Lecture 21: Neyman-Pearson lemma

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Neyman-Pearson test

Let $X_1, ..., X_n$ be a sample from some distribution with probability function f and parameter θ. To test

$$H_0 : \theta = \theta_0 \qquad H_A : \theta = \theta_1,$$

the **Neyman-Pearson** test rejects H_0 when

$$\frac{L(\theta_1|X)}{L(\theta_0|X)} = \frac{f(X_1, ..., X_n|\theta_1)}{f(X_1, ..., X_n|\theta_0)} > k$$

where k is chosen so that $\beta(\theta_0) = \alpha$.

Warm-up

Let $X_1, ..., X_n \overset{iid}{\sim} Poisson(\lambda)$, with pmf $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$. We want to test $H_0 : \lambda = \lambda_0$ vs. $H_A : \lambda = \lambda_1$, where $\lambda_1 > \lambda_0$. The Neyman-Pearson test rejects when

$$\frac{L(\lambda_1|\mathbf{X})}{L(\lambda_0|\mathbf{X})} > k.$$

1. Calculate $\frac{L(\lambda_1|\mathbf{X})}{L(\lambda_0|\mathbf{X})}$
2. Rearrange the ratio to show that $\frac{L(\lambda_1|\mathbf{X})}{L(\lambda_0|\mathbf{X})} > k$ if and only if $\sum_i X_i > c$ for some c
3. Using the fact that $\sum_i X_i \sim Poisson(n\lambda)$, find c such that $\beta(\lambda_0) = \alpha$

Uniformly most powerful tests

Big idea: can't do better than the Neyman-Pearson test for two simple hypotheses!

What does it mean for one test to be "better" than another?

Definition: Consider testing $H_0 : \theta \in \Theta_0$ vs. $H_A : \theta \in \Theta_1$. Let $\mathcal{C}_\alpha$ be the set of level α tests for these hypotheses. A test in C_α is the **uniformly most powerful** level α test if:

Neyman-Pearson lemma

Lemma: The Neyman-Pearson test is a *uniformly most powerful* level α test of $H_0 : \theta = \theta_0$ vs. $H_A : \theta = \theta_1$.