0	0.0	6.472471
5	1.0	0.0	1.0	0.0	0.376334
6	1.0	1.0	0.0	0.0	2.585938
7	1.0	1.0	0.0	1.0	0.859541

Formula Syntax

Code	Description	Example
`+`	unions terms on the left and right	`a+a` ⇒ `a`
`-`	removes terms on the right from terms on the left	`a+b-a` ⇒ `b`
`:`	constructs interactions between each term on the left and right	`(a+b):c` ⇒ `a:c + b:c`
`*`	short-hand for terms and their interactions	`a*b` ⇒ `a + b + a:b`
`/`	short-hand for left terms and their interactions with right terms	`a/b` ⇒ `a + a:b`
`I()`	used for calculating arithmetic calculations	`I(x1 + x2)`
`Q()`	used to quote column names, e.g. columns with spaces or symbols	`Q('bad name!')`
`C()`	used for categorical data coding	`C(a, Treatment('a2'))`

Examples

```python
dmatrix("x:y", demo_data("x","y","z"))
```

```
DesignMatrix with shape (5, 2)
  Intercept       x:y
          1  -1.72397
          1   0.38018
          1  -0.14814
          1  -0.23130
          1   0.76682
  Terms:
    'Intercept' (column 0)
    'x:y' (column 1)
```

```python
dmatrix("x*y", demo_data("x","y","z"))
```

```
DesignMatrix with shape (5, 4)
  Intercept        x         y       x:y
          1  1.76405  -0.97728  -1.72397
          1  0.40016   0.95009   0.38018
          1  0.97874  -0.15136  -0.14814
          1  2.24089  -0.10322  -0.23130
          1  1.86756   0.41060   0.76682
  Terms:
    'Intercept' (column 0)
    'x' (column 1)
    'y' (column 2)
    'x:y' (column 3)
```

```python
dmatrix("x/y", demo_data("x","y","z"))
```

```
DesignMatrix with shape (5, 3)
  Intercept        x       x:y
          1  1.76405  -1.72397
          1  0.40016   0.38018
          1  0.97874  -0.14814
          1  2.24089  -0.23130
          1  1.86756   0.76682
  Terms:
    'Intercept' (column 0)
    'x' (column 1)
    'x:y' (column 2)
```

```python
dmatrix("x*(y+z)", demo_data("x","y","z"))
```

```
DesignMatrix with shape (5, 6)
  Intercept        x         y        z       x:y
          1  1.76405  -0.97728  0.14404  -1.72397  0.
          1  0.40016   0.95009  1.45427   0.38018  0.
          1  0.97874  -0.15136  0.76104  -0.14814  0.
          1  2.24089  -0.10322  0.12168  -0.23130  0.
          1  1.86756   0.41060  0.44386   0.76682  0.
  Terms:
    'Intercept' (column 0)
    'x' (column 1)
    'y' (column 2)
    'z' (column 3)
    'x:y' (column 4)
    'x:z' (column 5)
```

Intercept Examples (-1)

```
dmatrix("x", demo_data("x","y","z"))
```

```
DesignMatrix with shape (5, 2)
  Intercept        x
          1  1.76405
          1  0.40016
          1  0.97874
          1  2.24089
          1  1.86756
  Terms:
    'Intercept' (column 0)
    'x' (column 1)
```

```
dmatrix("-1 + x", demo_data("x","y","z"))
```

```
DesignMatrix with shape (5, 1)
        x
  1.76405
  0.40016
  0.97874
  2.24089
  1.86756
  Terms:
    'x' (column 0)
```

```
dmatrix("x-1", demo_data("x","y","z"))
```

```
DesignMatrix with shape (5, 1)
        x
  1.76405
  0.40016
  0.97874
  2.24089
  1.86756
  Terms:
    'x' (column 0)
```

Intercept Examples (0)

```
dmatrix("x+0", demo_data("x","y","z"))
```

```
DesignMatrix with shape (5, 1)
        x
  1.76405
  0.40016
  0.97874
  2.24089
  1.86756
  Terms:
    'x' (column 0)
```

```
dmatrix("x-0", demo_data("x","y","z"))
```

```
DesignMatrix with shape (5, 2)
  Intercept        x
          1  1.76405
          1  0.40016
          1  0.97874
          1  2.24089
          1  1.86756
  Terms:
    'Intercept' (column 0)
    'x' (column 1)
```

```
dmatrix("x - (-0)", demo_data("x","y","z"))
```

```
DesignMatrix with shape (5, 1)
        x
  1.76405
  0.40016
  0.97874
  2.24089
  1.86756
  Terms:
    'x' (column 0)
```

Design Info

One of the keep features of the design matrix object is that it retains all the necessary details (including stateful transforms) that are necessary to apply to new data inputs (e.g. for prediction).

```
d = dmatrix("a + a:b + np.exp(x1)", data, return_type='dataframe')
d.design_info
```

```
DesignInfo(['Intercept',
            'a[T.a2]',
            'a[a1]:b[T.b2]',
            'a[a2]:b[T.b2]',
            'np.exp(x1)'],
           factor_infos={EvalFactor('a'): FactorInfo(factor=EvalFactor('a'),
                                    type='categorical',
                                    state=<factor state>,
                                    categories=('a1', 'a2')),
                         EvalFactor('b'): FactorInfo(factor=EvalFactor('b'),
                                    type='categorical',
                                    state=<factor state>,
                                    categories=('b1', 'b2')),
                         EvalFactor('np.exp(x1)'): FactorInfo(factor=EvalFactor('np.exp(x1)'),
                                    type='numerical',
                                    state=<factor state>,
                                    num_columns=1)},
           term_codings=OrderedDict([(Term([]),
```

```
 [SubtermInfo(factors=(),
              contrast_matrices={},
              num_columns=1)]),
(Term([EvalFactor('a')]),
```

Stateful transforms

```python
data = {"x1": np.random.normal(size=10)}
new_data = {"x1": np.random.normal(size=10)}
```

```python
d = dmatrix("scale(x1)", data)
d
```

```
DesignMatrix with shape (10, 2)
  Intercept  scale(x1)
          1   -1.90949
          1   -0.33140
          1    1.01561
          1    1.38457
          1   -0.73524
          1   -0.38595
          1    1.29286
          1   -0.79691
          1   -0.07595
          1    0.54190
  Terms:
    'Intercept' (column 0)
    'scale(x1)' (column 1)
```

```python
np.mean(d, axis=0)
```

```
array([1., 0.])
```

```python
pred = dmatrix(d.design_info, new_data)
pred
```

```
DesignMatrix with shape (10, 2)
  Intercept  scale(x1)
          1    0.11925
          1    1.78150
          1   -0.25861
          1    0.75752
          1    0.24595
          1   -0.29425
          1   -0.70571
          1   -0.55710
          1   -0.37134
          1   -1.95723
  Terms:
    'Intercept' (column 0)
    'scale(x1)' (column 1)
```

```python
np.mean(pred, axis=0)
```

```
array([ 1.   , -0.124])
```

scikit-lego PatsyTransformer

If you would like to use a Patsy formula in a scikitlearn pipeline, it is possible via the `PatsyTransformer` from the scikit-lego library (`sklego`).

