STA 711 Homework 9

Due: Monday, April 28, 10:00pm on Canvas.

Instructions: Submit your work as a single PDF. You may choose to either hand-write your work and submit a PDF scan, or type your work using LaTeX and submit the resulting PDF. See the course website for a homework template file and instructions on getting started with LaTeX and Overleaf.

Estimation

1. Let $X_1, ..., X_n \overset{iid}{\sim} Poisson(\lambda)$. One estimator for λ is $\widehat{\lambda} = \overline{X}$, which is unbiased. Use the fact that the variance of the Poisson distribution is also λ to provide a second unbiased estimator for λ. Which of the two estimators has a smaller variance?

2. Let $X_1, ..., X_n$ be an iid sample from a distribution with density

$$f(x|\theta) = \theta x^{-2} \qquad 0 < \theta \leq x < \infty$$

   Find the method of moments estimator for θ.

3. Let $X_1, ..., X_n \overset{iid}{\sim} Gamma(\alpha, \beta)$. Find the method of moments estimators for α and β.

4. Let f_θ be the density on $\mathbb{R}^2$ that is uniform on a disc of radius θ. Let $X_1, ..., X_n \overset{iid}{\sim} f_\theta$. Find a minimal sufficient statistic for θ.

5. Let $X_1, ..., X_n \overset{iid}{\sim} N(\mu, \sigma^2)$, and let

$$\widehat{\mu} = X_1$$

   Consider the sufficient statistic $T = \sum_{i=1}^{n} X_i$ and the Rao-Blackwellized estimator

$$\mu^* = \mathbb{E}[\widehat{\mu}|T]$$

   (a) Compute μ^*.

   (b) Compare the variance of μ^* to the variance of $\widehat{\mu}$.

   (c) Suppose instead you tried to use the estimator $\mathbb{E}[X_1|X_2]$. What is wrong with this estimator?