Minimal sufficiency and completeness

Recap: minimal sufficient statistics

Definition: A statistic $T(X_1, \ldots, X_n)$ is a *minimal sufficient statistic* if for any other sufficient statistic $T^*(X_1, \ldots, X_n)$, $T(X_1, \ldots, X_n)$ is a function of $T^*(X_1, \ldots, X_n)$.

Example

Suppose $X_1, \dots, X_n \overset{iid}{\sim} Uniform[\theta, \theta + 1]$.

Example

Suppose $X_1, \ldots, X_n \overset{iid}{\sim} Poisson(\lambda)$.

> Find a minimal sufficient statistic for λ.

Recap: Rao-Blackwell

Rao-Blackwell theorem: Let θ be a parameter of interest, and $\hat{\tau}$ an unbiased estimator of $\tau(\theta)$. If T is a sufficient statistic for θ, then $\tau^* = \mathbb{E}[\hat{\tau}|T]$ is an unbiased estimator of $\tau(\theta)$, and $Var(\tau^*) \leq Var(\hat{\tau})$.

> If we condition on the "right" sufficient statistic, does this process find the best unbiased estimator?

Completeness

Lehmann-Scheffe theorem