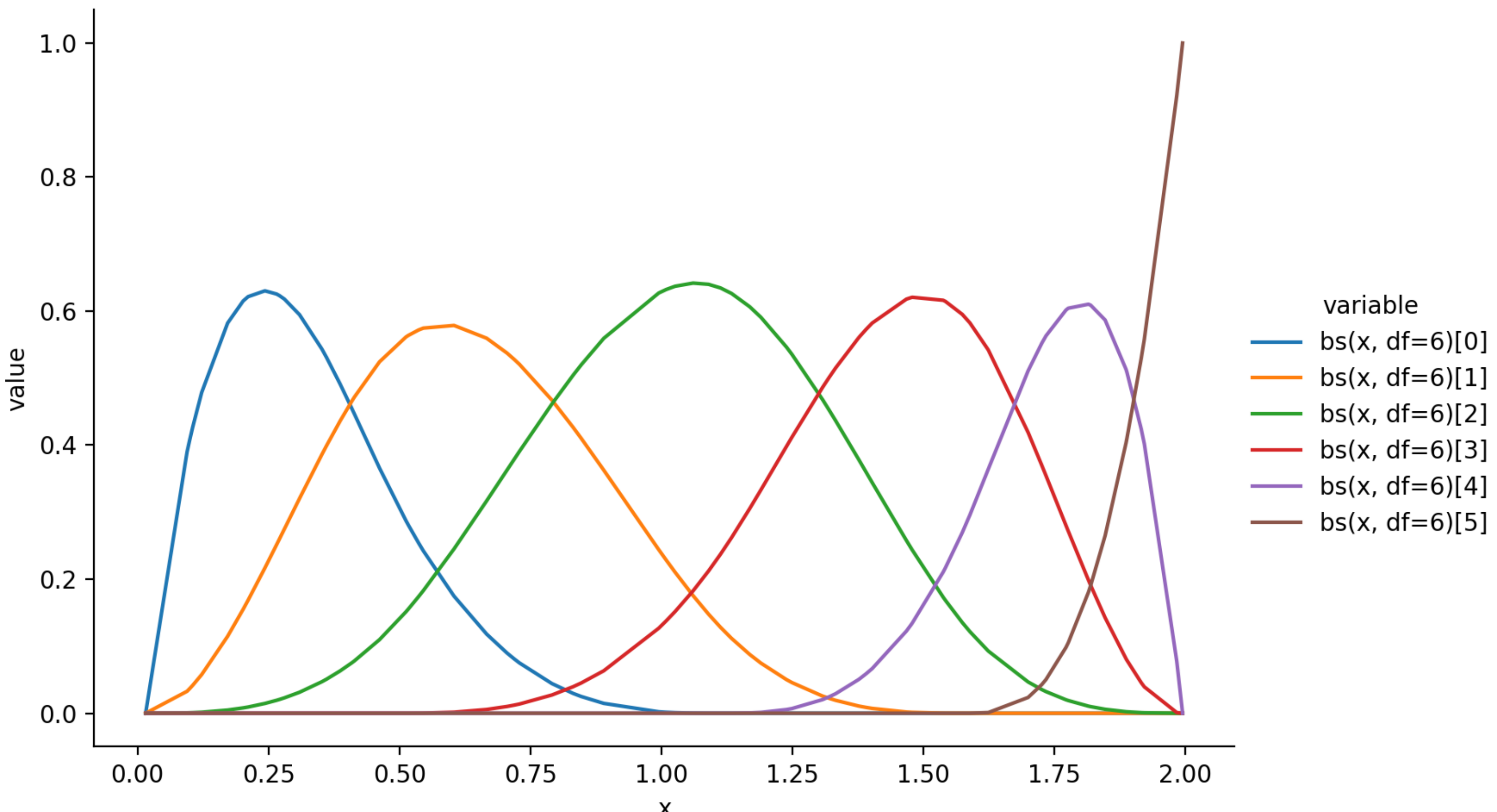
```
from sklego.preprocessing import PatsyTransformer
df = pd.DataFrame({
  "y": [2, 2, 4, 4, 6], "x": [1, 2, 3, 4, 5],
  "a": ["yes", "yes", "no", "no", "yes"]
})
X, y = df[["x", "a"]], df[["y"]].values
```

B-splines

Patsy also has support for B-splines and other related models,

What is `bs(x)[i]`?

```
bs_df = (
  dmatrix("bs(x, df=6)", data=d, return_type="dataframe")
  .drop(["Intercept"], axis = 1)
  .assign(x = d["x"])
  .melt(id_vars="x")
)
sns.relplot(x="x", y="value", hue="variable", kind="line", data = bs_df, aspect=1.5)
```

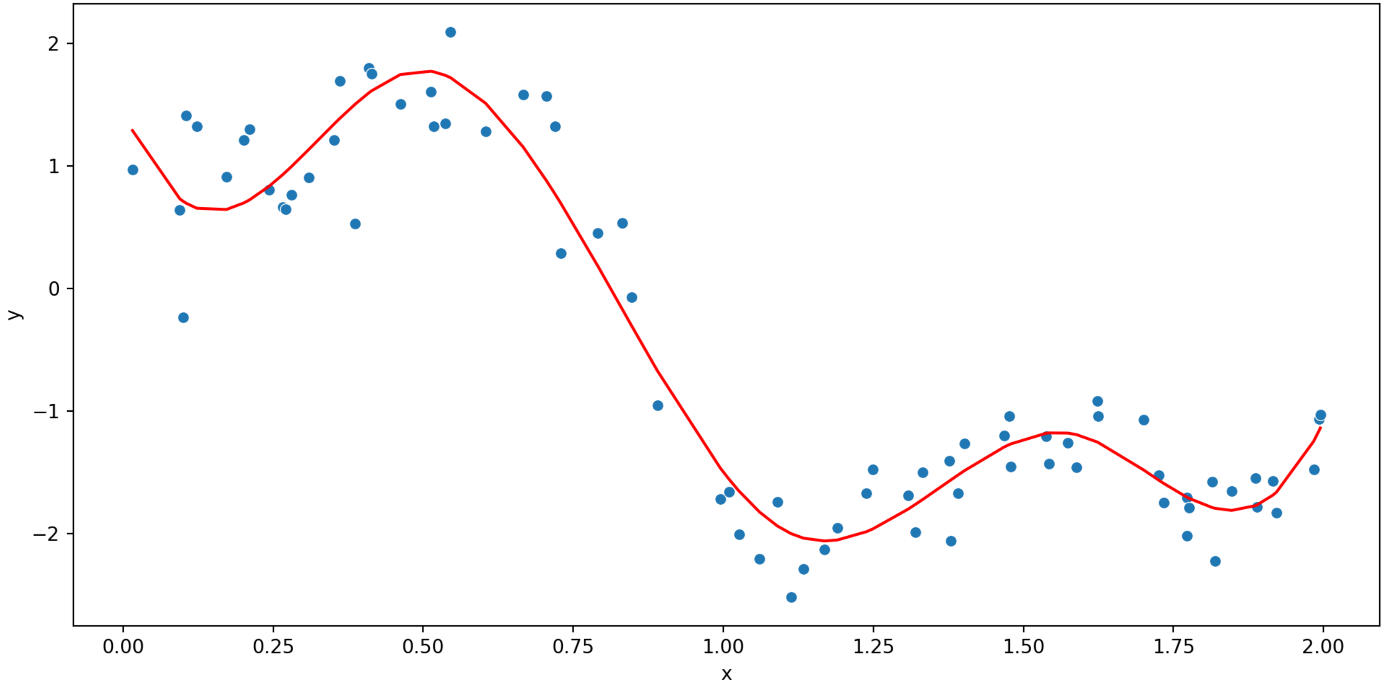


Fitting a model

```
from sklearn.linear_model import LinearRegression
lm = LinearRegression(fit_intercept=False).fit(X,y)
lm.coef_
```

```
array([[ 1.28955, -1.69132,  3.17914, -5.3865 , -1.18284, -3.8488 , -2.42867]])
```

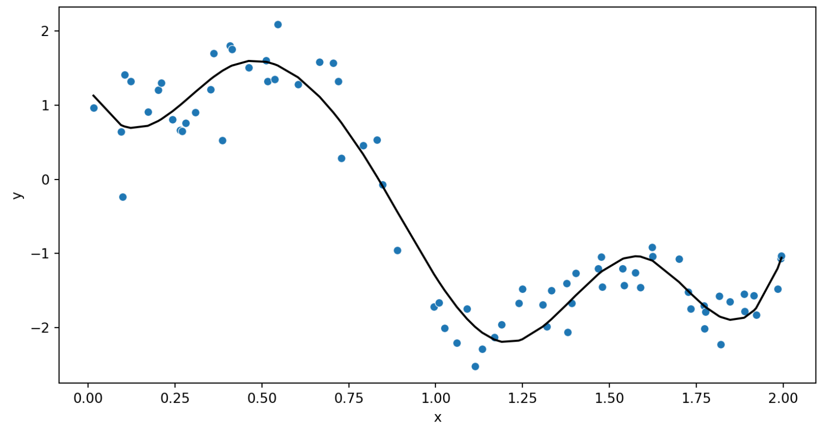
```
plt.figure(layout="constrained")
sns.lineplot(x=d["x"], y=lm.predict(X).ravel(), color="r")
sns.scatterplot(x="x", y="y", data=d)
plt.show()
```



