Lecture 8: Fisher information

Recap: Newton's method

To find β^* such that $U(\beta^*) = 0$, when there is no closed-form solution we use Newton's method:

- Begin with an initial guess $\beta^{(0)}$
- Iteratively update: $\beta^{(r+1)} = \beta^{(r)} - \mathbf{H}^{-1}(\beta^{(r)})U(\beta^{(r)})$
- Stop when the algorithm converges

Some intuition about Hessians

Example: Suppose that $\beta = (\beta_0, \beta_1)^T \in \mathbb{R}^2$, and

$$\ell(\beta) = -\beta_0^2 - 100\beta_1^2$$

Calculate

$$U(\beta) = \begin{bmatrix} \frac{\partial \ell}{\partial \beta_0} \\ \frac{\partial \ell}{\partial \beta_1} \end{bmatrix} \qquad \mathbf{H}(\beta) = \begin{bmatrix} \frac{\partial^2 \ell}{\partial \beta_0^2} & \frac{\partial^2 \ell}{\partial \beta_0 \partial \beta_1} \\ \frac{\partial^2 \ell}{\partial \beta_1 \partial \beta_0} & \frac{\partial^2 \ell}{\partial \beta_1^2} \end{bmatrix}$$

Fisher information

Example: Bernoulli sample

Suppose that $Y_1, \ldots, Y_n \overset{\text{iid}}{\sim} \text{Bernoulli}(p_i)$.

Properties

Example: Bernoulli sample

Suppose that $Y_1, \ldots, Y_n \overset{\text{iid}}{\sim} \text{Bernoulli}(p_i)$.