# pytorch - nn

**Lecture 18** 

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# Odds & Ends

### **Torch models**

#### Implementation details:

- Models are implemented as a class inheriting from torch.nn.Module
- Must implement constructor and forward() method
  - \_\_init\_\_() should call parent constructor via super()
    - Use torch.nn.Parameter() to indicate model parameters
  - forward() should implement the model constants + parameters -> return predictions

#### Fitting proceedure:

- For each iteration of solver:
  - Get current predictions via a call to forward() or equivalent.
  - Calculate a (scalar) loss or equivalent
  - Call backward() method on loss
  - Use built-in optimizer (step() and then zero\_grad() if necessary)

### From last time

```
class Model(torch.nn.Module):
       def __init__(self, X, y, beta=None):
            super().__init__()
 3
           self.X = X
           self_y = y
 6
           if beta is None:
              beta = torch.zeros(X.shape[1])
           beta.requires_grad = True
            self.beta = torch.nn.Parameter(beta)
 9
10
11
       def forward(self, X):
12
            return X @ self.beta
13
14
       def fit(self, opt, n=1000, loss_fn = torch.nn.MSELoss()):
15
         losses = []
16
         for i in range(n):
17
              loss = loss fn(
                self(self.X).squeeze(),
18
19
                self.y.squeeze()
20
              loss.backward()
21
```

### What is self(self.X)?

This is (mostly) just short hand for calling self.forward(X) to generate the output tensors from the current value(s) of the parameters.

This is done via the \_\_call\_\_() method in the torch.nn.Module class. \_\_call\_\_() allows python classes to be invoked like functions.

```
1 class greet:
2  def __init__(self, greeting):
3   self.greeting = greeting
4  def __call__(self, name):
5   return self.greeting + " " + name
```

```
1 hello = greet("Hello")
2 hello("Jane")

'Hello Jane'

1 gm = greet("Good morning")
2 gm("Bob")
```

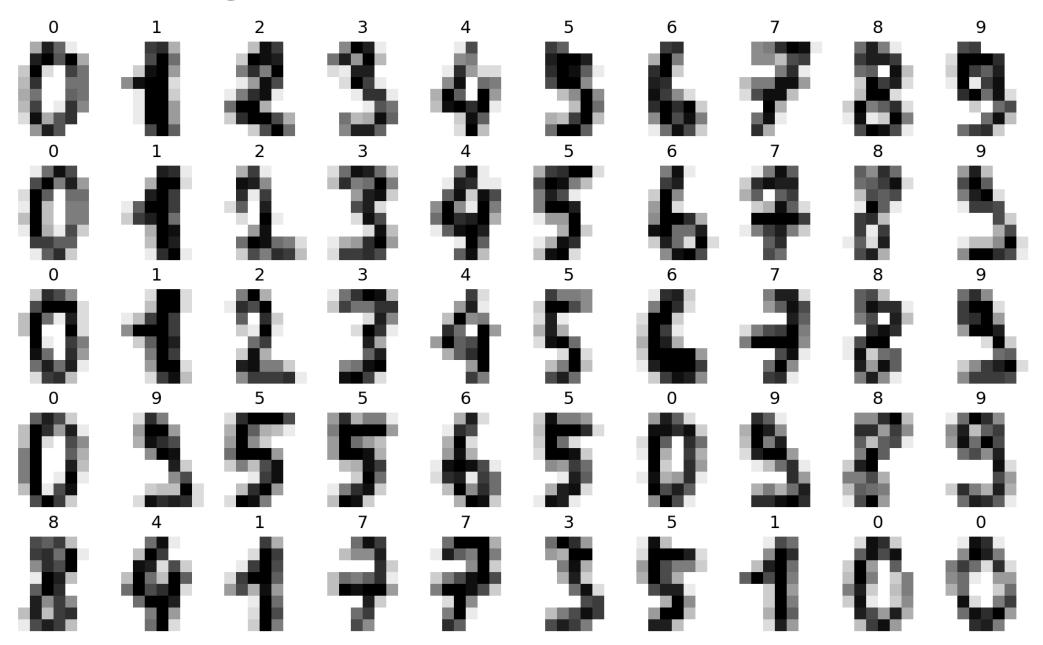
<sup>&#</sup>x27;Good morning Bob'

# MNIST & Logistic models

## MNIST handwritten digits - simplified

```
1 from sklearn.datasets import load_digits
 2 digits = load digits()
 1 X = digits.data
                                              1 y = digits.target
                                              2 y.shape
 2 X.shape
                                             (1797,)
(1797, 64)
 1 X[0:2]
                                              1 y[0:10]
array([[ 0., 0., 5., 13., 9., 1., 0.,
                                             array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
        0., 0., 0., 13., 15., 10., 15.,
        5., 0., 0., 3., 15., 2., 0.,
       11., 8., 0., 0., 4., 12., 0.,
        0., 8., 8., 0., 0., 5., 8.,
        0., 0., 9., 8., 0., 0., 4.,
       11., 0., 1., 12., 7., 0., 0.,
        2., 14., 5., 10., 12., 0., 0.,
        0., 0., 6., 13., 10., 0., 0.,
        0.],
      [0., 0., 0., 12., 13., 5., 0.,
        0., 0., 0., 11., 16., 9.,
        0., 0., 0., 0., 3., 15., 16.,
        6., 0., 0., 0., 7., 15., 16.,
       16., 2., 0., 0., 0., 0., 1.,
       16., 16., 3., 0., 0., 0., 0.,
        1., 16., 16., 6., 0., 0., 0.,
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```

## **Example digits**



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## Test train split