sklearn SplineTransformer

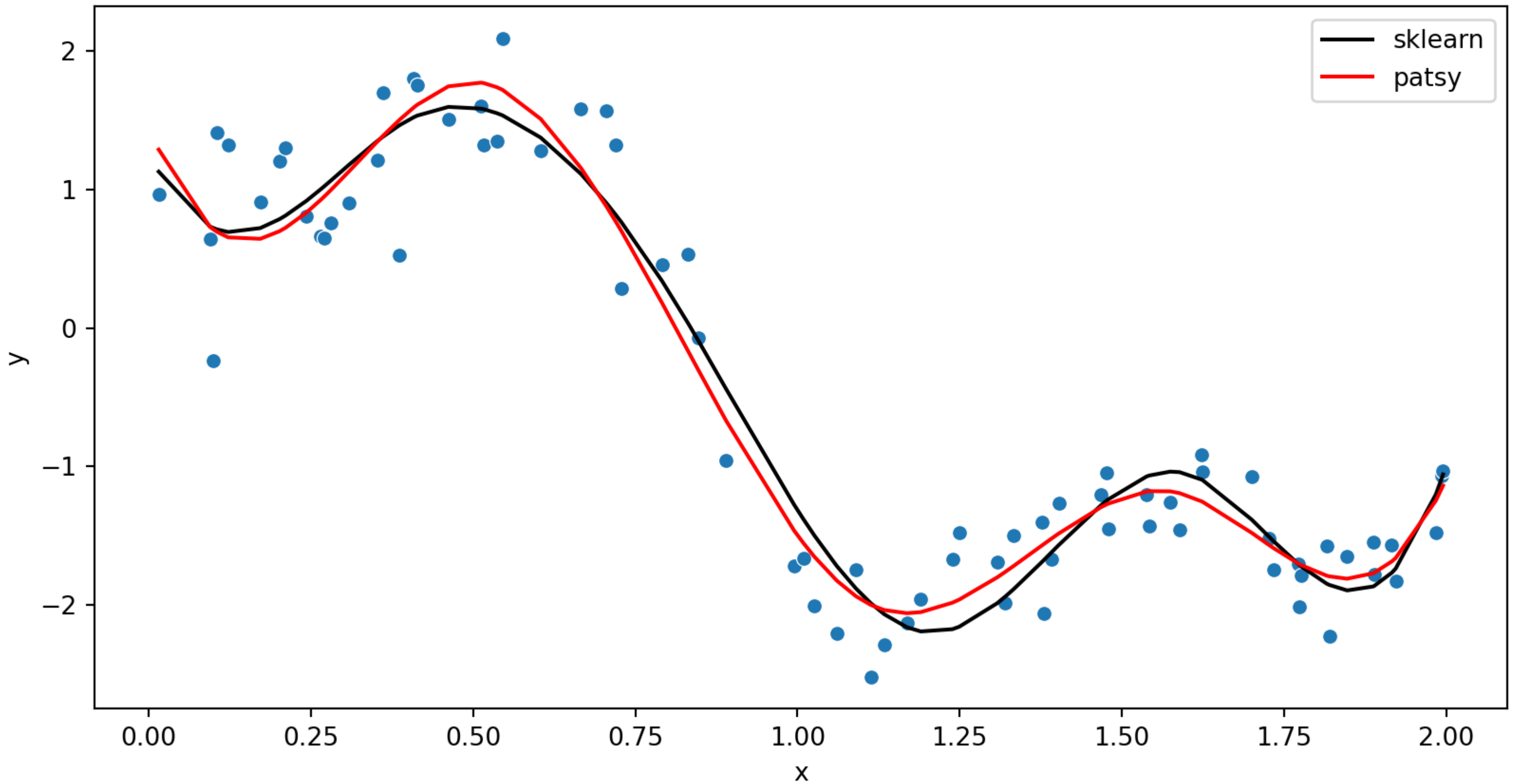
```python
from sklearn.preprocessing import SplineTransfor

p = make_pipeline(
  SplineTransformer(
    n_knots=6,
    degree=3,
    include_bias=True
  ),
  LinearRegression(fit_intercept=False)
).fit(
  d[["x"]], d["y"]
)
```

```python
plt.figure()
sns.lineplot(x=d["x"], y=p.predict(d[["x"]]).rav
sns.scatterplot(x="x", y="y", data=d)
plt.show()
```



Comparison

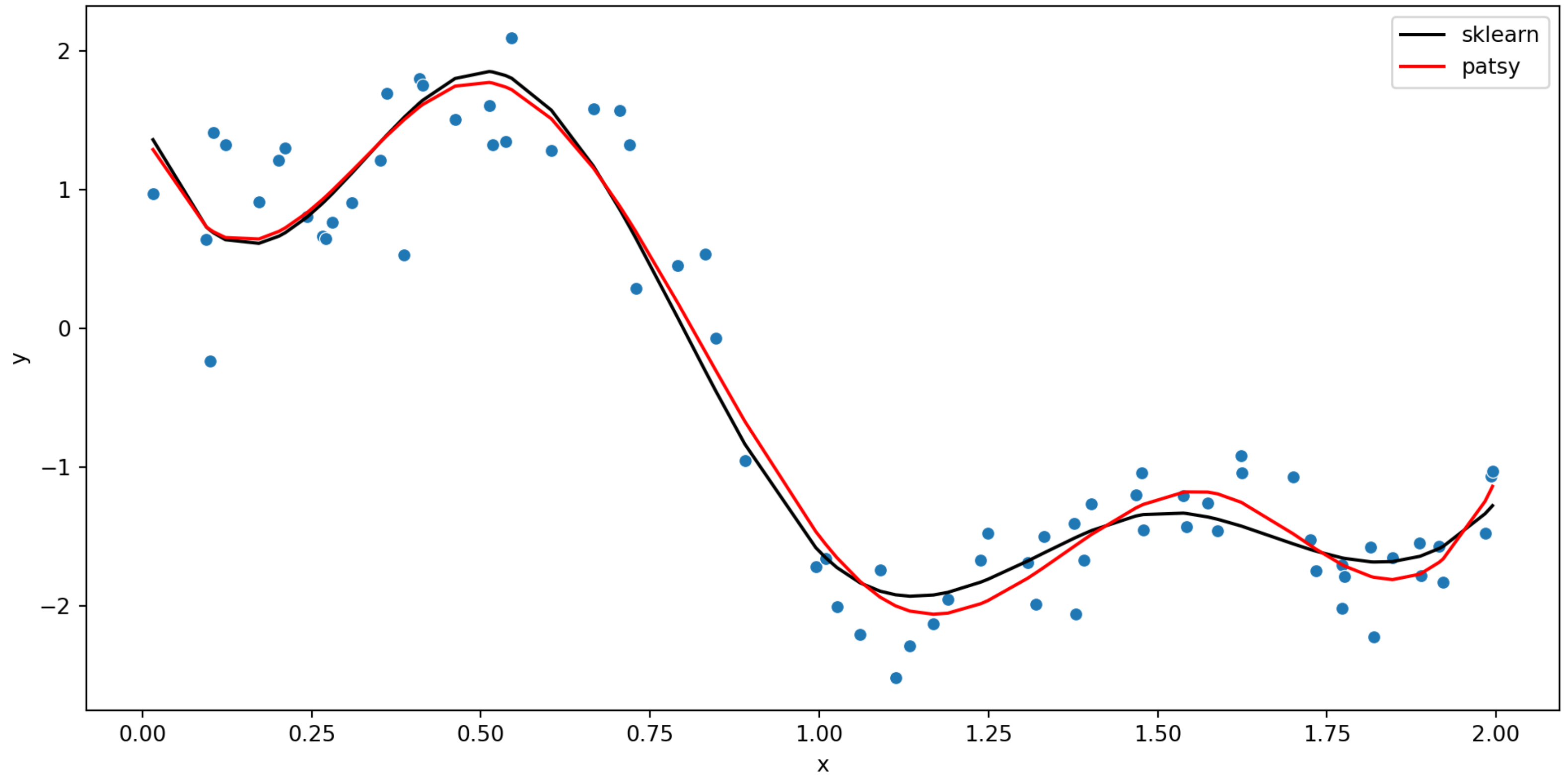
```
plt.figure()
sns.lineplot(x=d["x"], y=p.predict(d[["x"]]).ravel(), color="k", label = "sklearn")
sns.lineplot(x=d["x"], y=lm.predict(X).ravel(), color="r", label = "patsy")
sns.scatterplot(x="x", y="y", data=d)
plt.show()
```



Why different?

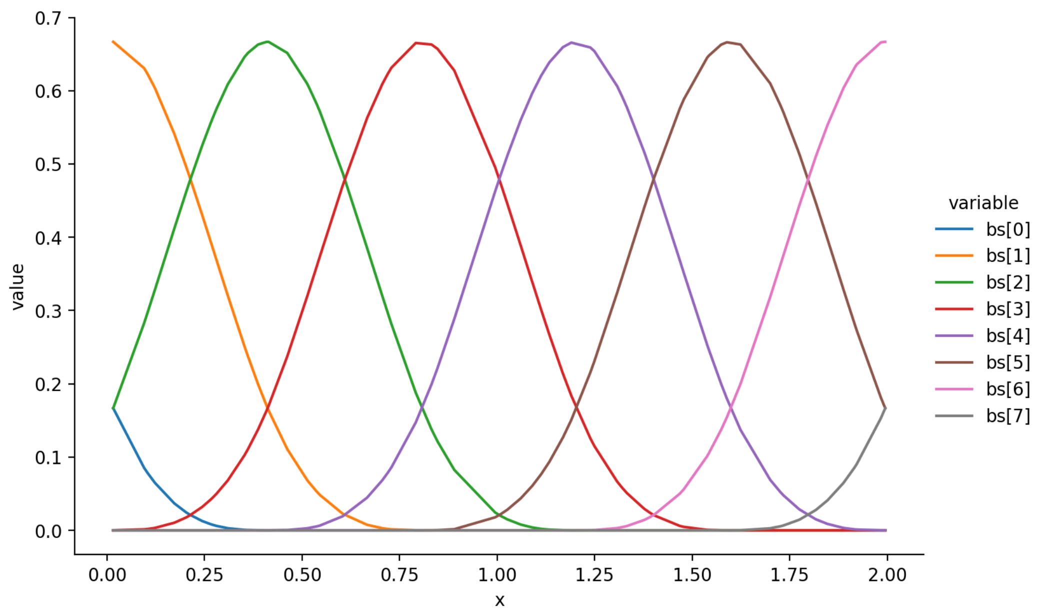
For patsy the number of splines is determined by `df` while for sklearn this is determined by `n_knots + degree - 1`.

```
p = p.set_params(splinetransformer__n_knots = 5).fit(d[["x"]], d["y"])

plt.figure(layout="constrained")
sns.lineplot(x=d["x"], y=p.predict(d[["x"]]).ravel(), color="k", label = "sklearn")
sns.lineplot(x=d["x"], y=lm.predict(X).ravel(), color="r", label = "patsy")
sns.scatterplot(x="x", y="y", data=d)
plt.show()
```


