Hypothesis testing framework

Recap: Constructing a test

$$H_0 : \theta \in \Theta_0 \qquad H_A : \theta \in \Theta_1$$

Observe data $X_1, \ldots, X_n$.

- Calculate a test statistic $T_n = T(X_1, \ldots, X_n)$
- Choose a rejection region $R = \{(x_1, \ldots, x_n) : \text{ reject } H_0\}$
- Reject H_0 if $(X_1, \ldots, X_n) \in R$

Power function

Suppose we reject H_0 when $(X_1, \ldots, X_n) \in R$. The **power function** $\beta(\theta)$ is

$$\beta(\theta) = P_\theta((X_1, \ldots, X_n) \in R)$$

Example

$X_1, \ldots, X_n$ iid from a population with mean μ and variance σ^2.

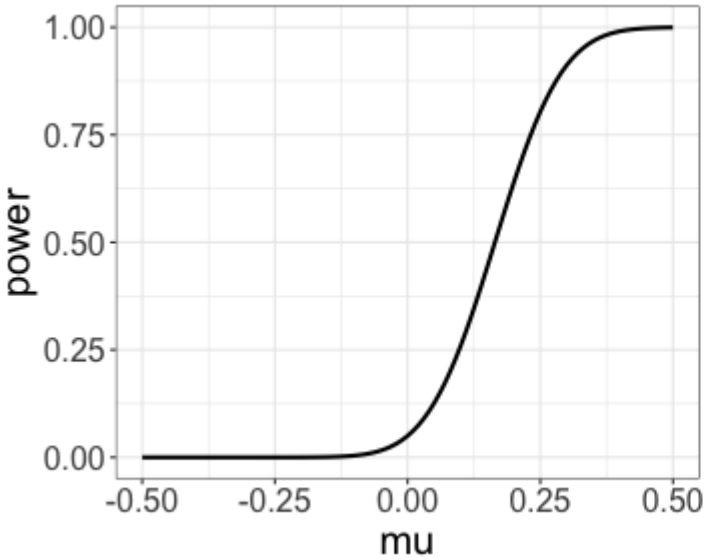
$$H_0 : \mu = \mu_0 \qquad H_A : \mu > \mu_0$$

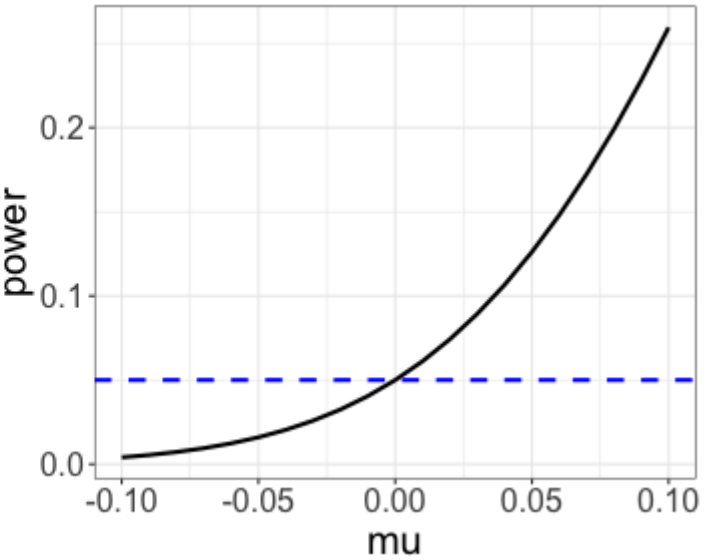
Class activity

$$\beta(\mu) \approx 1 - \Phi\left(z_\alpha - \frac{(\mu - \mu_0)}{\sigma/\sqrt{n}}\right)$$

- Suppose that $\mu_0 = 0, n = 100$, and $\sigma = 1$. Make a plot of $\beta(\mu)$ vs. μ for $\alpha = 0.05$.
- Now consider testing $H_0 : \mu \leq \mu_0$ vs. $H_A : \mu > \mu_0$. Will this change our rejection region if we want a size α test?

