Fisher information

Recap: Newton's method

To find β^* such that $U(\beta^*)=0$, when there is no closed-form solution we use Newton's method:

- lacktriangle Begin with an initial guess $eta^{(0)}$
- lacktriangle Iteratively update: $eta^{(r+1)}=eta^{(r)}-\mathbf{H}^{-1}(eta^{(r)})U(eta^{(r)})$
- Stop when the algorithm converges

Some intuition about Hessians

Example: Suppose that $eta=(eta_0,eta_1)^T\in\mathbb{R}^2$, and

$$\ell(\beta) = -\beta_0^2 - 100\beta_1^2$$

Calculate the score function

$$U(eta) = \left[egin{array}{c} rac{\partial \ell}{\partial eta_0} \ rac{\partial \ell}{\partial eta_1} \end{array}
ight]$$

and the Hessian

$$\mathbf{H}(eta) = \left[egin{array}{ccc} rac{\partial^2 \ell}{\partial eta_0^2} & rac{\partial^2 \ell}{\partial eta_0 \partial eta_1} \ rac{\partial^2 \ell}{\partial eta_1 \partial eta_0} & rac{\partial^2 \ell}{\partial eta_1^2} \end{array}
ight]$$

Fisher information

Example: Bernoulli sample

Suppose that $Y_1,\ldots,Y_n \overset{iid}{\sim} Bernoulli(p_i).$

Properties

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