STA 711 Homework 2

Due: Friday, January 27, 12:00pm (noon) on Canvas.

Instructions: Submit your work as a single PDF. For this assignment, you may include written work by scanning it and incorporating it into the PDF. Include all R code needed to reproduce your results in your submission.

Maximum likelihood estimation

1. Let $X_1, ..., X_n \overset{iid}{\sim} Poisson(\lambda)$, and let $X_1^n = (X_1, ..., X_n)$ denote the combined sample.
   (a) Write down the likelihood $L(\lambda | X_1^n)$.
   (b) Find the maximum likelihood estimate $\widehat{\lambda}$ of λ.

2. Let $X_1, ..., X_n \overset{iid}{\sim} Exponential(\theta)$, so $f(x|\theta) = \theta e^{-\theta x}$.
   (a) Write down the likelihood $L(\theta | X_1^n)$.
   (b) Find the maximum likelihood estimate $\widehat{\theta}$ of θ.
   (c) Show that $X_{(1)} \sim Exponential(n\theta)$.
   (d) Suppose that instead of observing $X_1, ..., X_n$, we only observe the minimum $X_{(1)}$. What would be the maximum likelihood estimate of θ?

3. Let $X_1, ..., X_n$ be iid from a distribution with pdf
$$f(x|\theta) = \theta x^{-2} \mathbb{1}\{x \geq \theta\},$$
   where $\theta > 0$. Find the maximum likelihood estimate of θ.

4. Let $X_1, ..., X_n$ be iid with one of two pdfs. If $\theta = 0$, then
$$f(x|\theta) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{else.} \end{cases}$$
   If $\theta = 1$, then
$$f(x|\theta) = \begin{cases} \frac{1}{2\sqrt{x}} & 0 < x < 1 \\ 0 & \text{else.} \end{cases}$$
   Find the maximum likelihood estimate of θ.

5. Let X be a single observation from a normal distribution with mean θ and variance θ^2, where $\theta > 0$. Find the maximum likelihood estimate of θ^2.

6. Let $X_1, ..., X_n$ be a random sample from a distribution with pdf
$$f(x|\mu, \sigma) = \frac{1}{\sigma} \exp\left\{ - \left(\frac{x - \mu}{\sigma} \right) \right\} \mathbb{1}\{x \geq \mu\},$$
   where $-\infty < \mu < \infty$, and $\sigma > 0$.
   (a) Find the maximum likelihood estimates of μ and σ. *(Hint: find $\widehat{\mu}$ first)*
   (b) Let $\tau(\mu, \sigma) = \mathbb{P}_{\mu,\sigma}(X_1 \geq t)$, where $t > \mu$, and $\mathbb{P}_{\mu,\sigma}$ denotes probability when μ, σ are the true parameters. Find the maximum likelihood estimate of $\tau(\mu, \sigma)$.