

Method of moments estimators

Course so far

- + Maximum likelihood estimation
- + Logistic regression
- + Asymptotics
- + Asymptotic properties of MLEs
- + Hypothesis testing
- + Confidence intervals

Common theme: Likelihoods and MLEs

Question: Why maximum likelihood estimation?

Example

Suppose $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Uniform}[0, \theta]$. How could I estimate θ ?

Example

Suppose $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Uniform}[a, b]$. How could I estimate a and b ?

Method of moments

Let X_1, \dots, X_n be a sample from a distribution with probability function $f(x|\theta_1, \dots, \theta_k)$, with k parameters $\theta_1, \dots, \theta_k$.

Example

Suppose $X_1, \dots, X_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$.

Find the method of moments estimates $\hat{\mu}$ and $\hat{\sigma}^2$.

Example

Suppose $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Gamma}(\alpha, \beta)$, i.e.

$$f(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}. \text{ Then}$$

$$\mu_1 = \mathbb{E}[X] = \frac{\alpha}{\beta} \quad \mu_2 = \mathbb{E}[X^2] = \left(\frac{\alpha}{\beta}\right)^2 + \frac{\alpha}{\beta^2}$$

Use the method of moments to estimate α and β .