

Comparing estimators

Example

Suppose $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Uniform}[0, \theta]$. Some possible estimates:

What properties might I want an estimator $\hat{\theta}$ to possess?

Bias, Variance and MSE

Example

Suppose $X_1, \dots, X_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$. On homework, we considered

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2 \quad s^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

and we showed that $\mathbb{E}\hat{\sigma}^2 = \frac{n-1}{n}\sigma^2$, $\mathbb{E}(s^2) = \sigma^2$, and $\frac{(n-1)s^2}{\sigma^2} \sim \chi_{n-1}^2$.

Calculate the MSE of both $\hat{\sigma}^2$ and s^2 . It may help that if $V \sim \chi_{\nu}^2$, then $E[V] = \nu$ and $Var(V) = 2\nu$.

MSE and consistency

Best unbiased estimators

Cramer-Rao lower bound