sklearn
patsy
2
1
0
−1
−2
y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x

but that is not the whole story, if we examine the bases we also see they differ slightly between implementations

```
bs_df = pd.DataFrame(
  SplineTransformer(n_knots=6, degree=3, include_bias=True).fit_transform(d[["x"]]),
  columns = ["bs["+ str(i) +"]" for i in range(8)]
).assign(
  x = d.x
).melt(
  id_vars = "x"
)
sns.relplot(x="x", y="value", hue="variable", kind="line", data = bs_df, aspect=1.5)
```



statsmodels

statsmodels

> statsmodels is a Python module that provides classes and functions for the estimation of many different statistical models, as well as for conducting statistical tests, and statistical data exploration. An extensive list of result statistics are available for each estimator. The results are tested against existing statistical packages to ensure that they are correct.

```python
import statsmodels.api as sm
import statsmodels.formula.api as smf
import statsmodels.tsa.api as tsa
```

`statsmodels` uses slightly different terminology for refering to `y` (dependent / response) and `x` (independent / explanatory) variables. Specifically it uses `endog` to refer to the `y` and `exog` to refer to the `x` variable(s).

This is particularly important when using the main API, less so when using the formula API.

OpenIntro Loans data

> This data set represents thousands of loans made through the Lending Club platform, which is a platform that allows individuals to lend to other individuals. Of course, not all loans are created equal. Someone who is a essentially a sure bet to pay back a loan will have an easier time getting a loan with a low interest rate than someone who appears to be riskier. And for people who are very risky? They may not even get a loan offer, or they may not have accepted the loan offer due to a high interest rate. It is important to keep that last part in mind, since this data set only represents loans actually made, i.e. do not mistake this data for loan applications!

For the full data dictionary see here. We have removed some of the columns to make the data set more reasonably sized and also droped any rows with missing values.

```
loans = pd.read_csv("data/openintro_loans.csv")
loans
```

```
     state  emp_length  term homeownership  annual_income  ... loan_amount  grade  interest_rate  public_rec
0       NJ           3    60      MORTGAGE        90000.0  ...       28000      C          14.07
1       HI          10    36          RENT        40000.0  ...        5000      C          12.61
2       WI           3    36          RENT        40000.0  ...        2000      D          17.09
3       PA           1    36          RENT        30000.0  ...       21600      A           6.72
4       CA          10    36          RENT        35000.0  ...       23000      C          14.07
...    ...         ...   ...           ...            ...  ...         ...    ...            ...
9177    TX          10    36          RENT       108000.0  ...       24000      A           7.35
9178    PA           8    36      MORTGAGE       121000.0  ...       10000      D          19.03
9179    CT          10    36      MORTGAGE        67000.0  ...       30000      E          23.88
9180    WI           1    36      MORTGAGE        80000.0  ...       24000      A           5.32
9181    CT           3    36          RENT        66000.0  ...       12800      B          10.91
```

```
[9182 rows x 16 columns]
```

```
print(loans.columns)
```

```
Index(['state', 'emp_length', 'term', 'homeownership', 'annual_income', 'verified_income', 'debt_to_income',
       'total_credit_utilized', 'num_cc_carrying_balance', 'loan_purpose', 'loan_amount', 'grade', 'interest
       'loan_status'],
      dtype='object')
```

OLS

```
y = loans["loan_amount"]
X = loans[["homeownership", "annual_income", "debt_to_income", "interest_rate", "public_record_bankrupt

model = sm.OLS(endog=y, exog=X)
```

```
Error: ValueError: Pandas data cast to numpy dtype of object. Check input data with np.asarray(data). The ty
<... omitted ...>0.0 21.33 17.09 0]
 ['RENT' 32000.0 37.06 18.45 0]
 ['MORTGAGE' 170000.0 10.4 6.08 0]
 ['RENT' 85000.0 12.4 9.43 0]
 ['MORTGAGE' 64000.0 36.49 9.93 0]
 ['MORTGAGE' 100000.0 21.71 9.93 1]
 ['MORTGAGE' 114000.0 14.6 9.93 0]
 ['MORTGAGE' 49000.0 36.2 21.45 0]
 ['MORTGAGE' 106000.0 22.26 7.35 0]
 ['MORTGAGE' 150000.0 6.26 6.07 0]
 ['MORTGAGE' 55000.0 22.19 9.43 0]
 ['RENT' 65000.0 9.77 9.92 0]
 ['RENT' 65000.0 27.1 15.05 0]
 ['OWN' 96774.0 0.04 9.44 0]
 ['MORTGAGE' 75000.0 28.45 11.99 0]
 ['RENT' 70000.0 15.31 9.93 0]
 ['MORTGAGE' 20000.0 23.23 7.97 0]
 ['RENT' 40000.0 12.07 10.41 0]
 ['RENT' 108000.0 22.28 7.35 1]
 ['MORTGAGE' 121000.0 32.38 19.03 0]
 ['MORTGAGE' 67000.0 45.26 23.88 0]
```

What do you think the issue is here?

The error occurs because `X` contains mixed types - specifically we have categorical data columns which cannot be directly converted to a numeric dtype so we need to take care of the dummy coding for statsmodels (with this interface).

```
X_dc = pd.get_dummies(X)
model = sm.OLS(endog=y, exog=X_dc)
model
```

```
<statsmodels.regression.linear_model.OLS object at 0x2d10c0820>
```

```
dir(model)
```

```
['__class__', '__delattr__', '__dict__', '__dir__', '__doc__', '__eq__', '__format__', '__ge__', '__getattri
```

Fitting and summary

```
res = model.fit()
print(res.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:            loan_amount   R-squared:                       0.135
Model:                            OLS   Adj. R-squared:                  0.135
Method:                 Least Squares   F-statistic:                     239.5
Date:                Tue, 21 Mar 2023   Prob (F-statistic):          2.33e-285
Time:                        12:14:38   Log-Likelihood:                -97245.
No. Observations:                9182   AIC:                         1.945e+05
Df Residuals:                    9175   BIC:                         1.946e+05
Df Model:                           6
Covariance Type:            nonrobust
============================================================================================
                              coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------------
annual_income               0.0505      0.002     31.952      0.000       0.047       0.054
debt_to_income             65.6641      7.310      8.982      0.000      51.334      79.994
interest_rate             204.2480     20.448      9.989      0.000     164.166     244.330
public_record_bankrupt -1362.3253    306.019     -4.452      0.000   -1962.191    -762.460
homeownership_MORTGAGE   1.002e+04    357.245     28.048      0.000    9319.724    1.07e+04
homeownership_OWN        8880.4144    422.296     21.029      0.000    8052.620    9708.209
homeownership_RENT       7446.5385    351.641     21.177      0.000    6757.243    8135.834
==============================================================================
Omnibus:                      481.833   Durbin-Watson:                   2.002
```

Formula interface

Most of the modeling interfaces are also provided by `smf` (`statsmodels.formula.api`) in which case patsy is used to construct the model matrices.

```
model = smf.ols(
  "loan_amount ~ homeownership + annual_income + debt_to_income + interest_rate + public_record_bankrup
  data = loans
)
res = model.fit()
print(res.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:            loan_amount   R-squared:                       0.135
Model:                            OLS   Adj. R-squared:                  0.135
Method:                 Least Squares   F-statistic:                     239.5
Date:                Tue, 21 Mar 2023   Prob (F-statistic):          2.33e-285
Time:                        12:14:38   Log-Likelihood:                -97245.
No. Observations:                9182   AIC:                         1.945e+05
Df Residuals:                    9175   BIC:                         1.946e+05
Df Model:                           6
Covariance Type:            nonrobust
==========================================================================================
                             coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------------
```

```
Intercept               1.002e+04    357.245     28.048      0.000    9319.724    1.07e+04
homeownership[T.OWN]   -1139.5893    322.361     -3.535      0.000   -1771.489    -507.690
homeownership[T.RENT]  -2573.4652    221.101    -11.639      0.000   -3006.873   -2140.057
annual_income              0.0505      0.002     31.952      0.000       0.047       0.054
debt_to_income            65.6641      7.310      8.982      0.000      51.334      79.994
interest_rate            204.2480     20.448      9.989      0.000     164.166     244.330
public_record_bankrupt -1362.3253    306.019     -4.452      0.000   -1962.191    -762.460
==============================================================================
```