```
from sklearn.model_selection import train_test_split
   X train, X test, y train, y test = train test split(
        X, y, test_size=0.20, shuffle=True, random_state=1234
 5
 1 X train.shape
                                                    1 from sklearn.linear model import LogisticRed
                                                    2 from sklearn.metrics import accuracy_score
(1437, 64)
                                                    1 lr = LogisticRegression(
 1 y_train.shape
                                                        penalty=None
(1437,)
                                                     ).fit(
                                                        X_train, y_train
 1 X_test.shape
                                                    5
(360, 64)
                                                    1 accuracy score(y train, lr.predict(X train)
 1 y test.shape
                                                  1.0
(360.)
                                                    1 accuracy_score(y_test, lr.predict(X_test))
                                                  0.9583333333333333
```

### As Torch tensors

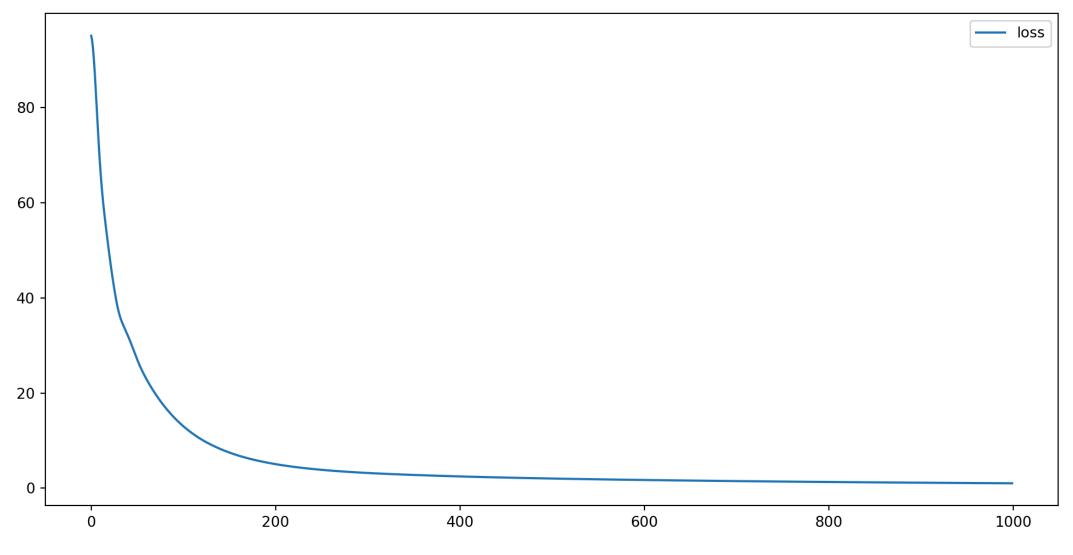
```
1 X_train = torch.from_numpy(X_train).float()
 2 y_train = torch.from_numpy(y_train)
 3 X_test = torch.from_numpy(X_test).float()
 4 y_test = torch.from_numpy(y_test)
 1 X_train.shape
                                              1 X_train.dtype
torch.Size([1437, 64])
                                            torch.float32
 1 y_train.shape
                                              1 y_train.dtype
torch.Size([1437])
                                            torch.int64
 1 X_test.shape
                                              1 X_test.dtype
torch.Size([360, 64])
                                            torch.float32
 1 y_test.shape
                                              1 y_test.dtype
torch.Size([360])
                                            torch.int64
```

## **PyTorch Model**

```
class mnist model(torch.nn.Module):
       def __init__(self, input_dim, output_dim):
 2
           super().__init__()
           self.beta = torch.nn.Parameter(
 4
             torch.randn(input_dim, output_dim, requires_grad=True)
 6
           self.intercept = torch.nn.Parameter(
             torch.randn(output dim, requires grad=True)
 9
10
11
       def forward(self, X):
12
            return (X @ self.beta + self.intercept).squeeze()
13
14
       def fit(self, X train, y train, X test, y test, lr=0.001, n=1000):
15
         opt = torch.optim.SGD(self.parameters(), lr=lr, momentum=0.9)
16
         losses = []
17
18
         for i in range(n):
19
             opt.zero_grad()
20
              loss = torch.nn.CrossEntropyLoss()(self(X train), y train)
             loss.backward()
21
```

## Cross entropy loss

```
1 model = mnist_model(64, 10)
2 l = model.fit(X_train, y_train, X_test, y_test)
```



## **Cross entropy loss**

From the pytorch documentation:

$$\ell(x,y) = L = \{l_1, \dots, l_N\}^{\top}, \quad l_n = -w_{y_n} \log \frac{\exp(x_{n,y_n})}{\sum_{c=1}^{C} \exp(x_{n,c})}$$

$$\ell(x,y) = \begin{cases} \sum_{n=1}^{N} \frac{1}{\sum_{n=1}^{N} w_{y_n} \cdot 1\{y_n \neq \text{ignore\_index}\}} l_n, & \text{if reduction} = \text{'mean'} \\ \sum_{n=1}^{N} l_n, & \text{if reduction} = \text{'sum'} \end{cases}$$

## **Out-of-sample accuracy**

```
model(X test)
tensor([[ 28.3264, -33.8749, -0.1270,
         32.7894. -10.8112. 8.8204.
         -6.9321, 50.0344, 17.1113,
         14.7006],
       [ 4.6819, 83.2354, 0.8541,
         56.6228, -11.0546, -18.0684,
         40.1415, -1.1687, 30.8871,
         73.6600],
       [ 25.8626, -8.2121, 2.2649,
         20.2590, -19.4369, -31.6355,
        -24.8725, 83.7362, 16.6910,
          9.7738],
       [ 58.6064, 8.9680, -2.1831,
        -28.0751, 16.9030, 24.5250,
        120.5182, -34.8430, 57.8929,
        -34.2762],
       [ 89.7358, 28.3815, 2.2295,
          1 6157
                  E4 677E
                            21 6604
```

```
1 val, index = torch.max(model(X test), dim=1
 2 index
tensor([7, 1, 7, 6, 0, 2, 4, 3, 6, 3, 7, 8, 7,
       9, 4, 3, 8, 7, 8, 4, 0, 3, 9, 9, 3, 6,
       6, 0, 5, 4, 1, 2, 1, 2, 3, 2, 7, 6, 5,
       8, 6, 4, 4, 0, 9, 2, 8, 5, 4, 4, 4, 1,
       7, 6, 8, 2, 9, 9, 9, 0, 1, 3, 1, 8, 8,
       8, 3, 9, 1, 3, 9, 6, 9, 5, 8, 1, 9, 2,
       1, 3, 8, 7, 3, 3, 8, 7, 8, 5, 8, 2, 6,
       1, 9, 1, 6, 4, 5, 2, 2, 4, 5, 4, 3, 6,
       5, 7, 2, 4, 1, 0, 7, 6, 1, 2, 9, 5, 2,
       5, 0, 3, 2, 7, 6, 4, 8, 2, 1, 1, 6, 9,
       6, 8, 2, 4, 7, 5, 0, 9, 1, 0, 5, 6, 7,
       6, 3, 1, 3, 2, 0, 4, 4, 3, 5, 4, 6, 1,
       1, 9, 6, 2, 7, 9, 0, 7, 9, 5, 4, 1, 3,
       8, 6, 4, 7, 1, 5, 7, 4, 7, 4, 3, 2, 2,
       1, 1, 4, 4, 3, 5, 5, 9, 4, 5, 5, 9, 3,
       9, 6, 1, 2, 0, 8, 2, 9, 9, 2, 4, 6, 8,
        3, 8, 1, 0, 8, 1, 8, 5, 6, 8, 7, 1, 8,
```

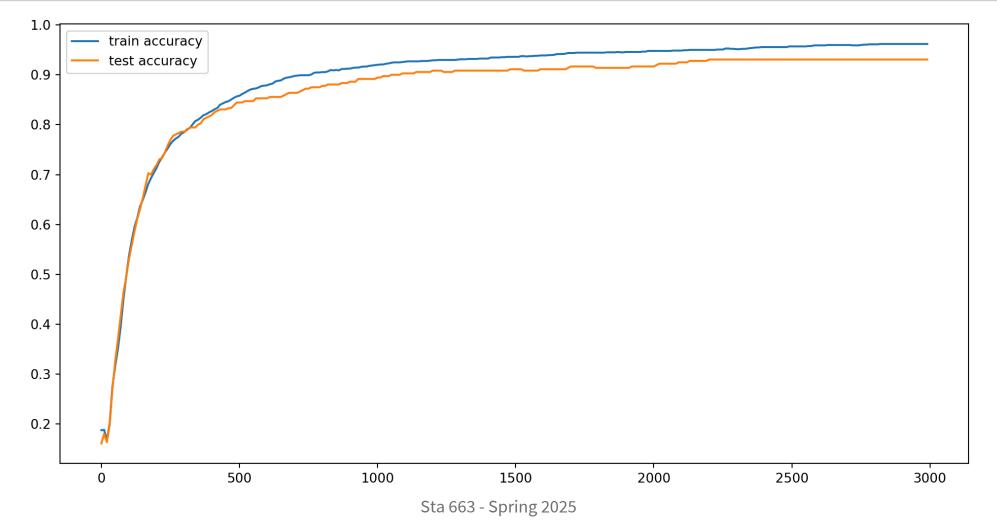
```
1 (index == y_test).sum()
tensor(324)
1 (index == y_test).sum() / len(y_test)
```

## **Calculating Accuracy**

