Fitting and interpreting logistic regression models

Last time: Dengue data

Data: Data on 5720 Vietnamese children, admitted to the hospital with possible dengue fever. Variables include:

- *Sex*: patient's sex (female or male)
- *Age*: patient's age (in years)
- *WBC*: white blood cell count
- *PLT*: platelet count
- other diagnostic variables...
- *Dengue*: whether the patient has dengue (0 = no, 1 = yes)

Logistic regression model

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> Why is there no noise term ε_i in the logistic regression model? Discuss for 1--2 minutes with your neighbor, then we will discuss as a class.

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```
m1 <- glm(Dengue ~ WBC, data = dengue,
          family = binomial)
summary(m1)
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```

```
...
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  1.73743    0.08499   20.44   <2e-16 ***
## WBC         -0.36085    0.01243  -29.03   <2e-16 ***
## ---
...
```

Making predictions

$$Y_i \sim Bernoulli(p_i)$$

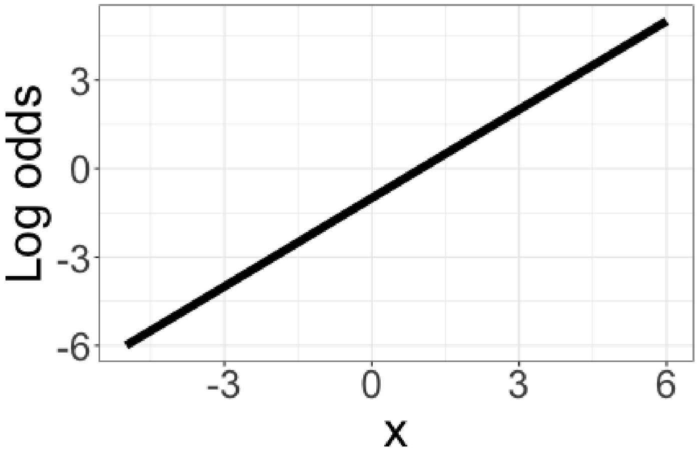
$$\log\left(\frac{\hat{p}_i}{1 - \hat{p}_i}\right) = 1.737 - 0.361\, WBC_i$$

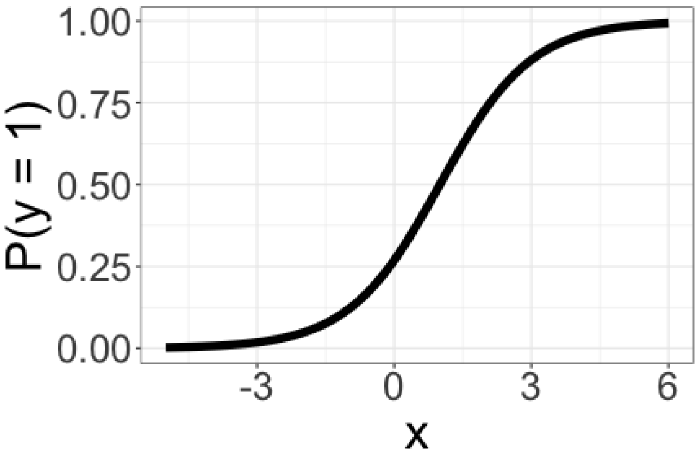
Work in groups of 2-3 for 5 minutes on the following questions:

- What is the predicted odds of dengue for a patient with a WBC of 10?
- For a patient with a WBC of 10, is the predicted probability of dengue > 0.5, < 0.5, or $= 0.5$?
- What is the predicted *probability* of dengue for a patient with a WBC of 10?