Result values and model parameters

```
res.params
```

```
Intercept                   10020.003630
homeownership[T.OWN]        -1139.589268
homeownership[T.RENT]       -2573.465175
annual_income                   0.050505
debt_to_income                 65.664103
interest_rate                 204.247993
public_record_bankrupt      -1362.325291
dtype: float64
```

```
res.bse
```

```
Intercept                   357.244896
homeownership[T.OWN]        322.361151
homeownership[T.RENT]       221.101300
annual_income                 0.001581
debt_to_income                7.310428
interest_rate                20.447644
public_record_bankrupt      306.019080
dtype: float64
```

```
res.rsquared
```

```
0.13542611095847523
```

```
res.aic
```

```
194503.99751598848
```

```
res.bic
```

```
194553.87251826216
```

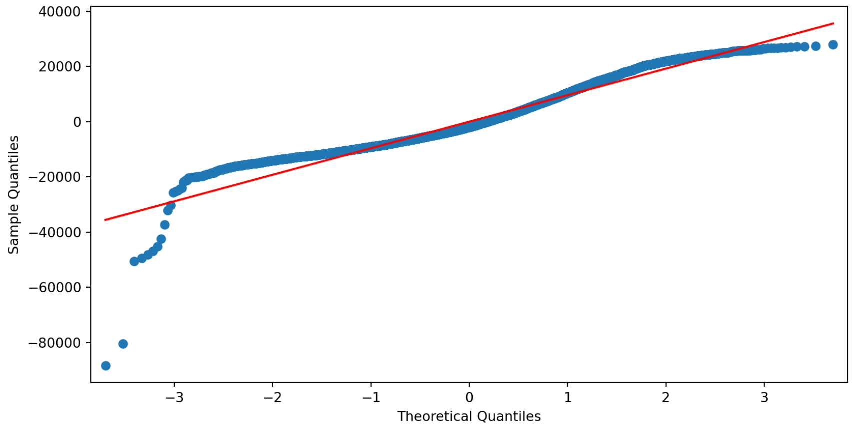
```
res.predict()
```

```
array([18621.86199, 11010.94015, 14346.14516, 11001.:
       18283.87492, 26719.61804, 18496.72337, 14811.4
       18845.26463, 20083.33976, ..., 19576.16932, 18
       17161.96037, 18764.48833, 19252.9242 , 18336.4
       22144.19006, 21253.25932, 15934.34097, 14375.:
```

Diagnostic plots

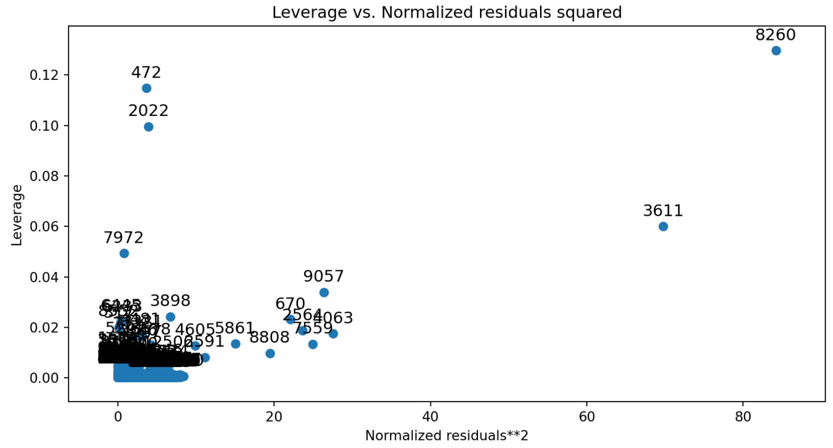
QQ Plot

```
plt.figure()
sm.graphics.qqplot(res.resid, line="s")
plt.show()
```



Leverage plot

```
plt.figure()
sm.graphics.plot_leverage_resid2(res)
plt.show()
```



Alternative model

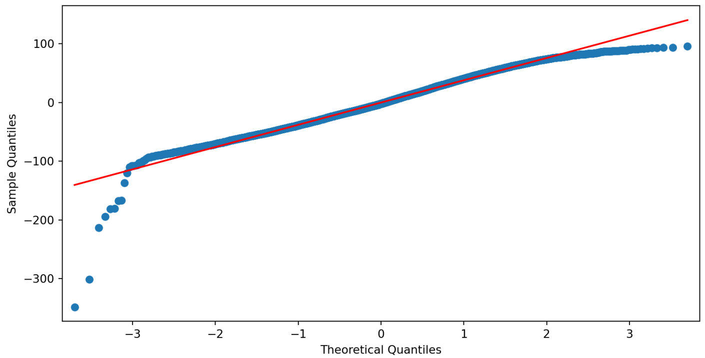
```
res = smf.ols(
  "np.sqrt(loan_amount) ~ homeownership + annual_income + debt_to_income + interest_rate + public_recor
  data = loans
).fit()
print(res.summary())
```

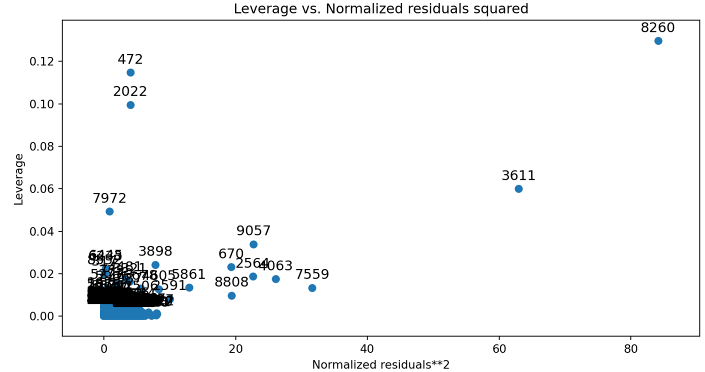
```
                            OLS Regression Results
================================================================================
Dep. Variable:     np.sqrt(loan_amount)   R-squared:                       0.132
Model:                              OLS   Adj. R-squared:                  0.132
Method:                   Least Squares   F-statistic:                     232.7
Date:                  Tue, 21 Mar 2023   Prob (F-statistic):          1.16e-277
Time:                          12:14:41   Log-Likelihood:                -46429.
No. Observations:                  9182   AIC:                         9.287e+04
Df Residuals:                      9175   BIC:                         9.292e+04
Df Model:                             6
Covariance Type:              nonrobust
==========================================================================================
                            coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------------
Intercept                95.4915      1.411     67.687      0.000      92.726      98.257
homeownership[T.OWN]     -4.4495      1.273     -3.495      0.000      -6.945      -1.954
homeownership[T.RENT]   -10.4225      0.873    -11.937      0.000     -12.134      -8.711
annual_income             0.0002   6.24e-06     30.916      0.000       0.000       0.000
debt_to_income            0.2720      0.029      9.421      0.000       0.215       0.329
interest_rate             0.8911      0.081     11.035      0.000       0.733       1.049
public_record_bankrupt   -4.6899      1.208     -3.881      0.000      -7.059      -2.321
```

```
==============================================================================
Omnibus:                      178.498   Durbin-Watson:                   2.011
```

```python
plt.figure()
sm.graphics.qqplot(res.resid, line="s")
plt.show()
```

```python
plt.figure()
sm.graphics.plot_leverage_resid2(res)
plt.show()
```





Bushtail Possums

> Data representing possums in Australia and New Guinea. This is a copy of the data set by the same name in the DAAG package, however, the data set included here includes fewer variables.
> `pop` - Population, either `Vic` (Victoria) or `other` (New South Wales or Queensland).

Logistic regression models (GLM)

```
y = pd.get_dummies( possum["pop"], drop_first = True )
X = pd.get_dummies( possum.drop(["site","pop"], axis=1) )

model = sm.GLM(y, X, family = sm.families.Binomial())
```

```
Error: statsmodels.tools.sm_exceptions.MissingDataError: exog contains inf or nan
```

What is wrong now?