```
class mnist model(torch.nn.Module):
       def __init__(self, input_dim, output_dim):
 2
           super().__init__()
           self.beta = torch.nn.Parameter(
 4
             torch.randn(input_dim, output_dim, requires_grad=True)
 6
           self.intercept = torch.nn.Parameter(
             torch.randn(output dim, requires grad=True)
 9
10
11
       def forward(self, X):
           return (X @ self.beta + self.intercept).squeeze()
12
13
14
       def fit(self, X train, y train, X test, y test, lr=0.001, n=1000, acc step=10)
15
         opt = torch.optim.SGD(self.parameters(), lr=lr, momentum=0.9)
16
         losses, train acc, test acc = [], [], []
17
18
         for i in range(n):
19
             opt.zero grad()
20
              loss = torch.nn.CrossEntropyLoss()(self(X train), y train)
21
             loss.backward()
```

### Performance

```
1 loss, train_acc, test_acc = mnist_model(
2   64, 10
3 ).fit(
4   X_train, y_train, X_test, y_test, acc_step=10, n=3000
5 )
```



### **NN Layers**

```
class mnist nn model(torch.nn.Module):
       def __init__(self, input_dim, output_dim):
 2
           super().__init__()
 3
           self.linear = torch.nn.Linear(input dim, output dim)
 4
       def forward(self, X):
 6
            return self.linear(X)
 9
       def fit(self, X train, y train, X test, y test, lr=0.001, n=1000, acc step=10)
         opt = torch.optim.SGD(self.parameters(), lr=lr, momentum=0.9)
10
11
         losses, train acc, test acc = [], [], []
12
13
         for i in range(n):
14
              opt.zero grad()
15
              loss = torch.nn.CrossEntropyLoss()(self(X_train), y_train)
16
              loss.backward()
17
             opt.step()
18
              losses.append(loss.item())
19
20
              if (i+1) % acc step == 0:
                val, train_pred = torch.max(self(X_train), dim=1)
21
```

## NN linear layer

Applies a linear transform to the incoming data (X):

$$y = XA^T + b$$

```
1 X.shape
(1797, 64)

1 model = mnist_nn_model(64, 10)
2 model.parameters()

<generator object Module.parameters at 0x3281919a0>

1 list(model.parameters())[0].shape # A - weights (betas)

torch.Size([10, 64])

1 list(model.parameters())[1].shape # b - bias

torch.Size([10])
```

### **Performance**

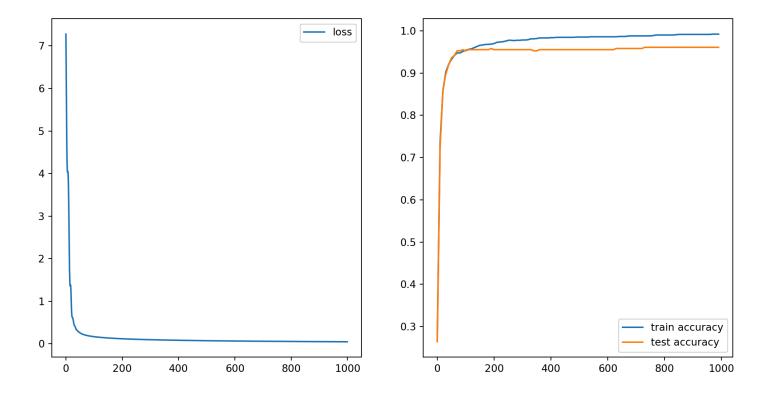
```
1 loss, train_acc, test_acc = model.fit(X_train, y_train, X_test, y_test, n=1000)

1 train_acc[-5:]

[tensor(0.9916), tensor(0.9916), tensor(0.9923), tensor(0.9923)]

[tensor(0.9923), tensor(0.9923), tensor(0.9923)]

[tensor(0.9611), tensor(0.9611), tensor(0.9611)]
```



## Feedforward Neural Network

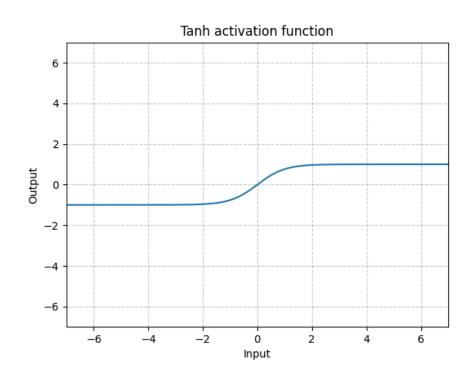
### **FNN Model**

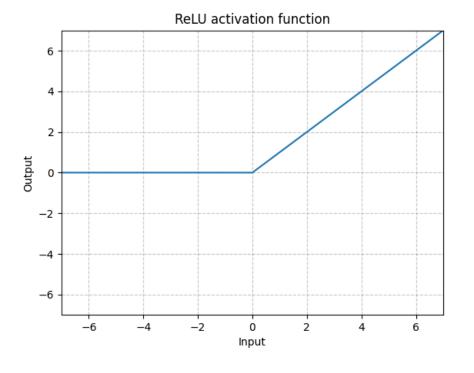
```
class mnist fnn model(torch.nn.Module):
       def __init__(self, input_dim, hidden_dim, output_dim, nl_step = torch.nn.ReLU(
 2
           super().__init__()
 3
           self.l1 = torch.nn.Linear(input dim, hidden dim)
 4
           self.nl = nl step
           self.l2 = torch.nn.Linear(hidden_dim, output_dim)
 6
       def forward(self, X):
 8
           out = self.l1(X)
 9
           out = self.nl(out)
10
11
           out = self.l2(out)
12
           return out
13
14
       def fit(self, X train, y train, X test, y test, lr=0.001, n=1000, acc step=10)
15
         opt = torch.optim.SGD(self.parameters(), lr=lr, momentum=0.9)
16
         losses, train acc, test acc = [], [], []
17
18
         for i in range(n):
19
             opt.zero grad()
20
             loss = torch.nn.CrossEntropyLoss()(self(X train), y train)
21
             loss.backward()
```

### Non-linear activation functions

$$Tanh(x) = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)}$$

$$ReLU(x) = max(0, x)$$





### Model parameters

