Unbiased estimators

Recap: Cramer-Rao lower bound

Let $X_1, \ldots, X_n$ be a sample from a distribution with probability function $f(x|\theta)$, and let $\hat{\theta}$ be an unbiased estimator of $\theta \in \mathbb{R}$. Then, under regularity conditions,

$$Var(\hat{\theta}) \geq \frac{1}{\mathcal{I}(\theta)}$$

Example

Suppose that $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$.

> Calculate the Cramer-Rao lower bound for the variance of an unbiased estimator of σ^2. Does the sample variance $s^2 = \frac{1}{n-1} \sum_i (X_i - \overline{X})^2$ attain the Cramer-Rao lower bound?

Attaining the CRLB

Sufficient statistics

> Given an unbiased estimator, can I improve its variance?

Rao-Blackwell

Example

Let $X_1, \ldots, X_n \overset{iid}{\sim} Poisson(\lambda)$.