Behavior for dealing with missing data can be handled via `missing`, possible values are `"none"`, `"drop"`, and `"raise"`.

```
model = sm.GLM(y, X, family = sm.families.Binomial(), missing="drop")
```

Fit and summary

```
res = model.fit()
print(res.summary())
```

```
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:                  other   No. Observations:                  102
Model:                            GLM   Df Residuals:                       95
Model Family:                Binomial   Df Model:                            6
Link Function:                  Logit   Scale:                          1.0000
Method:                          IRLS   Log-Likelihood:                -31.942
Date:                Tue, 21 Mar 2023   Deviance:                       63.885
Time:                        12:14:44   Pearson chi2:                     154.
No. Iterations:                     7   Pseudo R-squ. (CS):             0.5234
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
age           -0.1373      0.183     -0.751      0.453      -0.495       0.221
head_l         0.1972      0.158      1.247      0.212      -0.113       0.507
skull_w        0.2001      0.139      1.443      0.149      -0.072       0.472
total_l       -0.7569      0.176     -4.290      0.000      -1.103      -0.411
tail_l         2.0698      0.429      4.820      0.000       1.228       2.912
sex_f        -40.0148     13.077     -3.060      0.002     -65.645     -14.385
sex_m        -38.5395     12.941     -2.978      0.003     -63.904     -13.175
==============================================================================
```

Success vs failure

Note `endog` can be 1d or 2d for binomial models - in the case of the latter each row is interpreted as [success, failure].

```
y = pd.get_dummies( possum["pop"] )
X = pd.get_dummies( possum.drop(["site","pop"], axis=1) )

res = sm.GLM(y, X, family = sm.families.Binomial(), missing="drop").fit()
print(res.summary())
```

```
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:       ['Vic', 'other']   No. Observations:                  102
Model:                            GLM   Df Residuals:                       95
Model Family:                Binomial   Df Model:                            6
Link Function:                  Logit   Scale:                          1.0000
Method:                          IRLS   Log-Likelihood:                -31.942
Date:                Tue, 21 Mar 2023   Deviance:                       63.885
Time:                        12:14:44   Pearson chi2:                     154.
No. Iterations:                     7   Pseudo R-squ. (CS):             0.5234
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
age            0.1373      0.183      0.751      0.453      -0.221       0.495
head_l        -0.1972      0.158     -1.247      0.212      -0.507       0.113
skull_w       -0.2001      0.139     -1.443      0.149      -0.472       0.072
```

```
total_l        0.7569      0.176      4.290      0.000       0.411       1.103
tail_l        -2.0698      0.429     -4.820      0.000      -2.912      -1.228
sex_f         40.0148     13.077      3.060      0.002      14.385      65.645
sex_m         38.5395     12.941      2.978      0.003      13.175      63.904
==============================================================================
```

Fit and summary

```
res = model.fit()
print(res.summary())
```

```
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:                  other   No. Observations:                  102
Model:                            GLM   Df Residuals:                       95
Model Family:                Binomial   Df Model:                            6
Link Function:                  Logit   Scale:                          1.0000
Method:                          IRLS   Log-Likelihood:                -31.942
Date:                Tue, 21 Mar 2023   Deviance:                       63.885
Time:                        12:14:45   Pearson chi2:                     154.
No. Iterations:                     7   Pseudo R-squ. (CS):             0.5234
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
age           -0.1373      0.183     -0.751      0.453      -0.495       0.221
head_l         0.1972      0.158      1.247      0.212      -0.113       0.507
skull_w        0.2001      0.139      1.443      0.149      -0.072       0.472
total_l       -0.7569      0.176     -4.290      0.000      -1.103      -0.411
tail_l         2.0698      0.429      4.820      0.000       1.228       2.912
sex_f        -40.0148     13.077     -3.060      0.002     -65.645     -14.385
sex_m        -38.5395     12.941     -2.978      0.003     -63.904     -13.175
==============================================================================
```

Formula interface

```
res = smf.glm(
  "pop ~ sex + age + head_l + skull_w + total_l + tail_l-1",
  data = possum,
  family = sm.families.Binomial(),
  missing="drop"
).fit()
print(res.summary())
```

```
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:     ['pop[Vic]', 'pop[other]']   No. Observations:          102
Model:                                    GLM   Df Residuals:               95
Model Family:                        Binomial   Df Model:                    6
Link Function:                          Logit   Scale:                  1.0000
Method:                                  IRLS   Log-Likelihood:        -31.942
Date:                        Tue, 21 Mar 2023   Deviance:               63.885
Time:                                12:14:45   Pearson chi2:             154.
No. Iterations:                             7   Pseudo R-squ. (CS):     0.5234
Covariance Type:                    nonrobust
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
sex[f]        40.0148     13.077      3.060      0.002      14.385      65.645
sex[m]        38.5395     12.941      2.978      0.003      13.175      63.904
age            0.1373      0.183      0.751      0.453      -0.221       0.495
head_l        -0.1972      0.158     -1.247      0.212      -0.507       0.113
skull_w       -0.2001      0.139     -1.443      0.149      -0.472       0.072
```

```
total_l        0.7569      0.176      4.290      0.000       0.411       1.103
tail_l        -2.0698      0.429     -4.820      0.000      -2.912      -1.228
==============================================================================
```

sleepstudy data

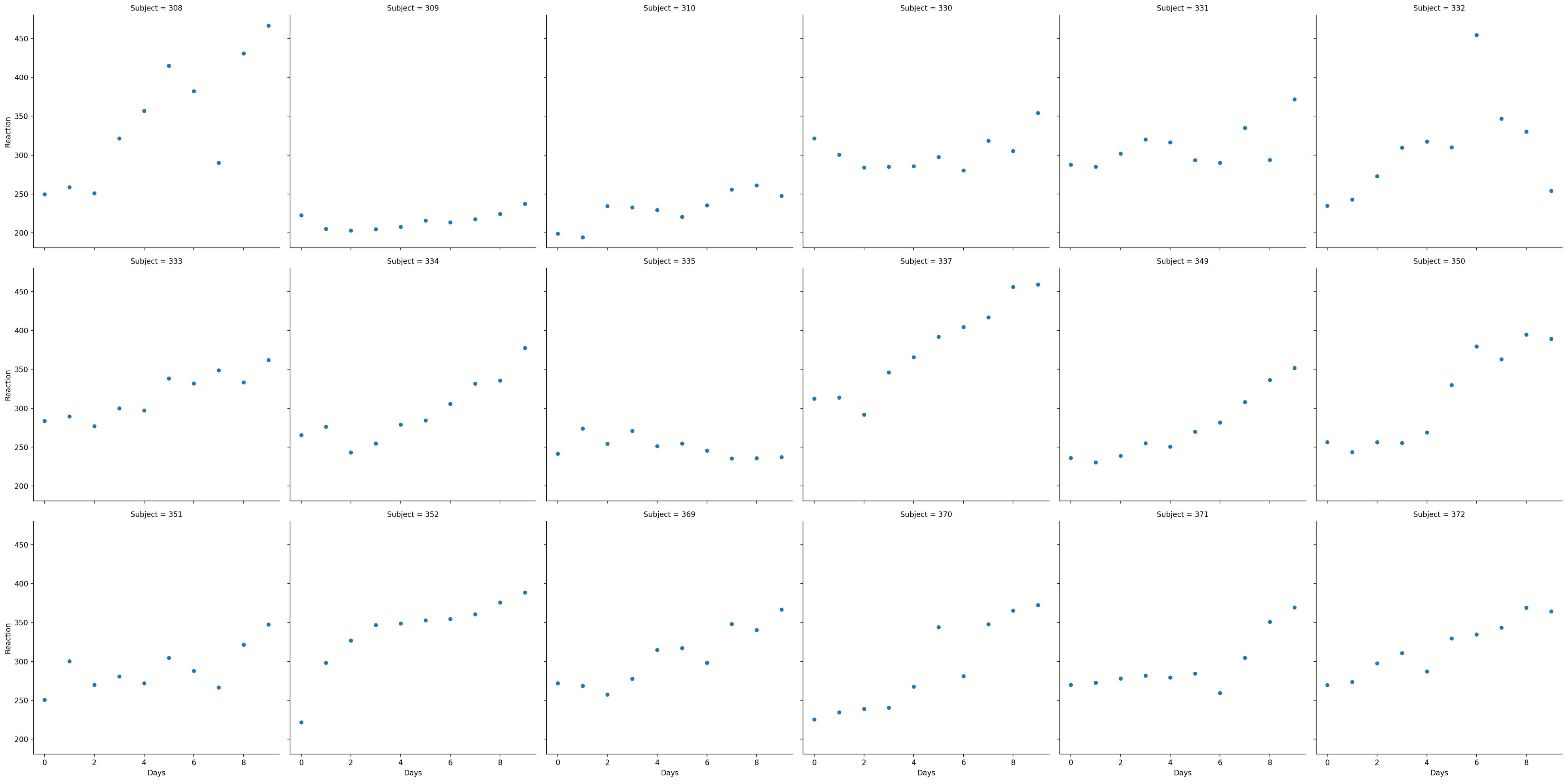
> These data are from the study described in Belenky et al. (2003), for the most sleep-deprived group (3 hours time-in-bed) and for the first 10 days of the study, up to the recovery period. The original study analyzed speed (1/(reaction time)) and treated day as a categorical rather than a continuous predictor.
> The average reaction time per day (in milliseconds) for subjects in a sleep deprivation study. Days 0-1 were adaptation and training (T1/T2), day 2 was baseline (B); sleep deprivation started after day 2.

```python
sleep = pd.read_csv("data/sleepstudy.csv")
sleep
```

```
     Reaction  Days  Subject
0    249.5600     0      308
1    258.7047     1      308
2    250.8006     2      308
3    321.4398     3      308
4    356.8519     4      308
..        ...   ...      ...
175  329.6076     5      372
176  334.4818     6      372
177  343.2199     7      372
178  369.1417     8      372
179  364.1236     9      372

[180 rows x 3 columns]
```