```
model = mnist_fnn_model(64,64,10)
 2 len(list(model.parameters()))
4
 1 for i, p in enumerate(model.parameters()):
      print("Param", i, p.shape)
Param 0 torch.Size([64, 64])
Param 1 torch Size([64])
Param 2 torch.Size([10, 64])
Param 3 torch.Size([10])
```

### **Performance - ReLU**

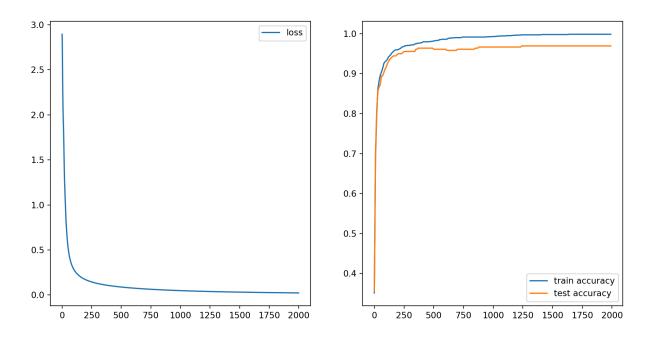
```
1 loss, train_acc, test_acc = mnist_fnn_model(64,64,10).fit(
2  X_train, y_train, X_test, y_test, n=2000
3 )
```

```
[0.9986082115518441, 0.9986082115518441, 0.9986082115518441, 0.9986082115518441, 0.9986082115518441]
```

train\_acc[-5:]

```
1 test_acc[-5:]
```

[0.9694444444444444, 0.9694444444444444, 0.9694444444444444, 0.9694444444444444, 0.9694444444444444]



### Performance - tanh

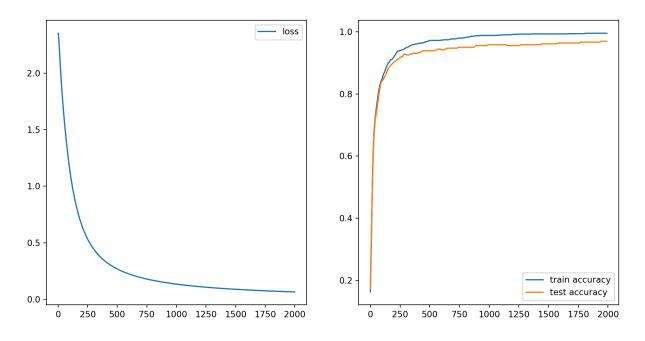
train\_acc[-5:]

```
1 loss, train_acc, test_acc = mnist_fnn_model(64,64,10, nl_step=torch.nn.Tanh()).fit(
2  X_train, y_train, X_test, y_test, n=2000
3 )
```

```
[0.9951287404314544, 0.9951287404314544, 0.9951287404314544, 0.9951287404314544, 0.9951287404314544]
```

```
1 test_acc[-5:]
```

[0.9694444444444444, 0.9694444444444444, 0.9694444444444444, 0.9694444444444444, 0.9694444444444444]



## Adding another layer

```
class mnist fnn2 model(torch.nn.Module):
       def init (self, input dim, hidden dim, output dim, nl step = torch.nn.ReLU(), seed=123
           super(). init__()
           self.l1 = torch.nn.Linear(input_dim, hidden_dim)
 4
 5
           self.nl = nl step
           self.l2 = torch.nn.Linear(hidden_dim, hidden_dim)
 6
           self.nl = nl_step
           self.l3 = torch.nn.Linear(hidden dim, output dim)
 8
9
       def forward(self, X):
10
           out = self.l1(X)
11
12
           out = self.nl(out)
13
           out = self.l2(out)
14
           out = self.nl(out)
           out = self.l3(out)
15
16
           return out
17
       def fit(self, X_train, y_train, X_test, y_test, lr=0.001, n=1000, acc_step=10):
18
19
         loss fn = torch.nn.CrossEntropyLoss()
         opt = torch.optim.SGD(self.parameters(), lr=lr, momentum=0.9)
20
21
         losses, train acc, test_acc = [], [], []
22
23
         for i in range(n):
             opt.zero_grad()
24
```

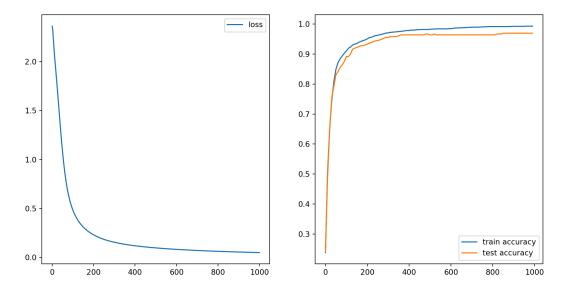
### Performance - relu

```
loss, train_acc, test_acc = mnist_fnn2_model(
    64,64,10, nl_step=torch.nn.ReLU()
    lit(
        X_train, y_train, X_test, y_test, n=1000
    lit()
```

```
[0.9923451635351427, 0.9930410577592206, 0.9930410577592206, 0.9930410577592206, 0.9930410577592206]
```

1 train acc[-5:]

```
1 test_acc[-5:]
```



### Performance - tanh

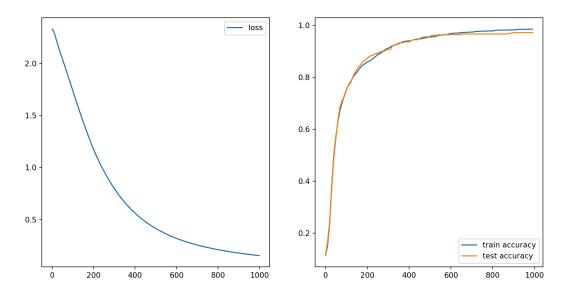
```
1 loss, train_acc, test_acc = mnist_fnn2_model(
2  64,64,10, nl_step=torch.nn.Tanh()
3 ).fit(
4  X_train, y_train, X_test, y_test, n=1000
5 )
```

