Maximum likelihood estimation for logistic regression

Recap: Newton's method

Example

Suppose that $\log\left(\dfrac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 X_i$, and we have

$$\beta^{(r)} = \begin{bmatrix} -3.1 \\ 0.9 \end{bmatrix}, \qquad U(\beta^{(r)}) = \begin{bmatrix} 9.16 \\ 31.91 \end{bmatrix},$$

$$\mathbf{H}(\beta^{(r)}) = -\begin{bmatrix} 17.834 & 53.218 \\ 53.218 & 180.718 \end{bmatrix}$$

> Use Newton's method to calculate $\beta^{(r+1)}$ (you may use R or a calculator, you do not need to do the matrix arithmetic by hand).

Newton's method for logistic regression

Checking the solution is a unique maximum

> Newton's method finds β^* such that $U(\beta^*) = 0$. How do we know that β^* maximizes the likelihood?

Some intuition about Hessians

Fisher information

Properties

Example