Lecture 30: Comparing estimators

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Course so far

- Maximum likelihood estimation
- Logistic regression
- Asymptotics
- Asymptotic properties of MLEs
- Hypothesis testing
- Confidence intervals

Common theme: Likelihoods and MLEs

Question: Why maximum likelihood estimation?

Today:

- Another approach to estimation: method of moments
- What makes a good estimator?

Example

Suppose $X_1, ..., X_n \overset{iid}{\sim} \mathit{Uniform}(0, \theta)$. How could I estimate θ?

Example

Suppose $X_1, ..., X_n \overset{iid}{\sim} Uniform(a, b)$. How could I estimate a and b?

Method of moments

Let $X_1, ..., X_n$ be a sample from a distribution with probability function $f(x|\theta_1, ..., \theta_k)$, with k parameters $\theta_1, ..., \theta_k$.

Example

Suppose $X_1, ..., X_n \stackrel{iid}{\sim} N(\mu, \sigma^2)$.

What makes a good estimator?

Suppose $X_1, ..., X_n \overset{iid}{\sim} \mathit{Uniform}(0, \theta)$. Two possible estimates:

$$\text{MLE: } \widehat{\theta} = X_{(n)} \qquad \text{MoM: } \widehat{\theta} = 2\overline{X}$$

Question: How would I choose between these estimators?

Bias, variance, and MSE

Example

Suppose $X_1, ..., X_n \overset{iid}{\sim} Uniform(0, \theta)$.

Example

Suppose $X_1, ..., X_n \overset{iid}{\sim} N(\mu, \sigma^2)$. Consider two estimates of σ^2:

$$\widehat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2 \qquad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$