```
[0.9846903270702854, 0.9846903270702854, 0.9846903270702854, 0.9853862212943633, 0.9853862212943633]
```

1 train acc[-5:]

#### 1 test\_acc[-5:]

[0.97222222222222, 0.97222222222222, 0.97222222222222, 0.972222222222222, 0.97222222222222]

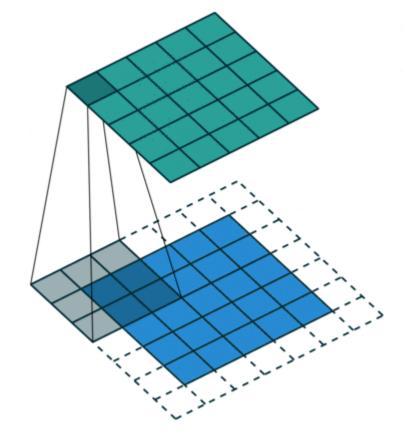


## **Convolutional NN**

## 2d convolutions

30	3	$2_2$	1	0
02	$0_2$	$1_0$	3	1
30	1,	22	2	3
2	0	0	2	2
2	0	0	0	1

12.0	12.0	17.0
10.0	17.0	19.0
9.0	6.0	14.0



### nn.Conv2d()

```
1 cv = torch.nn.Conv2d(
2  in_channels=1, out_channels=4,
3  kernel_size=3,
4  stride=1, padding=1
5 )
```

```
1 list(cv.parameters())[0] # kernel weights
Parameter containing:
tensor([[[-0.1167, -0.1832, 0.0059],
         [0.2188, -0.0119, -0.0130],
         [0.2176, 0.2637, -0.1447]]
       [[-0.0134, 0.1201, 0.0502],
         [-0.1689, 0.0637, 0.0928],
         [-0.0445, -0.1124, 0.1476]]
       [[-0.0168, -0.2369, 0.1777],
         [0.1322, -0.1596, 0.0169],
         [-0.1928, 0.3038, -0.1910]],
       [[[0.2350, -0.1067, 0.0957],
         [-0.2044, 0.0171, 0.3302],
         [0.0489, 0.2266, -0.2085]]]],
      requires grad=True)
```

```
1 list(cv.parameters())[1] # biases
```

```
Parameter containing:
tensor([ 0.1893,  0.2009, -0.2069, -0.2023],
requires_grad=True)
```

## Applying Conv2d()

```
1 X train[[0]]
tensor([[ 0., 0., 0., 10., 11., 0., 0.,
         0., 0., 0., 9., 16., 6.,
             0., 0., 0., 15., 13.,
             0., 0., 0., 0., 14., 10.,
         0., 0., 0., 0., 0., 1., 15.,
        12., 8., 2., 0., 0., 0., 0.,
        12., 16., 16., 16., 10., 1.,
         0., 7., 16., 12., 12., 16.,
         0., 0., 0., 9., 15., 12., 5.,
         0.]])
 1 X_train[[0]].shape
torch.Size([1, 64])
 1 cv(X train[[0]])
RuntimeError: Expected 3D (unbatched) or 4D
(batched) input to conv2d, but got input of
size: [1, 64]
```

```
tensor([[[ 0., 0., 0., 10., 11., 0., 0.,
                  0.],
                 [0., 0., 9., 16., 6., 0., 0.,
                  0.],
                 [ 0., 0., 15., 13., 0., 0., 0.,
                  0.],
                 [ 0., 0., 14., 10., 0., 0., 0.,
                  0.],
                 [ 0., 1., 15., 12., 8., 2., 0.,
                  0.],
                 [ 0., 0., 12., 16., 16., 16., 10.,
                  1.],
                 [ 0., 0., 7., 16., 12., 12., 16.,
                  4.],
                 [0., 0., 0., 9., 15., 12., 5.,
                  0.]]])
         1 cv(X train[[0]].view(1,8,8))
        tensor([[[ 0.1893, -1.1129, 0.1171, 5.2364,
                  7.3098, 3.9011, 0.1893, 0.1893],
                 [0.1893, -2.0983, 2.0070, 6.8142,
                  3.2647, 0.2183, 0.1893, 0.1893],
                 [0.1893, -1.9791, 0.5317,
                                            5.0533.
                  2.2427, -0.5109, 0.1893,
                                            0.1893],
                 [0.0446, -1.8120, -0.3422, 4.2718,
                  5.2909, 2.4572, 0.6244, 0.1893],
                 [0.1762, -1.6723, -1.5831, 4.2735,
                  6.9118, 8.1689, 6.6000, 2.6286],
                 [0.1951, -1.0753, -3.4250, 2.5202,
                  5.3464, 5.5298, 9.5757, 6.9009],
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                 [ 0.1893.  0.1683. -3.5091. -2.6602.
```

1 X train[[0]].view(1,8,8)

## **Pooling**

```
1 x = torch.tensor(
      [[[0,0,0,0]]]
      [0,1,2,0],
      [0,3,4,0],
 5
      [0,0,0,0]],
      dtype=torch.float
 6
 8 x.shape
torch.Size([1, 4, 4])
 1 torch.nn.MaxPool2d(
                                                    1 torch.nn.AvgPool2d(
      kernel size=2, stride=1
                                                        kernel size=2
 3 )(x)
                                                    3 )(x)
tensor([[[1., 2., 2.],
                                                  tensor([[[0.2500, 0.5000],
         [3., 4., 4.],
                                                           [0.7500, 1.0000]]])
         [3., 4., 4.]]])
                                                    1 torch.nn.AvgPool2d(
 1 torch.nn.MaxPool2d(
                                                        kernel size=2, padding=1
      kernel size=3, stride=1, padding=1
                                                    3 )(x)
 3 )(x)
                                                  tensor([[[0.0000, 0.0000, 0.0000],
tensor([[[1., 2., 2., 2.],
                                                           [0.0000, 2.5000, 0.0000],
         [3., 4., 4., 4.],
                                                           [0.0000, 0.0000, 0.0000]]
         [3., 4., 4., 4.],
         [3., 4., 4., 4.]]])
```

### Convolutional model

