Convergence of the MLE, Wald tests

Recap: convergence of the MLE

Under regularity conditions,

- $\hat{\theta}_n \xrightarrow{p} \theta$
- $\sqrt{n}(\hat{\theta}_n - \theta) \xrightarrow{d} N(0, \mathcal{I}_1^{-1}(\theta))$

Regularity conditions

Application to logistic regression

Wald tests for single parameters

Logistic regression model for the dengue data:

$$Y_i \sim Bernoulli(p_i)$$

$$\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 WBC_i + \beta_2 PLT_i$$

$H_0 : \beta_1 = 0 \quad H_A : \beta_1 \neq 0$

Class activity

https://sta711-s23.github.io/class_activities/ca_lecture_16.html

Class activity

```
m1 <- glm(Dengue ~ WBC + PLT, data = dengue, family = binomial)
X <- model.matrix(m1)
solve(t(X) %*% diag(m1$weights) %*% X)
```

```
##                (Intercept)           WBC           PLT
## (Intercept)  1.471934e-02 -4.937020e-04 -5.125888e-05
## WBC         -4.937020e-04  1.804972e-04 -3.221337e-06
## PLT         -5.125888e-05 -3.221337e-06  3.518938e-07
```

```
vcov(m1)
```

```
##                (Intercept)           WBC           PLT
## (Intercept)  1.471934e-02 -4.937020e-04 -5.125888e-05
## WBC         -4.937020e-04  1.804972e-04 -3.221337e-06
## PLT         -5.125888e-05 -3.221337e-06  3.518938e-07
```

```
summary(m1)$coefficients[,2]^2
```

```
##  (Intercept)          WBC          PLT
## 1.471934e-02 1.804972e-04 3.518938e-07
```

Class activity