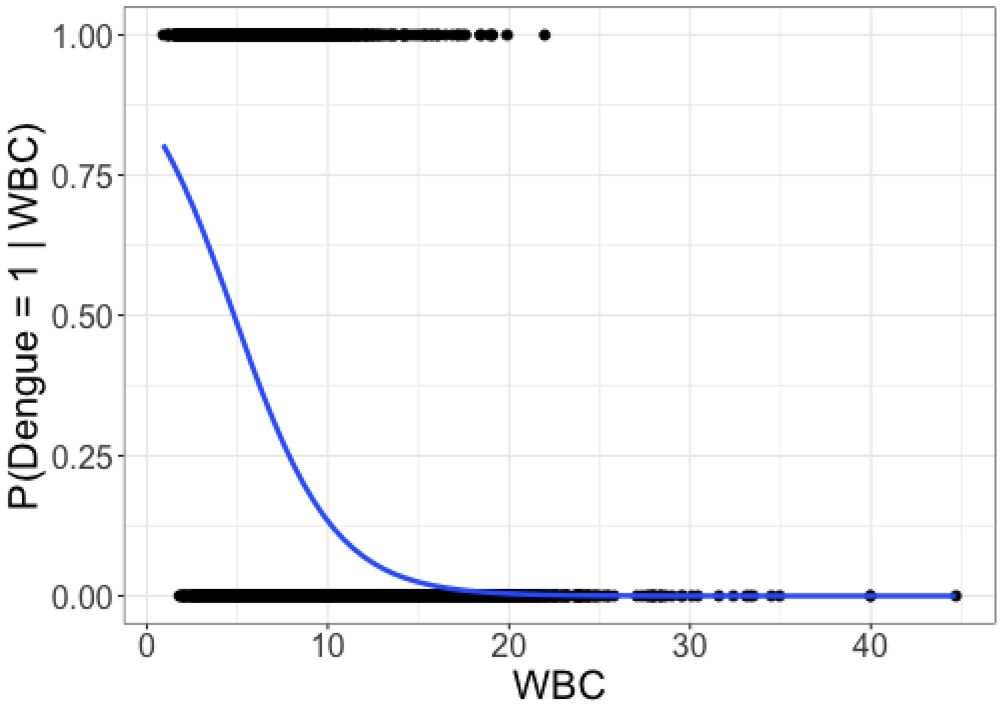
Shape of the regression curve

$$\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1\ X_i \qquad p_i = \frac{e^{\beta_0+\beta_1\ X_i}}{1+e^{\beta_0+\beta_1\ X_i}}$$





Plotting the fitted model for dengue data

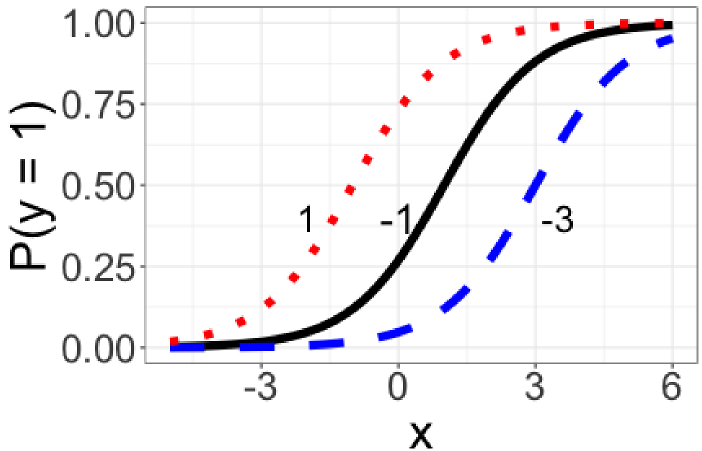


Shape of the regression curve

How does the shape of the fitted logistic regression depend on β_0 and β_1?

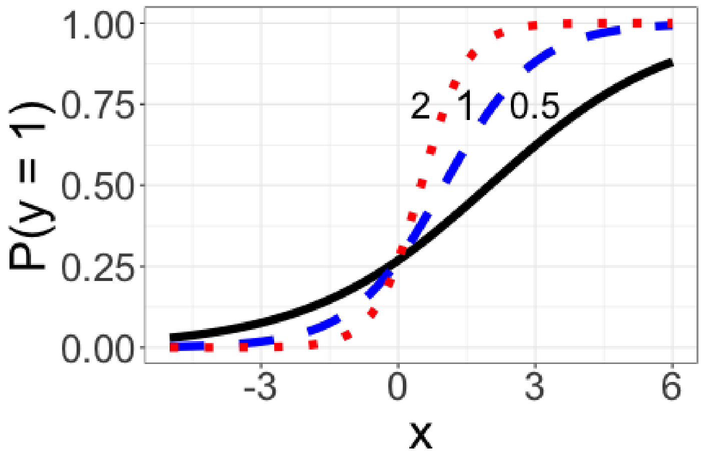
$$p_i = \frac{\exp\{\beta_0 + X_i\}}{1 + \exp\{\beta_0 + X_i\}}$$

for $\beta_0 = -3, -1, 1$



$$p_i = \frac{\exp\{-1 + \beta_1\, X_i\}}{1 + \exp\{-1 + \beta_1\, X_i\}}$$

for $\beta_1 = 0.5, 1, 2$



Interpretation

$$Y_i \sim Bernoulli(p_i)$$

$$\log\left(\frac{\hat{p}_i}{1-\hat{p}_i}\right) = 1.737 - 0.361\, WBC_i$$

Work in groups of 2-3 for 5 minutes on the following questions:

- Are patients with a higher WBC more or less likely to have dengue?
- What is the change in *log odds* associated with a unit increase in WBC?
- What is the change in *odds* asociated with a unit increase in WBC?

Recap: ways of fitting a *linear* regression model

$$Y_i = \beta_0 + \beta_1 X_{i,1} + \beta_2 X_{i,2} + \cdots + \beta_k X_{i,k} + \varepsilon_i \qquad \varepsilon_i \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

> Suppose we observe data $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$, where $X_i = (1, X_{i,1}, \ldots, X_{i,k})^T$.
>
> How do we fit this linear regression model? That is, how do we estimate
>
> $$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}$$

Summary: three ways of fitting linear regression models

- Minimize SSE, via derivatives of $\sum_{i=1}^{n}(Y_i - \beta_0 - \beta_1 X_{i,1} - \cdots - \beta_k X_{i,k})^2$
- Minimize $||Y - \widehat{Y}||$ (equivalent to minimizing SSE)
- Maximize likelihood (for *normal* data, equivalent to minimizing SSE)

> Which of these three methods, if any, is appropriate for fitting a logistic regression model? Do any changes need to be made for the logistic regression setting?