Lecture 4: Maximum likelihood estimation

Reminders

- HW 1 due Friday 11am on Canvas
- You are always welcome to work together on assignments, just make sure to write up your own solutions
- You have a bank of 5 extension days; you can use 1 or 2 on any assignment or exam

Recap: maximum likelihood estimation

Definition: Let $\mathbf{Y} = (Y_1, \ldots, Y_n)$ be a sample of n observations, and let $f(\mathbf{y}|\theta)$ denote the joint pdf or pmf of $\mathbf{Y}$, with parameter(s) θ. The *likelihood function* is

$$L(\theta|\mathbf{Y}) = f(\mathbf{Y}|\theta)$$

Definition: Let $\mathbf{Y} = (Y_1, \ldots, Y_n)$ be a sample of n observations. The *maximum likelihood estimator* (MLE) is

$$\widehat{\theta} = \text{argmax}_{\theta} \, L(\theta|\mathbf{Y})$$

Example: $\mathrm{N}(\theta, 1)$

Example: $Uniform(0, \theta)$

Let $Y_1, \ldots, Y_n \overset{iid}{\sim} Uniform(0, \theta)$, where $\theta > 0$. We want the maximum likelihood estimator of θ.

Discuss with your neighbors what the MLE of θ might be.

Hint: focus on finding and sketching the likelihood function $L(\mathbf{Y}|\theta)$

Example: $N(\mu, \sigma^2)$

Logistic regression

$$Y_i \sim \text{Bernoulli}(p_i)$$

$$\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 X_{i,1} + \cdots + \beta_k X_{i,k}$$

Suppose we observe independent samples $(X_1, Y_1), \ldots, (X_n, Y_n)$. Write down the likelihood function

$$L(\beta|\mathbf{X}, \mathbf{Y}) = \prod_{i=1}^{n} f(Y_i|\beta, X_i)$$

for the logistic regression problem.