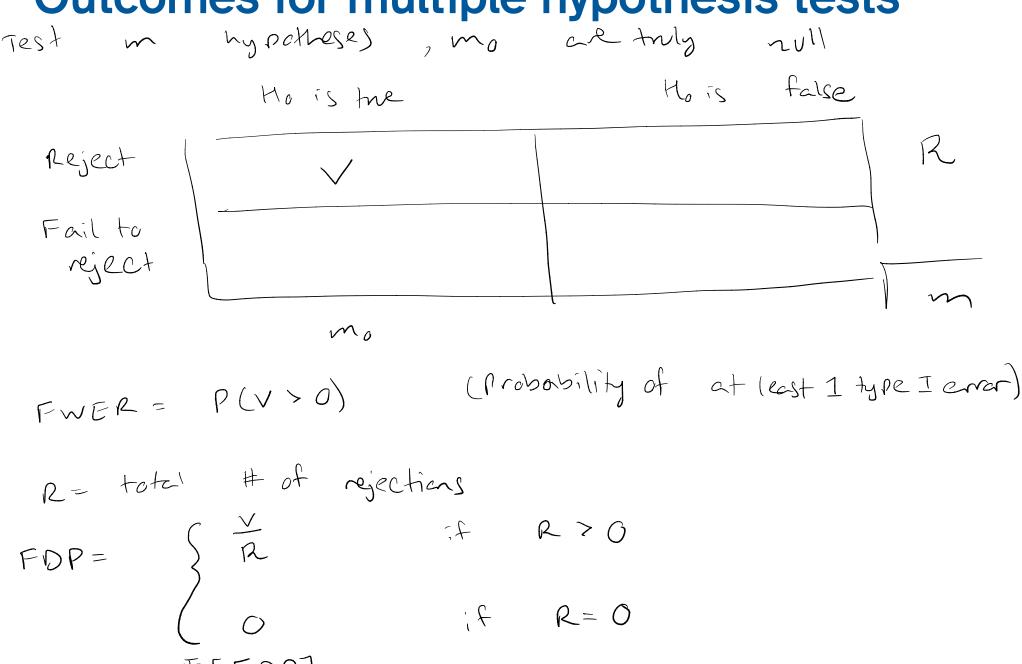
Lecture 31: False discovery rate

Outcomes for multiple hypothesis tests



False discovery rate

Suppose we test m hypotheses, m_0 of which are truly null. Let V denote the number of type I errors, and R the total number of rejections.

$$FWER = P(V > 0)$$
 $FDR = \mathbb{E}[FDP]$

O If
$$m_0 = m$$
, then FWER = FDR

PS: Since $m_0 = m$, either $R = 0$, or $R > 0$ and $V = R$

$$= 7 FDP = \begin{cases} 1 & R > 0 \\ R = 0 \end{cases}$$

$$= 7 E[FDP] = P(R > 0) * 1 + P(R = 0) * 0$$

$$= P(R > 0) = P(V > 0) = FWER$$

D) In general, FOR $L = FWER$ (controlling FOR does not recessoring

(2) In general, FDR = FWER (controlling FWER, but controlling FWER obes control FOR)

FOR = $E[FOP] \perp E[15 \vee 703] = P(\vee 70) = FWER$

The Benjamini-Hochberg procedure

Suppose we test m null hypotheses $H_{0,1}, \ldots, H_{0,m}$. Let p_i be the corresponding p-value for test i.

- Order the p-values $p_{(1)} \leq p_{(2)} \leq \cdots \leq p_{(m)}$
- Reject $H_{0,(i)}$ for all $i \leq i^*$

• Let
$$i^* = \max\left\{i: p_{(i)} < \frac{i\alpha}{m}\right\}$$
 it hypotheses ejected total • Reject $H_{0,(i)}$ for all $i \leq i^*$

Claim: If the hypotheses are independent, BH controls FDR at level $\frac{m_0}{m} \alpha \leq \alpha$

Summary

- BH controls FDR at level $\frac{m_0}{m} \alpha$
- If $m_0 = m$, then controlling FDR is equivalent to controlling FWER
- If $m_0 < m$, then controlling FDR provides more power to reject H_0 when H_0 is false