These data come from the `sleepstudy` dataset in the `lme4` R package

```python
1  sns.relplot(x="Days", y="Reaction", col="Subject", col_wrap=6, data=sleep)
```



Random intercept model

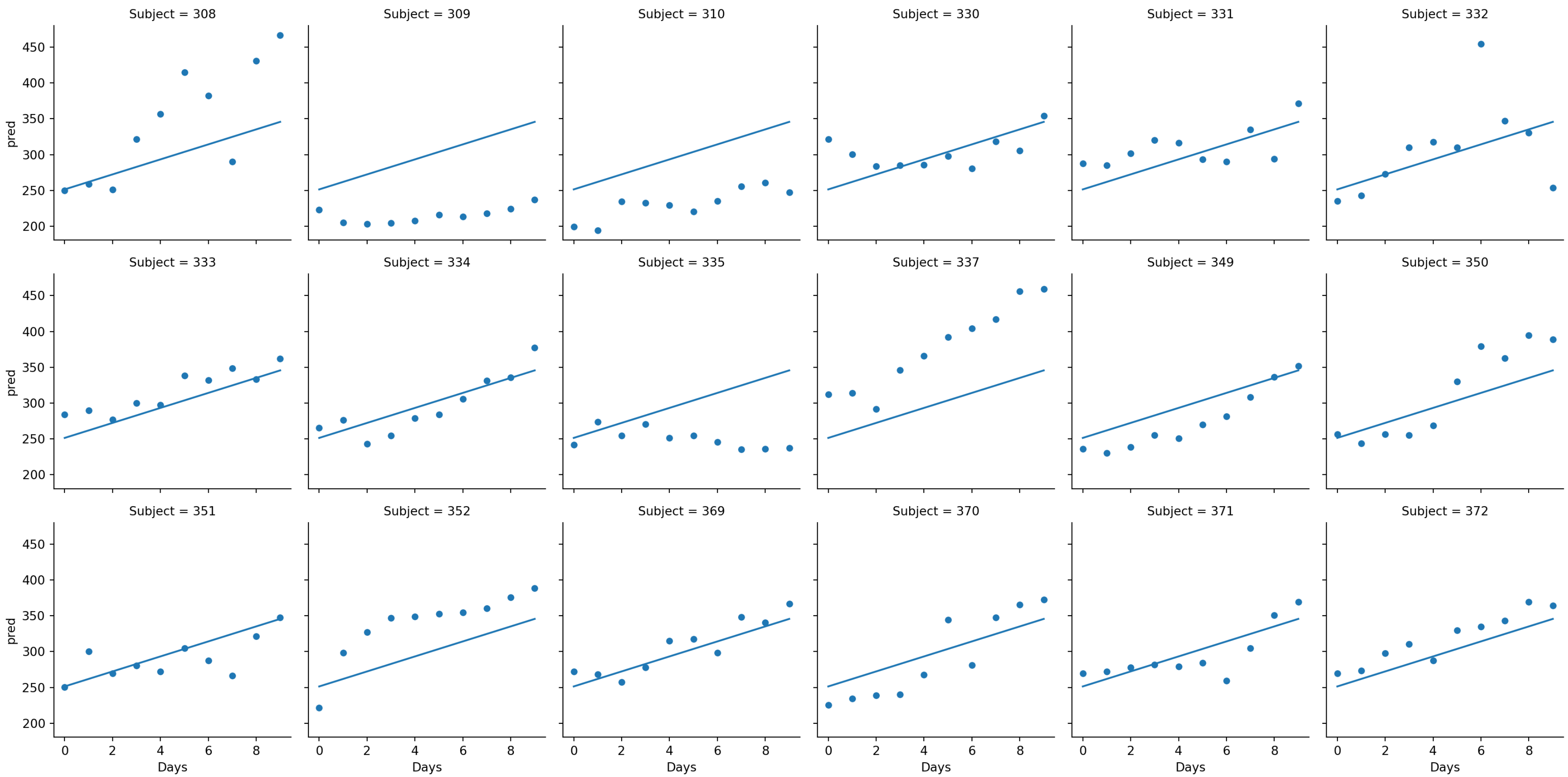
```
me_rand_int = smf.mixedlm(
  "Reaction ~ Days", data=sleep, groups=sleep["Subject"],
  subset=sleep.Days >= 2
)
res_rand_int = me_rand_int.fit(method=["lbfgs"])
print(res_rand_int.summary())
```

```
         Mixed Linear Model Regression Results
=========================================================
Model:             MixedLM Dependent Variable: Reaction
No. Observations:  180     Method:             REML
No. Groups:        18      Scale:              960.4529
Min. group size:   10      Log-Likelihood:     -893.2325
Max. group size:   10      Converged:          Yes
Mean group size:   10.0
---------------------------------------------------------
           Coef.   Std.Err.   z    P>|z|  [0.025  0.975]
---------------------------------------------------------
Intercept  251.405    9.747 25.793 0.000 232.302 270.509
Days        10.467    0.804 13.015 0.000   8.891  12.044
Group Var 1378.232   17.157
=========================================================
```

```r
summary(
  lmer(Reaction ~ Days + (1|Subject), data=sleepstudy)
)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: Reaction ~ Days + (1 | Subject)
   Data: sleepstudy

REML criterion at convergence: 1786.5

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.2257 -0.5529  0.0109  0.5188  4.2506

Random effects:
 Groups   Name        Variance Std.Dev.
 Subject  (Intercept) 1378.2   37.12
 Residual              960.5   30.99
Number of obs: 180, groups:  Subject, 18

Fixed effects:
            Estimate Std. Error t value
(Intercept) 251.4051     9.7467   25.79
Days         10.4673     0.8042   13.02

Correlation of Fixed Effects:
     (Intr)
```

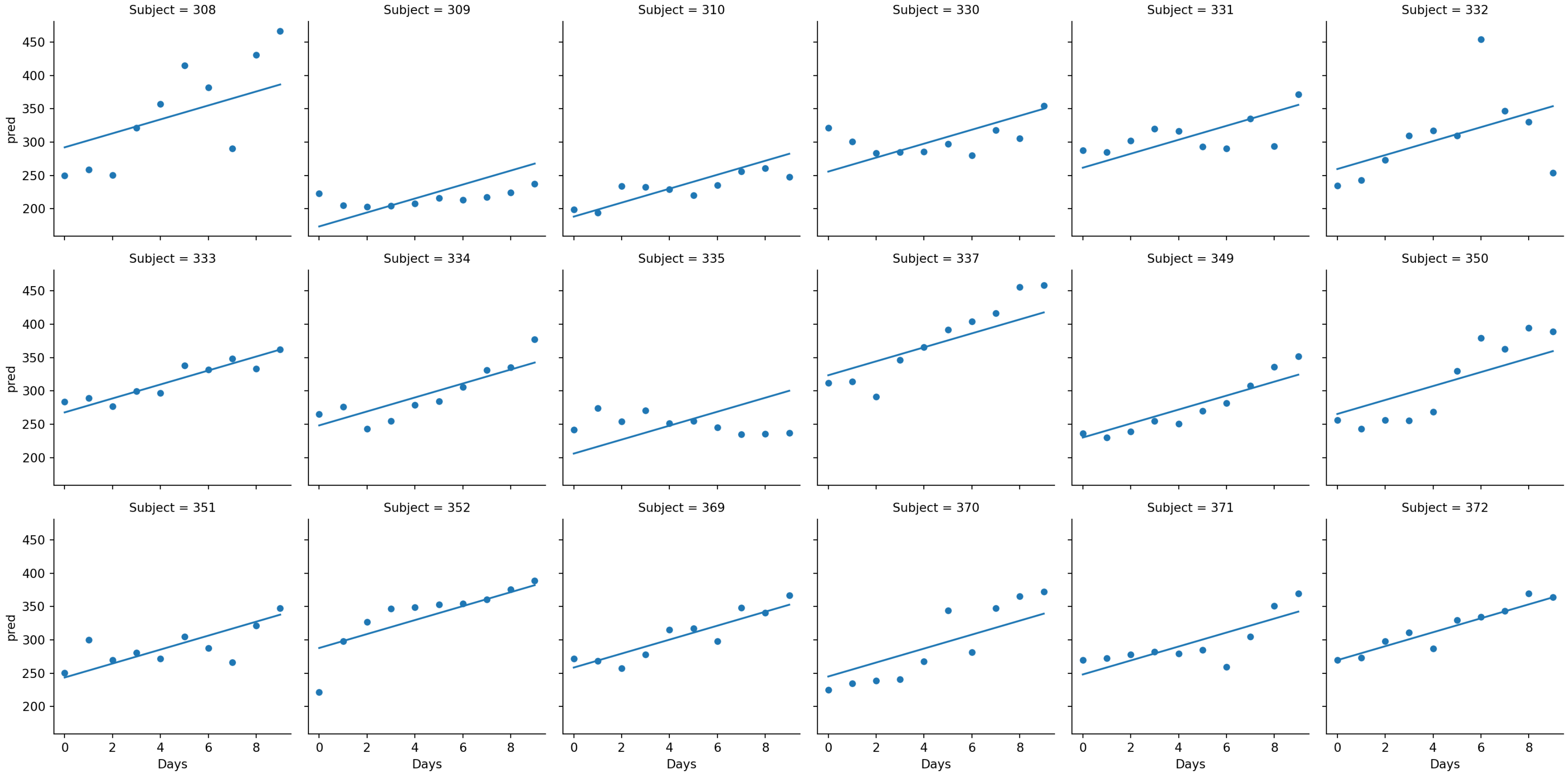
Predictions



The prediction is only taking into account the fixed effects here, not the group random effects.

