Lecture 2: Fitting and interpreting logistic regression models

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Last time: Dengue data

Data: Data on 5720 Vietnamese children, admitted to the hospital with possible dengue fever. Variables include:

- *Sex*: patient's sex (female or male)
- *Age*: patient's age (in years)
- *WBC*: white blood cell count
- *PLT*: platelet count
- other diagnostic variables. . .
- *Dengue*: whether the patient has dengue (0 = no, 1 = yes)

Logistic regression model

$$Y_i \sim Bernoulli(p_i)$$

$$\log\left(\frac{p_i}{1 - p_i}\right) = \beta_0 + \beta_1 WBC_i$$

Why is there no noise term ε_i in the logistic regression model? Discuss for 1–2 minutes with your neighbor, then we will discuss as a class.

Fitting the logistic regression model

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```
m1 <- glm(Dengue ~ WBC, data = dengue,
          family = binomial)
summary(m1)
```

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```
m1 <- glm(Dengue ~ WBC, data = dengue,
          family = binomial)
summary(m1)
```

```
##
## Call:
## glm(formula = Dengue ~ WBC, family = binomial, data = de
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  1.73743    0.08499   20.44   <2e-16 ***
## WBC         -0.36085    0.01243  -29.03   <2e-16 ***
##
```

Making predictions

$$\log\left(\frac{\widehat{p}_i}{1-\widehat{p}_i}\right) = 1.737 - 0.361\ WBC_i$$

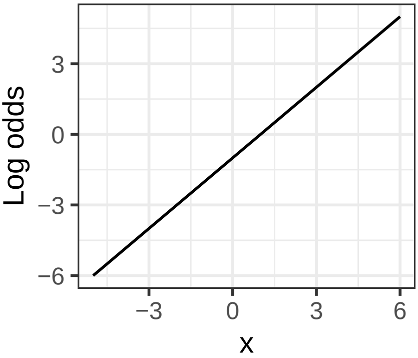
Work in groups of 2-3 on the following questions:

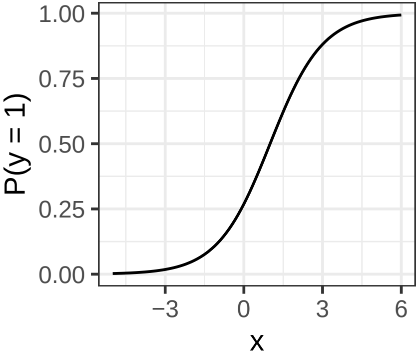
- What is the predicted odds of dengue for a patient with a WBC of 10?
- For a patient with a WBC of 10, is the predicted probability of dengue > 0.5, < 0.5, or $= 0.5$?
- What is the predicted *probability* of dengue for a patient with a WBC of 10?

Shape of the regression curve

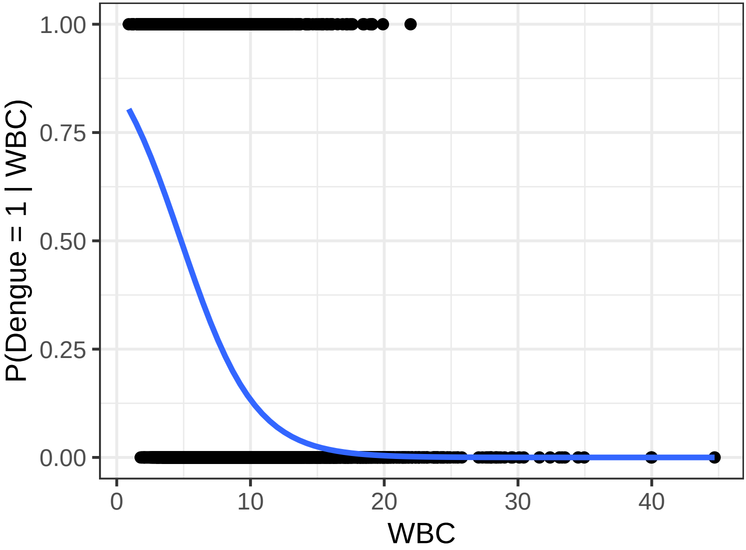
$$\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 \ X_i$$

$$p_i = \frac{e^{\beta_0+\beta_1 \ X_i}}{1+e^{\beta_0+\beta_1 \ X_i}}$$





Plotting the fitted model for dengue data

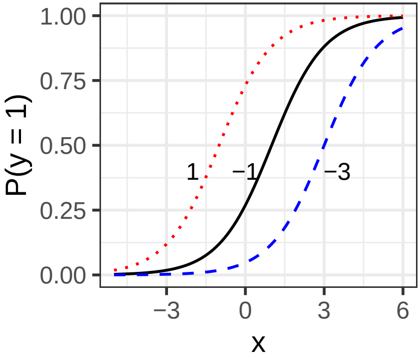


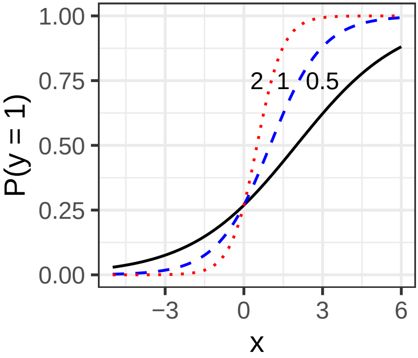
Shape of the regression curve

How does the shape of the fitted logistic regression depend on β_0 and β_1?

$p_i = \dfrac{\exp\{\beta_0 + X_i\}}{1 + \exp\{\beta_0 + X_i\}}$ for $\beta_0 = -3, -1, 1$

$p_i = \dfrac{\exp\{-1 + \beta_1\ X_i\}}{1 + \exp\{-1 + \beta_1\ X_i\}}$ for $\beta_1 = 0.5, 1, 2$





Interpretation

$$\log\left(\frac{\widehat{p}_i}{1-\widehat{p}_i}\right) = 1.737 - 0.361\ WBC_i$$

Work in groups of 2-3 for on the following questions:

- Are patients with a higher WBC more or less likely to have dengue?
- What is the change in *log odds* associated with a unit increase in WBC?
- What is the change in *odds* asociated with a unit increase in WBC?