

Hypothesis testing framework

Last time

We have the model

$$Y_i \sim \text{Bernoulli}(p_i)$$

$$\log\left(\frac{p_i}{1 - p_i}\right) = \beta_0 + \beta_1 \text{Sex}_i + \beta_2 \text{Age}_i + \beta_3 \text{SecondClass}_i + \beta_4 \text{ThirdClass}_i$$

We want to test whether there is a difference in the chance of survival for second and third class passengers, holding age and sex fixed.

What hypotheses should we test?

Contrasts

Class activity

Work on Part II from last class:

https://sta711-s23.github.io/class_activities/ca_lecture_18.html

Class activity

```
a <- c(0, 0, 0, -1, 1)
test_stat <- (t(a) %*% coef(m1))/sqrt(t(a) %*% vcov(m1) %*% a)
test_stat
```

```
##           [,1]
## [1,] -5.207289
```

Class activity

```
a <- c(0, 0, 0, -1, 1)
test_stat <- (t(a) %*% coef(m1))/sqrt(t(a) %*% vcov(m1) %*% a)
test_stat
```

```
##           [,1]
## [1,] -5.207289
```

```
# rejection region for alpha = 0.05
qnorm(0.025, lower.tail=F)
```

```
## [1] 1.959964
```

```
# p-value
2*pnorm(abs(test_stat), lower.tail=F)
```

```
##           [,1]
## [1,] 1.916191e-07
```

Summary of Wald tests

Let $\theta \in \mathbb{R}^p$ be some parameter of interest. We wish to test the hypotheses

$$H_0 : C\theta = \gamma_0 \quad H_A : C\theta \neq \gamma_0$$

for some $C \in \mathbb{R}^{q \times p}$. Given an estimator $\hat{\theta}_n$ such that

$$V_n^{-\frac{1}{2}}(\hat{\theta}_n - \theta) \xrightarrow{d} N(0, I),$$

the **Wald test** rejects when

$$(C\hat{\theta}_n - \gamma_0)^T V_n^{-1} (C\hat{\theta}_n - \gamma_0) > \chi_{q,\alpha}^2$$

General framework for hypothesis tests

Definition: Let $\theta \in \mathbb{R}^p$ be some parameter of interest. A **hypothesis** is a statement about θ . The goal of a hypothesis test is to compare two competing hypotheses:

$$H_0 : \theta \in \Theta_0 \quad H_A : \theta \in \Theta_1$$

Outcomes

$$H_0 : \theta \in \Theta_0 \quad H_A : \theta \in \Theta_1$$

The outcome of the test is a decision to either **reject** H_0 or **fail to reject** H_0 .

Constructing a test

$$H_0 : \theta \in \Theta_0 \quad H_A : \theta \in \Theta_1$$

Power function

Suppose we reject H_0 when $(X_1, \dots, X_n) \in R$. The **power function** $\beta(\theta)$ is

$$\beta(\theta) = P_{\theta}((X_1, \dots, X_n) \in R)$$

Example

X_1, \dots, X_n iid from a population with mean μ and variance σ^2 .

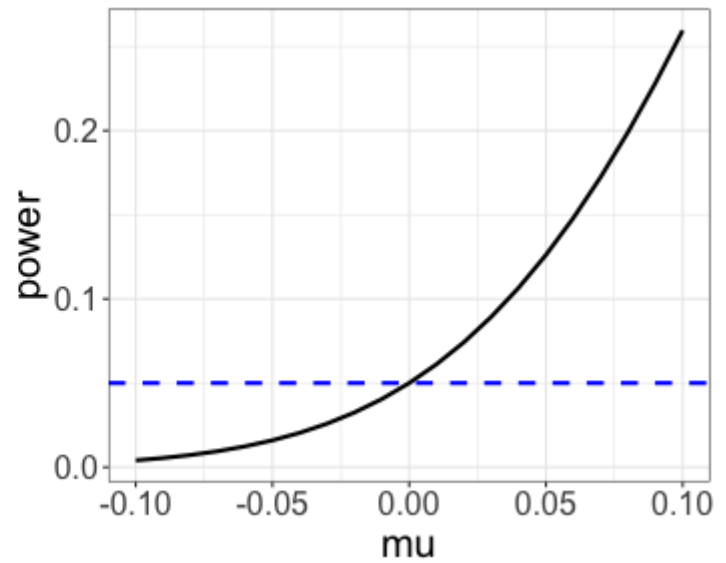
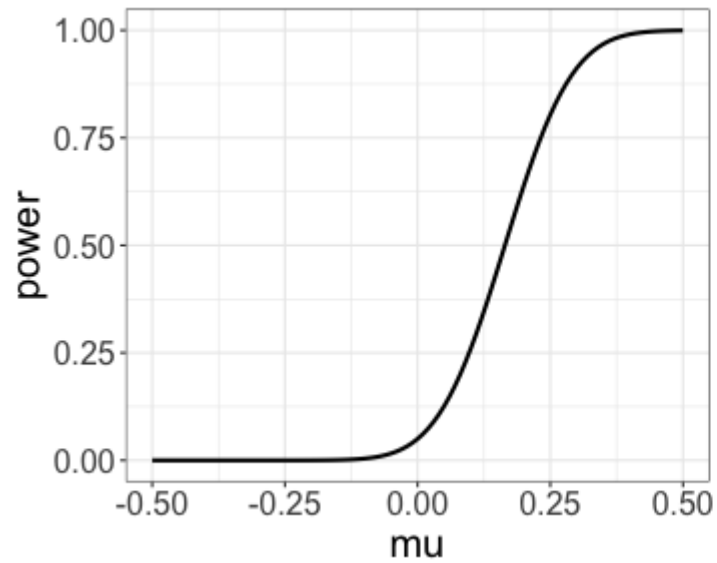
$$H_0 : \mu = \mu_0 \quad H_A : \mu > \mu_0$$

Class activity

$$\beta(\mu) \approx 1 - \Phi \left(z_\alpha - \frac{(\mu - \mu_0)}{\sigma/\sqrt{n}} \right)$$

- + Suppose that $\mu_0 = 0$, $n = 100$, and $\sigma = 1$. Make a plot of $\beta(\mu)$ vs. μ for $\alpha = 0.05$.
- + Now consider testing $H_0 : \mu \leq \mu_0$ vs. $H_A : \mu > \mu_0$. Will this change our rejection region if we want a size α test?

Class activity



Simulation verification

```
n <- 100
sigma <- 1
mu0 <- 0
nreps <- 5000
test_stats <- rep(0, nreps)
for(i in 1:nreps){
  x <- rnorm(n, mu0, sigma)
  test_stats[i] <- (mean(x) - mu0)/(sigma/sqrt(n))
}
mean(test_stats > qnorm(0.05, lower.tail=F))
```

```
## [1] 0.0548
```

Simulation verification

```
n <- 100
sigma <- 1
mu0 <- 0
nreps <- 5000
test_stats <- rep(0, nreps)
for(i in 1:nreps){
  x <- rnorm(n, 0.1, sigma)
  test_stats[i] <- (mean(x) - mu0)/(sigma/sqrt(n))
}
mean(test_stats > qnorm(0.05, lower.tail=F))
```

```
## [1] 0.2646
```