Class activity





Rejecting H_0

$$H_0 : \theta \in \Theta_0 \qquad H_A : \theta \in \Theta_1$$

A hypothesis test rejects H_0 if $(X_1, \ldots, X_n)$ is in the rejection region R. Are there any issues if we only use a rejection region to test hypotheses?

p-values

$$H_0 : \theta \in \Theta_0 \qquad H_A : \theta \in \Theta_1$$

Given α, we construct a rejection region R and reject H_0 when $(X_1, \ldots, X_n) \in R$. Let $(x_1, \ldots, x_n)$ be an observed set of data.

Definition: The **p-value** for the observed data $(x_1, \ldots, x_n)$ is the smallest α for which we reject H_0.

Example

$X_1, \ldots, X_n$ iid from a population with mean μ and variance σ^2.

$$H_0 : \mu = \mu_0 \qquad H_A : \mu > \mu_0$$

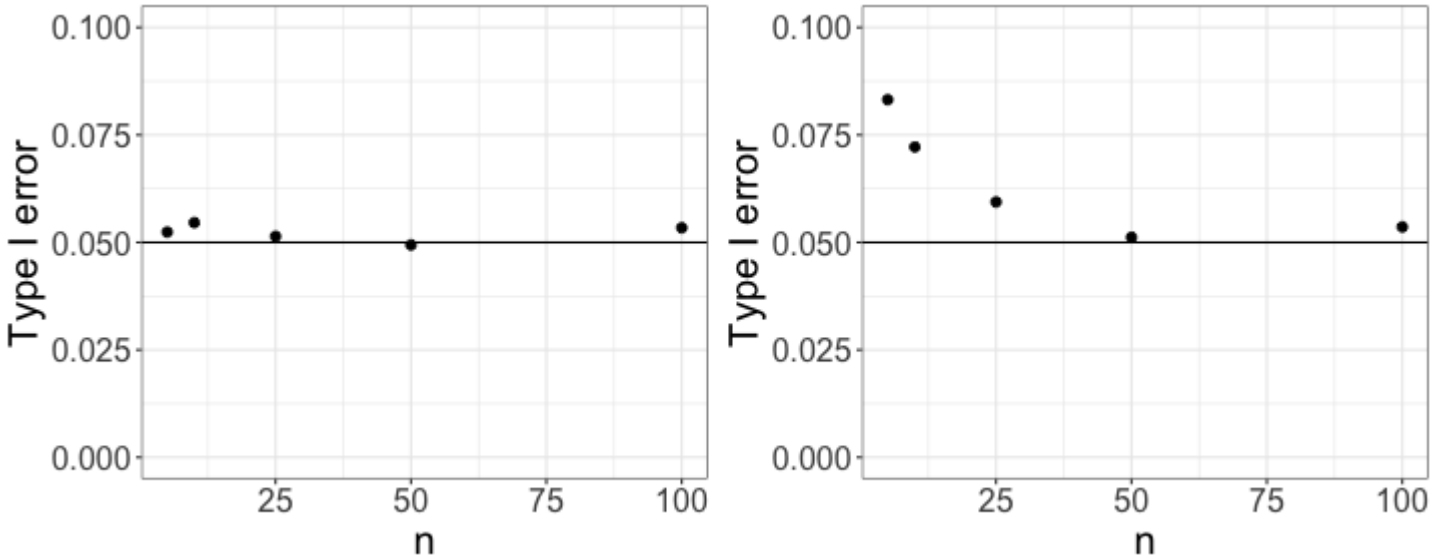
Next steps

> So far, we have discussed the Wald test in detail. What other hypothesis tests have you seen in statistics courses?

Class activity

https://sta711-s23.github.io/class_activities/ca_lecture_20.html

Class activity



If we reject $H_0 : \mu = \mu_0$ when $\frac{\sqrt{n}(\overline{X}_n - \mu_0)}{s} > z_\alpha$, why does type I error increase as n decreases?