```
class mnist_conv_model(torch.nn.Module):
       def __init__(self):
 2
           super().__init__()
 3
           self.cnn = torch.nn.Conv2d(
 4
              in_channels=1, out_channels=8,
 6
              kernel size=3, stride=1, padding=1
           self.relu = torch.nn.ReLU()
           self.pool = torch.nn.MaxPool2d(kernel_size=2)
 9
            self.lin = torch.nn.Linear(8 * 4 * 4, 10)
10
11
       def forward(self, X):
12
           out = self.cnn(X.view(-1, 1, 8, 8))
13
14
           out = self.relu(out)
15
           out = self.pool(out)
           out = self.lin(out.view(-1, 8 * 4 * 4))
16
17
            return out
18
19
       def fit(self, X train, y train, X test, y test, lr=0.001, n=1000, acc step=10)
20
         loss fn = torch.nn.CrossEntropyLoss()
         opt = torch.optim.SGD(self.parameters(), lr=lr, momentum=0.9)
21
```

#### Performance

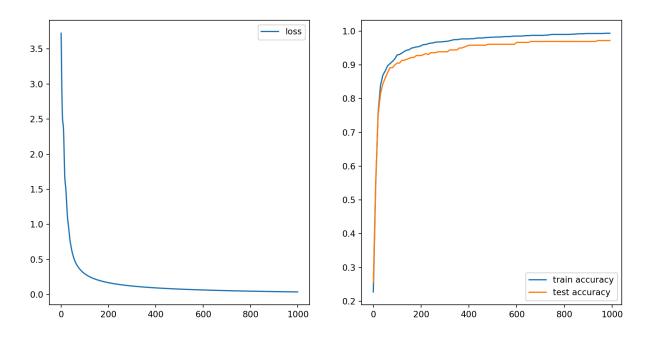
```
1 loss, train_acc, test_acc = mnist_conv_model().fit(
2  X_train, y_train, X_test, y_test, n=1000
3 )
```

```
1 train_acc[-5:]
[0.9930410577592206, 0.9930410577592206,
```

0.9937369519832986, 0.9937369519832986,
0.9937369519832986]

#### 1 test\_acc[-5:]

[0.97222222222222, 0.972222222222222, 0.972222222222222, 0.972222222222222, 0.972222222222222]



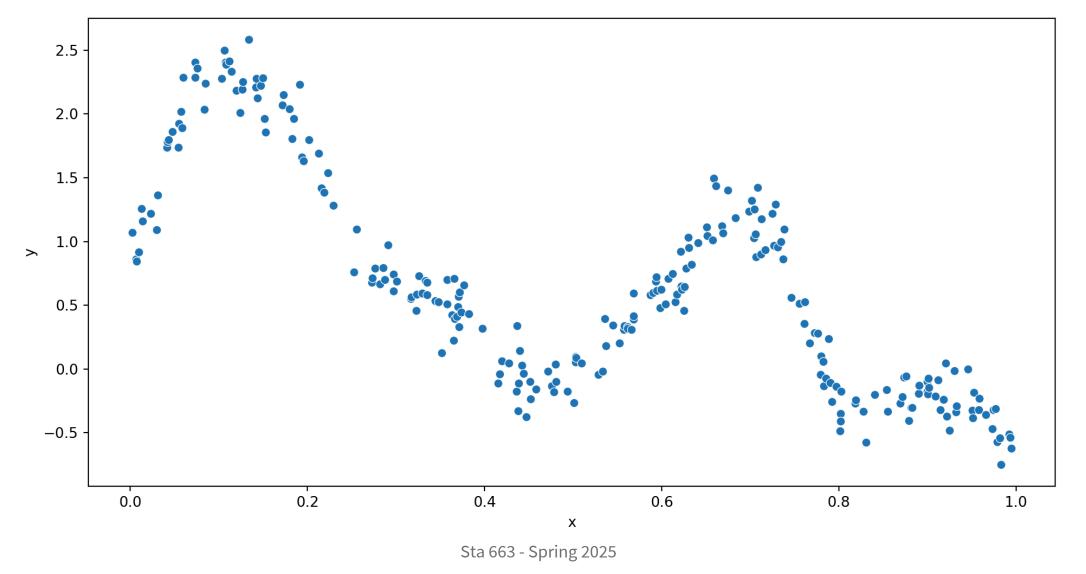
## Organizing models

```
class mnist conv model2(torch.nn.Module):
       def init (self):
           super().__init__()
 3
           self.model = torch.nn.Sequential(
 4
 5
             torch.nn.Unflatten(1, (1,8,8)),
             torch.nn.Conv2d(
 6
                in channels=1, out channels=8,
                kernel size=3, stride=1, padding=1
 8
9
             torch.nn.ReLU(),
10
             torch.nn.MaxPool2d(kernel size=2),
11
             torch.nn.Flatten(),
12
             torch.nn.Linear(8 * 4 * 4, 10)
13
14
15
       def forward(self, X):
16
           return self.model(X)
17
18
19
       def fit(self, X_train, y_train, X_test, y_test, lr=0.001, n=1000, acc_step=10):
         opt = torch.optim.SGD(self.parameters(), lr=lr, momentum=0.9)
20
21
         losses, train acc, test acc = [], [], []
22
23
         for i in range(n):
24
             opt.zero grad()
```

# A bit more on non-linear activation layers

## Non-linear functions

```
1 df = pd.read_csv("data/gp.csv")
2 X = torch.tensor(df["x"], dtype=torch.float32).reshape(-1,1)
3 y = torch.tensor(df["y"], dtype=torch.float32)
```

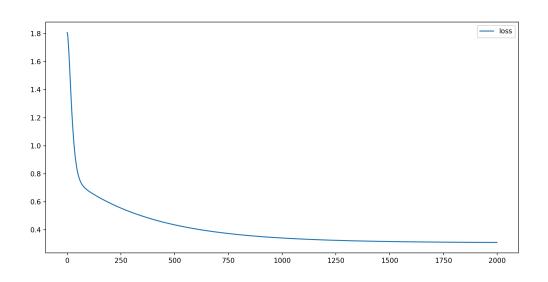


## **Linear regression**

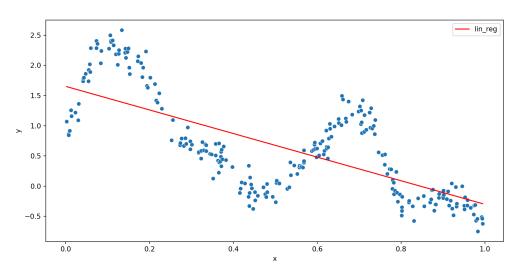
```
class lin_reg(torch.nn.Module):
       def __init__(self, X):
            super().__init__()
           self.n = X.shape[0]
 4
           self.p = X.shape[1]
 5
            self.model = torch.nn.Sequential(
 6
              torch.nn.Linear(self.p, self.p)
 8
 9
       def forward(self, X):
10
            return self.model(X)
11
12
13
       def fit(self, X, y, n=1000):
         losses = []
14
15
         opt = torch.optim.SGD(self.parameters(), lr=0.001, momentum=0.9)
         for i in range(n):
16
              loss = torch.nn.MSELoss()(self(X).squeeze(), y)
17
              loss.backward()
18
19
              opt.step()
              opt.zero_grad()
20
21
              losses.append(loss.item())
22
          return losses
23
```