Recovering random effects for prediction

```
# Dictionary of random effects estimates
re = res_rand_int.random_effects
# Multiply each RE by the random effects design matrix for each group
rex = [np.dot(me_rand_int.exog_re_li[j], re[k]) for (j, k) in enumerate(me_rand_int.group_labels)]
# Add the fixed and random terms to get the overall prediction
y_hat = res_rand_int.predict() + np.concatenate(rex)
```


Subject = 308
Subject = 309
Subject = 310
Subject = 330
Subject = 331
Subject = 332
Subject = 333
Subject = 334
Subject = 335
Subject = 337
Subject = 349
Subject = 350
Subject = 351
Subject = 352
Subject = 369
Subject = 370
Subject = 371
Subject = 372
pred
Days
200
250
300
350
400
450
0
2
4
6
8

Random intercept and slope model

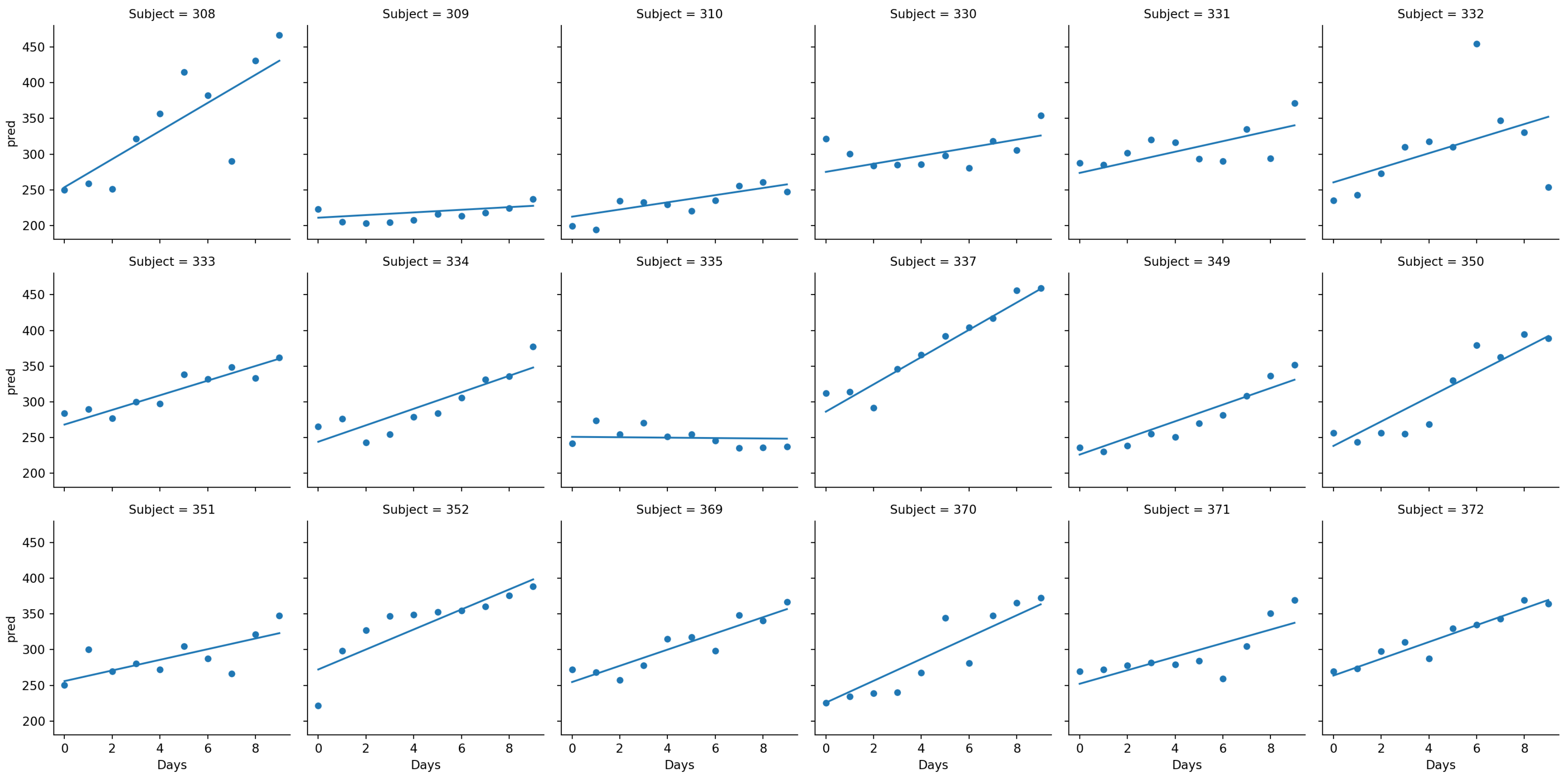
```
me_rand_sl= smf.mixedlm(
  "Reaction ~ Days", data=sleep, groups=sleep["Subject"],
  subset=sleep.Days >= 2,
  re_formula="~Days"
)
res_rand_sl = me_rand_sl.fit(method=["lbfgs"])
print(res_rand_sl.summary())
```

```
            Mixed Linear Model Regression Results
==============================================================
Model:               MixedLM  Dependent Variable:  Reaction
No. Observations:    180      Method:              REML
No. Groups:          18       Scale:               654.9412
Min. group size:     10       Log-Likelihood:      -871.8141
Max. group size:     10       Converged:           Yes
Mean group size:     10.0
--------------------------------------------------------------
                  Coef.  Std.Err.   z    P>|z|  [0.025  0.975]
--------------------------------------------------------------
Intercept        251.405    6.825 36.838 0.000 238.029 264.781
Days              10.467    1.546  6.771 0.000   7.438  13.497
Group Var        612.089   11.881
Group x Days Cov   9.605    1.820
Days Var          35.072    0.610
==============================================================
```

```r
summary(
  lmer(Reaction ~ Days + (Days|Subject), data=sleepstudy)
)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: Reaction ~ Days + (Days | Subject)
   Data: sleepstudy

REML criterion at convergence: 1743.6

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.9536 -0.4634  0.0231  0.4634  5.1793

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Subject  (Intercept) 612.10   24.741
          Days         35.07    5.922   0.07
 Residual             654.94   25.592
Number of obs: 180, groups:  Subject, 18

Fixed effects:
            Estimate Std. Error t value
(Intercept)  251.405      6.825  36.838
Days          10.467      1.546   6.771

Correlation of Fixed Effects:
```

Prediction



We are using the same approach described previously to obtain the RE estimates and use them in the

Odds and ends

t-test and z-test for equality of means

```
books = pd.read_csv("data/daag_books.csv")
cm = sm.stats.CompareMeans(
  sm.stats.DescrStatsW( books.weight[books.cover == "hb"] ),
  sm.stats.DescrStatsW( books.weight[books.cover == "pb"] )
)
```

```
print(cm.summary())
```

```
                          Test for equality of means
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
subset #1    168.3036    136.636      1.232      0.240    -126.880     463.487
==============================================================================
```

```
print(cm.summary(use_t=False))
```

```
                          Test for equality of means
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
subset #1    168.3036    136.636      1.232      0.218     -99.497     436.104
==============================================================================
```

```python
print(cm.summary(usevar="unequal"))
```

```
                          Test for equality of means
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
subset #1    168.3036    136.360      1.234      0.239    -126.686     463.293
==============================================================================
```

Contigency tables

Below are data from the GSS and a survery of Duke students in a intro stats class - the question asked about how concerned the respondent was about the effect of global warming on polar ice cap melt.

```
gss = pd.DataFrame({"US": [454, 226], "Duke": [56,32]}, index=["A great deal", "Not a great deal"])
gss
```

```
                   US  Duke
A great deal      454    56
Not a great deal  226    32
```

```
tbl = sm.stats.Table2x2(gss.to_numpy())
print(tbl.summary())
```

```
               Estimate   SE   LCB    UCB  p-value
--------------------------------------------------
Odds ratio        1.148        0.723 1.823   0.559
Log odds ratio    0.138 0.236 -0.325 0.601   0.559
Risk ratio        1.016        0.962 1.074   0.567
Log risk ratio    0.016 0.028 -0.039 0.071   0.567
--------------------------------------------------
```

```
print(tbl.test_nominal_association())
```

```
df          1
pvalue      0.5587832913935942
```