Lecture 11: Convergence in probability

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Convergence in probability

Definition: A sequence of random variables $X_1, X_2, \dots$ *converges in probability* to a random variable X if, for every $\varepsilon > 0$,

$$\lim_{n \to \infty} P(|X_n - X| \geq \varepsilon) = 0$$

We write $X_n \xrightarrow{p} X$.

Example

Let $U \sim \mathit{Uniform}(0, 1)$, and let $X_n = \sqrt{n}\, \mathbb{I}\{U \leq 1/n\}$.

Then $X_n \xrightarrow{p} 0$.

Weak Law of Large Numbers (WLLN)

Theorem: Let $X_1, X_2, ...$ be iid random variables with $\mathbb{E}[X_i] = \mu$ and $Var(X_i) = \sigma^2 < \infty$. Then

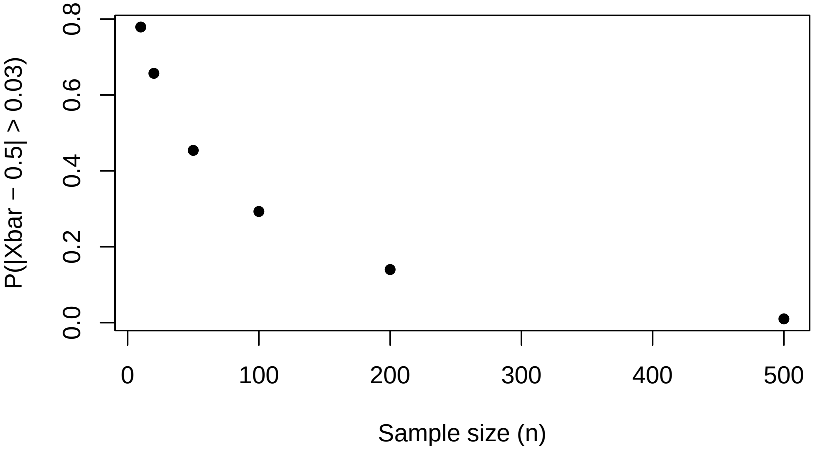
$$\overline{X}_n \xrightarrow{p} \mu$$

Activity Part I

Conduct a simulation to see the WLLN in action:

https://sta711-s25.github.io/class_activities/ca_lecture_11.html

Activity Part I



Another example

Suppose that $X_1, X_2, ... \overset{iid}{\sim} Uniform(0, 1)$, and let $X_{(n)} = \max\{X_1, ..., X_n\}$. Then $X_{(n)} \xrightarrow{p} 1$.

Activity Part II

Use a simulation to verify the Uniform example from the previous slide:

https://sta711-s25.github.io/class_activities/ca_lecture_11.html