Likelihood ratio tests

Recap: likelihood ratio test

Let $X_1, \ldots, X_n$ be a sample from a distribution with parameter $\theta \in \mathbb{R}^d$. We wish to test $H_0 : \theta \in \Theta_0$ vs. $H_A : \theta \in \Theta_1$.

The **likelihood ratio test** (LRT) rejects H_0 when

$$\frac{\sup\limits_{\theta \in \Theta_1} L(\theta|\mathbf{X})}{\sup\limits_{\theta \in \Theta_0} L(\theta|\mathbf{X})} > k,$$

where k is chosen such that $\sup\limits_{\theta \in \Theta_0} \beta_{LR}(\theta) \leq \alpha$.

Example: linear regression with normal data

Suppose we observe $(X_1, Y_1), \ldots, (X_n, Y_n)$, where $Y_i = \beta^T X_i + \varepsilon_i$ and $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$. Partition $\beta = (\beta_{(1)}, \beta_{(2)})^T$. We wish to test $H_0 : \beta_{(2)} = 0$ vs. $H_A : \beta_{(2)} \neq 0$.

Example: Poisson sample

Let $X_1, \ldots, X_n \overset{iid}{\sim} Poisson(\lambda)$. We wish to test $H_0 : \lambda = \lambda_0$ vs. $H_A : \lambda \neq \lambda_0$.

> Write down the LRT statistic, and simplify as much as possible.

Asymptotics of the LRT

Generalization to higher dimensions