

## Lecture 4: Maximum likelihood estimation

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## Recap: maximum likelihood estimation

**Definition:** Let  $\mathbf{Y} = (Y_1, \dots, Y_n)$  be a sample of  $n$  observations, and let  $f(\mathbf{y}|\theta)$  denote the joint pdf or pmf of  $\mathbf{Y}$ , with parameter(s)  $\theta$ . The *likelihood function* is

$$L(\theta|\mathbf{Y}) = f(\mathbf{Y}|\theta)$$

**Definition:** Let  $\mathbf{Y} = (Y_1, \dots, Y_n)$  be a sample of  $n$  observations. The *maximum likelihood estimator* (MLE) is

$$\hat{\theta} = \operatorname{argmax}_{\theta} L(\theta|\mathbf{Y})$$

Example:  $N(\theta, 1)$

## Example: $Uniform(0, \theta)$

Let  $Y_1, \dots, Y_n \stackrel{iid}{\sim} Uniform(0, \theta)$ , where  $\theta > 0$ . We want the maximum likelihood estimator of  $\theta$ .

Discuss with your neighbors what the MLE of  $\theta$  might be. *Hint: focus on finding and sketching the likelihood function  $L(\mathbf{Y}|\theta)$*

Example:  $N(\mu, \sigma^2)$

## Linear regression with normal errors

$$Y_i \sim N(\mu_i, \sigma^2)$$

$$\mu_i = \beta_0 + \beta_1 X_{i,1} + \cdots + \beta_k X_{i,k}$$

Suppose we observe independent samples  $(X_1, Y_1), \dots, (X_n, Y_n)$ .  
Write down the likelihood function

$$L(\beta | \mathbf{X}, \mathbf{Y}) \propto \prod_{i=1}^n f(Y_i | \beta, X_i)$$

for the linear regression problem.