Lecture 20: Hypothesis testing framework

General framework for hypothesis tests

Definition: Let $\theta \in \mathbb{R}^p$ be some parameter of interest. A **hypothesis** is a statement about θ. The goal of a hypothesis test is to compare two competing hypotheses:

$$H_0 : \theta \in \Theta_0 \quad H_A : \theta \in \Theta_1$$

Outcomes

$$H_0 : \theta \in \Theta_0 \quad H_A : \theta \in \Theta_1$$

The outcome of the test is a decision to either **reject** H_0 or **fail to reject** H_0.

Constructing a test

$$H_0 : \theta \in \Theta_0 \quad H_A : \theta \in \Theta_1$$

Power function

Suppose we reject H_0 when $(X_1, \ldots, X_n) \in R$. The **power function** $\beta(\theta)$ is

$$\beta(\theta) = P_\theta((X_1, \ldots, X_n) \in R)$$

Example

$X_1, \ldots, X_n$ iid from a population with mean μ and variance σ^2.

$$H_0 : \mu = \mu_0 \quad H_A : \mu \neq \mu_0$$

Rejecting H_0

$$H_0 : \theta \in \Theta_0 \quad H_A : \theta \in \Theta_1$$

Question: A hypothesis test rejects H_0 if $(X_1, \ldots, X_n)$ is in the rejection region R. Are there any issues if we only use a rejection region to test hypotheses?

p-values

$$H_0 : \theta \in \Theta_0 \quad H_A : \theta \in \Theta_1$$

Given α, we construct a rejection region R and reject H_0 when $(X_1, \ldots, X_n) \in R$. Let $(x_1, \ldots, x_n)$ be an observed set of data.

Definition: The **p-value** for the observed data $(x_1, \ldots, x_n)$ is the smallest α for which we reject H_0.

Example

$X_1, \ldots, X_n$ iid from a population with mean μ and variance σ^2.

$$H_0 : \mu = \mu_0 \quad H_A : \mu \neq \mu_0$$