Maximum likelihood estimation

Recap: maximum likelihood estimation

Definition: Let $\mathbf{Y} = (Y_1, \ldots, Y_n)$ be a sample of n observations, and let $f(\mathbf{y}|\theta)$ denote the joint pdf or pmf of $\mathbf{Y}$, with parameter(s) θ. The *likelihood function* is

$$L(\theta|\mathbf{Y}) = f(\mathbf{Y}|\theta)$$

Definition: Let $\mathbf{Y} = (Y_1, \ldots, Y_n)$ be a sample of n observations. The *maximum likelihood estimator* (MLE) is

$$\hat{\theta} = \operatorname{argmax}_\theta L(\theta|\mathbf{Y})$$

Continuing $N(\theta, 1)$ example

Example: $Uniform(0, \theta)$

Let $Y_1, \ldots, Y_n \overset{iid}{\sim} Uniform(0, \theta)$, where $\theta > 0$. We want the maximum likelihood estimator of θ.

> Discuss with your neighbors what the MLE of θ might be. *Hint: focus on finding and sketching the likelihood function* $L(\mathbf{Y}|\theta)$

Example: $N(\mu, \sigma^2)$

Invariance of the MLE

Maximum likelihood estimation for logistic regression

$$Y_i \sim Bernoulli(p_i)$$

$$\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 X_{i,1} + \cdots + \beta_k X_{i,k}$$

Suppose we observe independent samples $(X_1, Y_1), \ldots, (X_n, Y_n)$. Write down the likelihood function

$$L(\beta | \mathbf{X}, \mathbf{Y}) = \prod_{i=1}^{n} f(Y_i | \beta, X_i)$$

for the logistic regression problem.