Lecture 21: More hypothesis testing

Rejecting H_0

$$H_0 : \theta \in \Theta_0 \qquad H_A : \theta \in \Theta_1$$

Question: A hypothesis test rejects H_0 if $(X_1, \ldots, X_n)$ is in the rejection region R. Are there any issues if we only use a rejection region to test hypotheses?

p-values

$$H_0 : \theta \in \Theta_0 \qquad H_A : \theta \in \Theta_1$$

Given α, we construct a rejection region R and reject H_0 when $(X_1, \ldots, X_n) \in R$. Let $(x_1, \ldots, x_n)$ be an observed set of data.

Definition: The **p-value** for the observed data $(x_1, \ldots, x_n)$ is the smallest α for which we reject H_0.

Example

$X_1, \ldots, X_n$ iid from a population with mean μ and variance σ^2.

$$H_0 : \mu = \mu_0 \qquad H_A : \mu \neq \mu_0$$

Issue: Wald tests with small n

The Wald test for a population mean μ relies on

$$Z_n = \frac{\sqrt{n}(\overline{X}_n - \mu)}{s} \approx N(0,1)$$

- $Z_n \xrightarrow{d} N(0,1)$ as $n \to \infty$
- But for small n, Z_n is not normal, even if $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

What is the exact distribution of $\frac{\sqrt{n}(\overline{X}_n - \mu)}{s}$?

t-tests

If $X_1, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, then

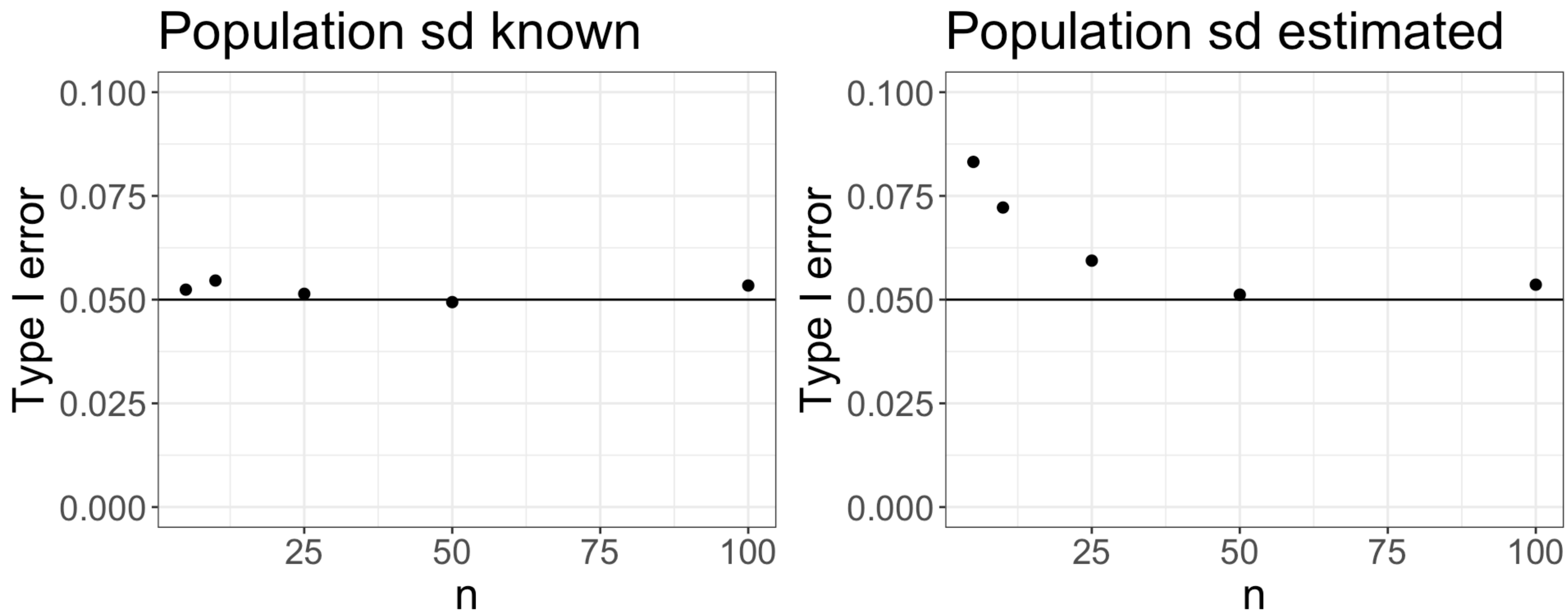
$$\frac{\sqrt{n}(\overline{X}_n - \mu)}{s} \sim t_{n-1}$$

