Maximum likelihood estimation for logistic regression

Invariance of the MLE

Maximum likelihood estimation for logistic regression

$$Y_i \sim Bernoulli(p_i)$$

$$\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 X_{i,1} + \cdots + \beta_k X_{i,k}$$

Suppose we observe independent samples $(X_1, Y_1), \ldots, (X_n, Y_n)$. Write down the likelihood function

$$L(\beta|\mathbf{X}, \mathbf{Y}) = \prod_{i=1}^{n} f(Y_i|\beta, X_i)$$

for the logistic regression problem.

Iterative methods for maximizing likelihood

Newton's method

Newton's method for logistic regression

Example

Suppose that $\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 X_i$, and we have

$$\beta^{(r)} = \begin{bmatrix} -3.1 \\ 0.9 \end{bmatrix}, \qquad U(\beta^{(r)}) = \begin{bmatrix} 9.16 \\ 31.91 \end{bmatrix},$$

$$\mathbf{H}(\beta^{(r)}) = -\begin{bmatrix} 17.834 & 53.218 \\ 53.218 & 180.718 \end{bmatrix}$$

> Use Newton's method to calculate $\beta^{(r+1)}$ (you may use R or a calculator, you do not need to do the matrix arithmetic by hand).