```
1 m1 = lin_reg(X)
2 loss = m1.fit(X,y, n=2000)
```

#### Training loss:



#### Predictions

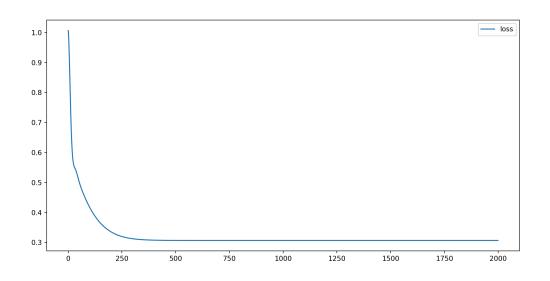


## Double linear regression

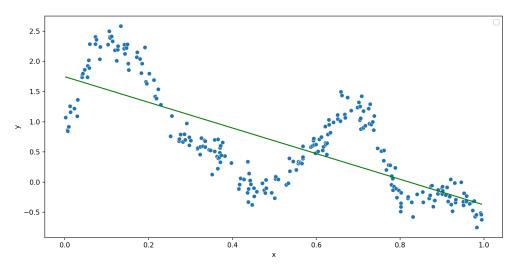
```
class dbl_lin_reg(torch.nn.Module):
       def __init__(self, X, hidden_dim=10):
            super(). init ()
           self.n = X.shape[0]
 4
            self.p = X.shape[1]
 5
            self.model = torch.nn.Sequential(
 6
              torch.nn.Linear(self.p, hidden_dim),
              torch.nn.Linear(hidden dim, 1)
 9
10
       def forward(self, X):
11
            return self.model(X)
12
13
14
       def fit(self, X, y, n=1000):
         losses = []
15
         opt = torch.optim.SGD(self.parameters(), lr=0.001, momentum=0.9)
16
         for i in range(n):
17
              loss = torch.nn.MSELoss()(self(X).squeeze(), y)
18
19
              loss.backward()
20
              opt.step()
21
              opt.zero grad()
22
              losses.append(loss.item())
23
24
         return losses
```

```
1 m2 = dbl_lin_reg(X, hidden_dim=10)
```

#### Training loss:



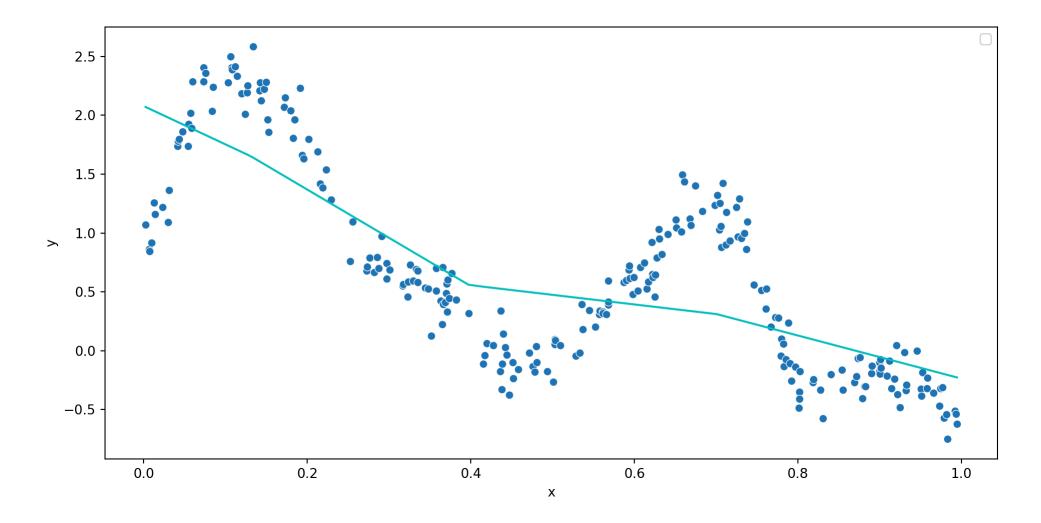
#### **Predictions**



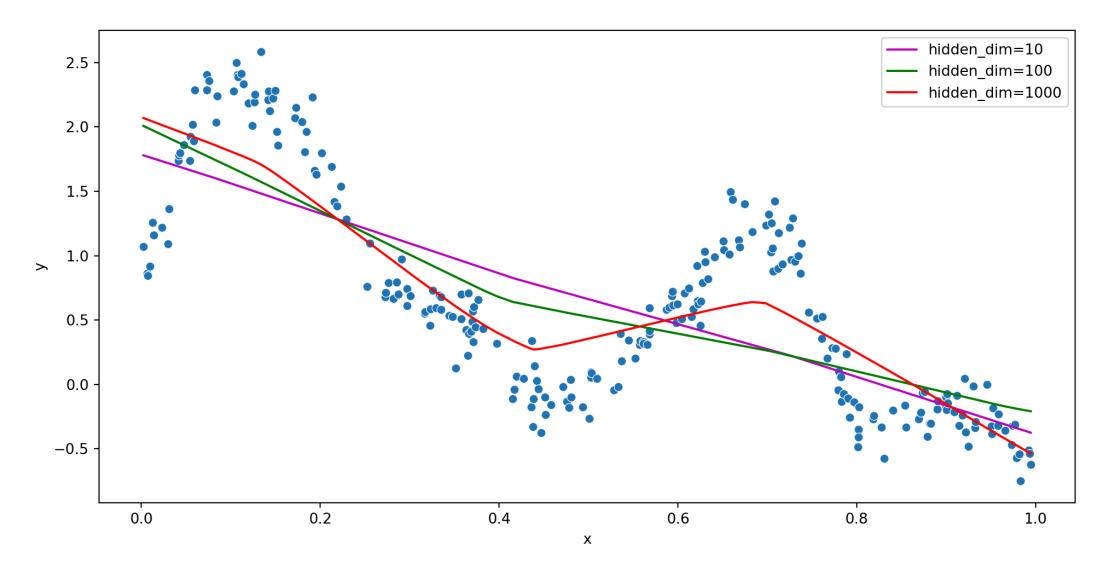
<sup>2</sup> loss = m2.fit(X,y, n=2000)

## Non-linear regression w/ ReLU

