

## Lecture 28: Pivotal quantities

Ciaran Evans

## Recap

**Confidence set:** Let  $\theta \in \Theta$  be a parameter of interest, and  $X_1, \dots, X_n$  a sample. A set  $C(X_1, \dots, X_n) \subseteq \Theta$  is a  $1 - \alpha$  **confidence set** for  $\theta$  if

$$\inf_{\theta \in \Theta} P_{\theta}(\theta \in C(X_1, \dots, X_n)) = 1 - \alpha$$

**Inverting a test:** Create a confidence set by inverting a test:

$$C(X_1, \dots, X_n) = \{\theta_0 : (X_1, \dots, X_n) \notin \mathcal{R}(\theta_0)\}$$

## Using confidence sets to test hypotheses

## t interval

Suppose  $X_1, \dots, X_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$ . We want to construct a  $1 - \alpha$  CI for  $\mu$ .

## Pivotal quantities

Let  $X_1, \dots, X_n$  be a sample and  $\theta$  be an unknown parameter. A function  $Q(X_1, \dots, X_n, \theta)$  is called a **pivot** if the distribution of  $Q(X_1, \dots, X_n, \theta)$  does not depend on  $\theta$ .

## Example

Let  $X_1, \dots, X_n \stackrel{iid}{\sim} \text{Uniform}(0, \theta)$ . We want to construct a  $1 - \alpha$  CI for  $\theta$  using a pivotal quantity.