Lecture 31: Comparing estimators

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Recap: MSE

Let $\widehat{\theta}$ be an estimator of θ. The **mean squared error** (MSE) of $\widehat{\theta}$ is

$$MSE(\widehat{\theta}) = \mathbb{E}_\theta[(\widehat{\theta} - \theta)^2] = Var(\widehat{\theta}) + Bias^2(\widehat{\theta})$$

MSE and consistent estimators

MSE example

Suppose $X_1, ..., X_n \overset{iid}{\sim} N(\mu, \sigma^2)$. Consider two estimates of σ^2:

$$\widehat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2 \qquad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

Activity: Compute the MSE for $\widehat{\sigma}^2$ and s^2 (see handout).

Best unbiased estimators

Suppose we restrict ourselves to **unbiased** estimators.

Definition (best unbiased estimator):

Cramer-Rao lower bound

Example

Suppose $X_1, ..., X_n \overset{iid}{\sim} Poisson(\lambda)$

Why MLEs are nice

Let θ be a parameter of interest, and $\widehat{\theta}$ be the maximum likelihood estimator from a sample of size n. Under regularity conditions, $\widehat{\theta}$ satisfies the following properties:

- $\widehat{\theta} \xrightarrow{p} \theta$
- $\sqrt{n}(\widehat{\theta} - \theta) \xrightarrow{d} N(0, \mathcal{I}_1^{-1}(\theta))$