```
class lin_reg_relu(torch.nn.Module):
       def __init__(self, X, hidden_dim=100):
            super(). init ()
           self.n = X.shape[0]
 4
            self.p = X.shape[1]
 5
            self.model = torch.nn.Sequential(
 6
              torch.nn.Linear(self.p, hidden_dim),
              torch.nn.ReLU(),
 8
              torch.nn.Linear(hidden dim, 1)
 9
10
11
12
       def forward(self, X):
13
            return self.model(X)
14
15
       def fit(self, X, y, n=1000):
         losses = []
16
         opt = torch.optim.SGD(self.parameters(), lr=0.001, momentum=0.9)
17
         for i in range(n):
18
19
              loss = torch.nn.MSELoss()(self(X).squeeze(), y)
              loss.backward()
20
21
              opt.step()
22
              opt.zero grad()
              losses.append(loss.item())
23
24
```



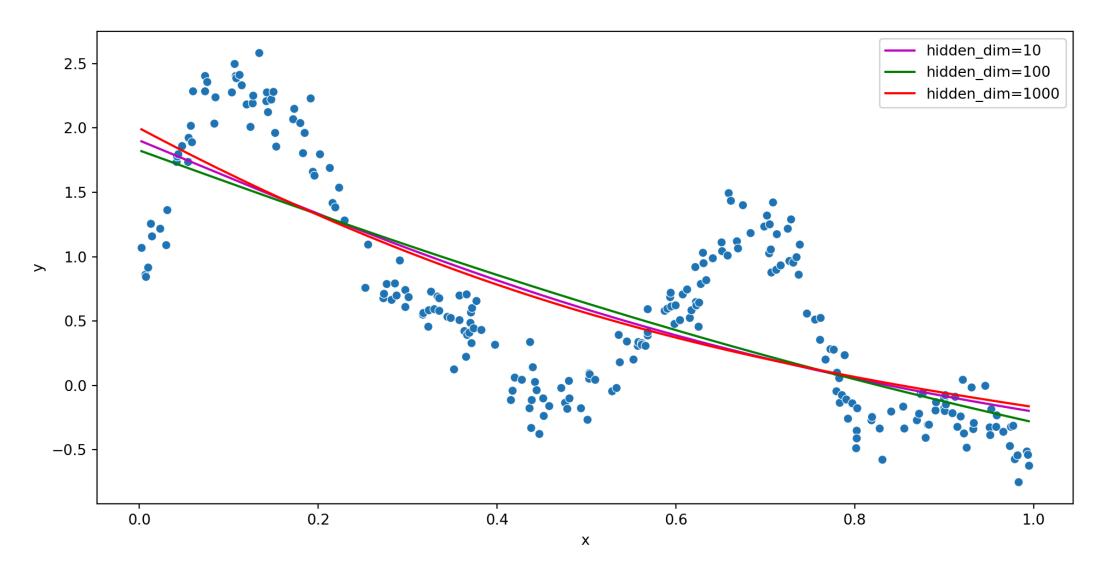
## **Hidden dimensions**



## Non-linear regression w/ Tanh

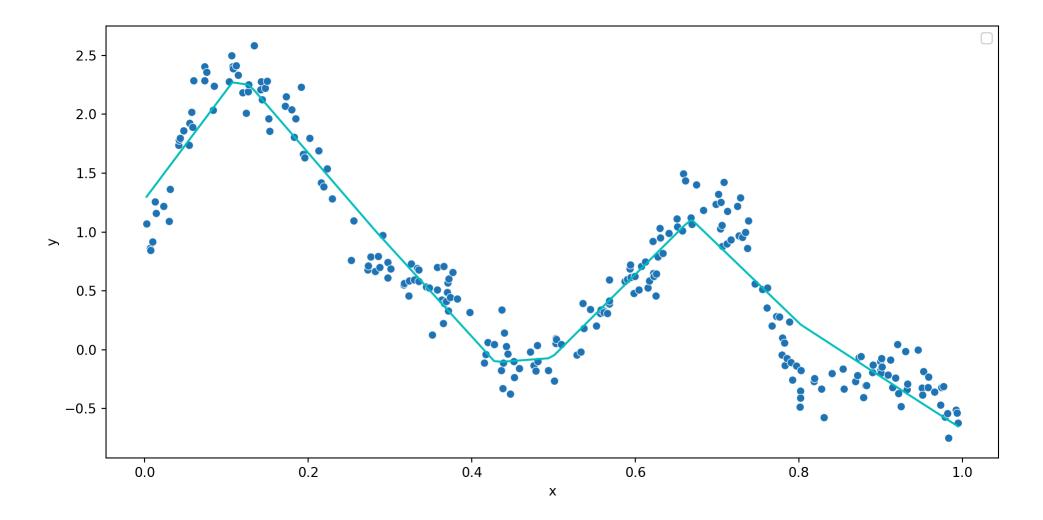
```
class lin_reg_tanh(torch.nn.Module):
       def __init__(self, X, hidden dim=10):
            super(). init ()
           self.n = X.shape[0]
 4
            self.p = X.shape[1]
 5
            self.model = torch.nn.Sequential(
 6
              torch.nn.Linear(self.p, hidden_dim),
              torch.nn.Tanh(),
              torch.nn.Linear(hidden dim, 1)
 9
10
11
12
       def forward(self, X):
13
            return self.model(X)
14
15
       def fit(self, X, y, n=1000):
         losses = []
16
         opt = torch.optim.SGD(self.parameters(), lr=0.001, momentum=0.9)
17
         for i in range(n):
18
19
              loss = torch.nn.MSELoss()(self(X).squeeze(), y)
              loss.backward()
20
21
              opt.step()
22
              opt.zero grad()
              losses.append(loss.item())
23
24
```

## Tanh & hidden dimension



## Three layers

```
class three_layers(torch.nn.Module):
       def __init__(self, X, hidden_dim=100):
           super(). init ()
           self.n = X.shape[0]
 4
           self.p = X.shape[1]
 5
           self.model = torch.nn.Sequential(
 6
              torch.nn.Linear(self.p, hidden_dim),
              torch.nn.ReLU(),
 8
              torch.nn.Linear(hidden_dim, hidden_dim),
 9
              torch.nn.ReLU(),
10
              torch.nn.Linear(hidden dim, 1)
11
12
13
       def forward(self, X):
14
           return self.model(X)
15
16
17
       def fit(self, X, y, n=1000):
         losses = []
18
19
         opt = torch.optim.SGD(self.parameters(), lr=0.001, momentum=0.9)
         for i in range(n):
20
              loss = torch.nn.MSELoss()(self(X).squeeze(), y)
21
22
              loss.backward()
23
             opt.step()
24
              opt.zero grad()
```



## Five layers

```
class five_layers(torch.nn.Module):
       def init (self, X, hidden dim=100):
           super(). init ()
           self.n = X.shape[0]
 4
           self.p = X.shape[1]
 5
           self.model = torch.nn.Sequential(
 6
             torch.nn.Linear(self.p, hidden_dim),
             torch.nn.ReLU(),
 8
             torch.nn.Linear(hidden_dim, hidden_dim),
 9
             torch.nn.ReLU(),
10
              torch.nn.Linear(hidden_dim, hidden_dim),
11
             torch.nn.ReLU(),
12
13
             torch.nn.Linear(hidden_dim, hidden_dim),
             torch.nn.ReLU(),
14
             torch.nn.Linear(hidden dim, 1)
15
16
17
       def forward(self, X):
18
19
           return self.model(X)
20
21
       def fit(self, X, y, n=1000):
22
         losses = []
23
         opt = torch.optim.SGD(self.parameters(), lr=0.001, momentum=0.9)
24
         for i in range(n):
```

