Neyman-Pearson and likelihood ratio tests

Recap: Neyman-Pearson test

Let $X_1, \ldots, X_n$ be a sample from a distribution with probability function f, and parameter θ. To test

$$H_0 : \theta = \theta_0 \qquad H_A : \theta = \theta_1,$$

the Neyman-Pearson test rejects H_0 when

$$\frac{L(\theta_1|\mathbf{X})}{L(\theta_0|\mathbf{X})} > k,$$

where k is chosen so that $\beta(\theta_0) = \alpha$.

Recap: Neyman-Pearson lemma

The Neyman-Pearson test is a *uniformly most power* level α test of $H_0 : \theta = \theta_0$ vs. $H_A : \theta = \theta_1$.

Composite hypotheses with a UMP test

Let $\mathbf{X} = X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$ with σ^2 known. We wish to test

$$H_0 : \mu = \mu_0 \qquad H_A : \mu > \mu_0$$

Claim: the Wald test is a uniformly most powerful level α test for these hypotheses.

Composite hypotheses without a UMP test

Let $\mathbf{X} = X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$ with σ^2 known. We wish to test

$$H_0 : \mu = \mu_0 \qquad H_A : \mu \neq \mu_0$$

Claim: there is no uniformly most powerful level α test for these hypotheses.

The likelihood ratio test

Example

Let $X_1, \ldots, X_n \overset{iid}{\sim} Poisson(\lambda)$. We wish to test $H_0 : \lambda = \lambda_0$ vs. $H_A : \lambda \neq \lambda_0$.