Lecture 24: Neyman-Pearson and Likelihood ratio tests

Recap: Neyman-Pearson test

Let $X_1, \ldots, X_n$ be a sample from a distribution with probability function f, and parameter θ. To test

$$H_0 : \theta = \theta_0 \qquad H_A : \theta = \theta_1,$$

the Neyman-Pearson test rejects H_0 when

$$\frac{L(\theta_1 | \mathbf{X})}{L(\theta_0 | \mathbf{X})} > k,$$

where k is chosen so that $\beta(\theta_0) = \alpha$.

Warm-up

Let $X_1, \ldots, X_n \overset{iid}{\sim} Poisson(\lambda)$. We wish to test $H_0 : \lambda = \lambda_0$ vs. $H_A : \lambda = \lambda_1$, where $\lambda_1 > \lambda_0$. Find a most powerful test for these hypotheses (that is, determine a rejection region such that the test has a desired type I error rate α).

Recap: Neyman-Pearson lemma

The Neyman-Pearson test is a *uniformly most power* level α test of $H_0 : \theta = \theta_0$ vs. $H_A : \theta = \theta_1$.

Another question

Let $X_1, \ldots, X_n \overset{iid}{\sim} Poisson(\lambda)$. We wish to test $H_0 : \lambda = \lambda_0$ vs. $H_A : \lambda \neq \lambda_0$.

Likelihood ratio tests

Back to the Poisson example

Let $X_1, \ldots, X_n \overset{iid}{\sim} Poisson(\lambda)$. We wish to test $H_0 : \lambda = \lambda_0$ vs. $H_A : \lambda \neq \lambda_0$.

Asymptotics of the LRT