Lecture 32: Variance and unbiased estimators

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Best unbiased estimators

Suppose we restrict ourselves to **unbiased** estimators.

Definition (best unbiased estimator):

Cramer-Rao lower bound

Example

Suppose $X_1, ..., X_n \overset{iid}{\sim} Poisson(\lambda)$

Example

Suppose that $X_1, ..., X_n \overset{iid}{\sim} N(\mu, \sigma^2)$.

Why MLEs are nice

Let θ be a parameter of interest, and $\widehat{\theta}$ be the maximum likelihood estimator from a sample of size n. Under regularity conditions, $\widehat{\theta}$ satisfies the following properties:

- $\widehat{\theta} \xrightarrow{p} \theta$
- $\sqrt{n}(\widehat{\theta} - \theta) \xrightarrow{d} N(0, \mathcal{I}_1^{-1}(\theta))$

Sufficient statistics

Question: Given an unbiased estimator, can I improve its variance?

- Answering this requires us to introduce a new concept: sufficient statistics

Definition (sufficient statistic):

Example

Suppose $X_1, ..., X_n \overset{iid}{\sim} Poisson(\lambda)$

Example

Suppose $X_1, ..., X_n \overset{iid}{\sim} Bernoulli(p)$