

# Method of moments estimators

## Course so far

- + Maximum likelihood estimation
- + Logistic regression
- + Asymptotics
- + Asymptotic properties of MLEs
- + Hypothesis testing
- + Confidence intervals

**Common theme:** Likelihoods and MLEs

**Question:** Why maximum likelihood estimation?

## Example

Suppose  $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Uniform}[0, \theta]$ . How could I estimate  $\theta$ ?

## Example

Suppose  $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Uniform}[a, b]$ . How could I estimate  $a$  and  $b$ ?

# Method of moments

Let  $X_1, \dots, X_n$  be a sample from a distribution with probability function  $f(x|\theta_1, \dots, \theta_k)$ , with  $k$  parameters  $\theta_1, \dots, \theta_k$ .

## Example

Suppose  $X_1, \dots, X_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$ .

Find the method of moments estimates  $\hat{\mu}$  and  $\hat{\sigma}^2$ .

## Example

Suppose  $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Gamma}(\alpha, \beta)$ , i.e.

$$f(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}. \text{ Then}$$

$$\mu_1 = \mathbb{E}[X] = \frac{\alpha}{\beta} \quad \mu_2 = \mathbb{E}[X^2] = \left(\frac{\alpha}{\beta}\right)^2 + \frac{\alpha}{\beta^2}$$

Use the method of moments to estimate  $\alpha$  and  $\beta$ .