Class activity

https://sta711-s24.github.io/class_activities/ca_lecture_21.html

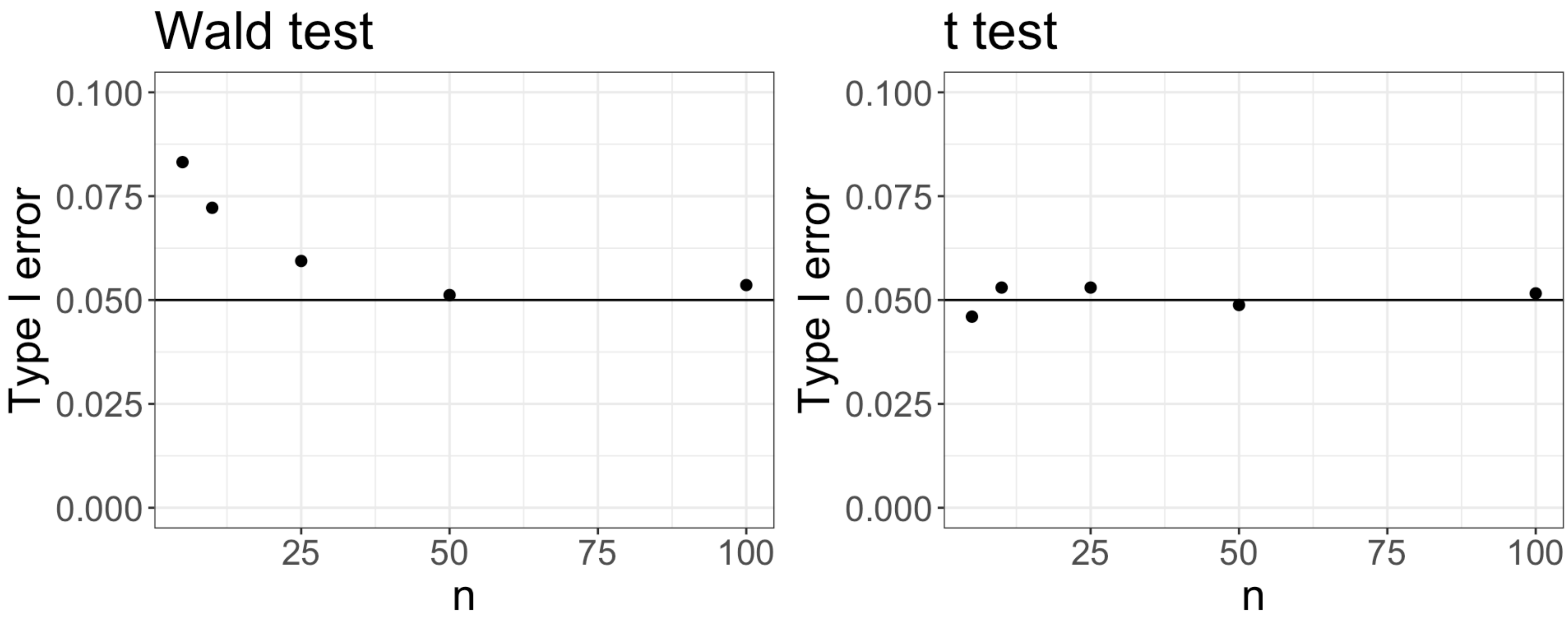
Class activity

Type I error rate with Normal distribution:



Class activity

Wald test vs. t-test:



Philosophical question

- **Position 1:** We should always use a Wald test to test hypotheses about a population mean
- **Position 2:** We should always use a t-test to test hypotheses about a population mean

With which position do you agree?