Lecture 28: Pivotal quantities

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Recap

Confidence set: Let $\theta \in \Theta$ be a parameter of interest, and $X_1, ..., X_n$ a sample. A set $C(X_1, ..., X_n) \subseteq \Theta$ is a $1 - \alpha$ **confidence set** for θ if

$$\inf_{\theta \in \Theta} P_\theta(\theta \in C(X_1, ..., X_n)) = 1 - \alpha$$

Inverting a test: Create a confidence set by inverting a test:

$$C(X_1, ..., X_n) = \{\theta_0 : (X_1, ..., X_n) \notin \mathcal{R}(\theta_0)\}$$

Using confidence sets to test hypotheses

t interval

Suppose $X_1, ..., X_n \overset{iid}{\sim} N(\mu, \sigma^2)$. We want to construct a $1 - \alpha$ CI for μ.

Pivotal quantities

Let $X_1, ..., X_n$ be a sample and θ be an unknown parameter. A function $Q(X_1, ..., X_n, \theta)$ is called a **pivot** if the distribution of $Q(X_1, ..., X_n, \theta)$ does not depend on θ.

Example

Let $X_1, ..., X_n \overset{iid}{\sim} \textit{Uniform}(0, \theta)$. We want to construct a $1 - \alpha$ CI for θ using a pivotal quantity.

Example

Let $X_1, ..., X_n \overset{iid}{\sim} Exponential(\theta)$, with pdf $f(x|\theta) = \theta e^{-\theta x}$. Find a pivotal quantity $Q(X_1, ..., X_n, \theta)$ and construct a $1 - \alpha$ confidence interval for θ using that quantity.

- Begin with the MLE, $\widehat{\theta} = \dfrac{n}{\sum_{i=1}^{n} X_i}$
- If $X \sim Exponential(\theta)$, then $cX \sim Exponential\left(\dfrac{\theta}{c}\right)$