Inequalities and Asymptotics

Wald tests for single parameters

Logistic regression model for the dengue data:

$$Y_i \sim Bernoulli(p_i)$$

$$\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 WBC_i + \beta_2 PLT_i$$

> Researchers want to know if there is a relationship between white blood cell count and the probability a patient has dengue, after accounting for platelet count. What hypotheses should the researchers test?

Wald tests for single parameters

```
m1 <- glm(Dengue ~ WBC + PLT, data = dengue,
          family = binomial)
summary(m1)
```

```
...
##                Estimate Std. Error z value Pr(>|z|)
## (Intercept)  2.6415063  0.1213233   21.77   <2e-16 ***
## WBC         -0.2892904  0.0134349  -21.53   <2e-16 ***
## PLT         -0.0065615  0.0005932  -11.06   <2e-16 ***
## ---
...
```

What this requires

What we need to do

Markov's inequality

Theorem: Let Y be a non-negative random variable, and suppose that $\mathbb{E}[Y]$ exists. Then for any $t > 0$,

$$P(Y > t) \leq \frac{\mathbb{E}[Y]}{t}$$

Chebyshev's inequality

Theorem: Let Y be a random variable, and let $\mu = \mathbb{E}[Y]$ and $\sigma^2 = Var(Y)$. Then

$$P(|Y - \mu| \geq t) \leq \frac{\sigma^2}{t^2}$$

With your neighbor, apply Markov's inequality to prove Chebyshev's inequality.

Cauchy-Schwarz inequality

Theorem: For any two random variables X and Y,

$$|\mathbb{E}[XY]| \leq \mathbb{E}|XY| \leq (\mathbb{E}[X^2])^{1/2}(\mathbb{E}[Y^2])^{1/2}$$

Example: The *correlation* between X and Y is defined by

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$$

Working with a neighbor, use the Cauchy-Schwarz inequality to prove that $-1 \leq \rho(X,Y) \leq 1$.

Jensen's inequality

Theorem: For any random variable Y, if g is a convex function, then

$$\mathbb{E}[g(Y)] \geq g(\mathbb{E}[Y])$$