Lecture 38: False discovery rate (FDR)

Ciaran Evans

Outcomes for multiple hypothesis tests

False discovery rate

Suppose we test m hypotheses, m_0 of which are truly null. Let V denote the number of type I errors, and R the total number of rejections.

$$FWER = P(V > 0) \qquad FDR = \mathbb{E}[FDP]$$

The Benjamini-Hochberg procedure

Suppose we test m null hypotheses $H_{0,1}, ..., H_{0,m}$. Let p_i be the corresponding p-value for test i.

- Order the p-values $p_{(1)} \leq p_{(2)} \leq \cdots \leq p_{(m)}$
- Let $i^* = \max\left\{i : p_{(i)} < \frac{i\alpha}{m}\right\}$
- Reject $H_{0,(i)}$ for all $i \leq i^*$

Claim: If the hypotheses are independent, BH controls FDR at level $\frac{m_0}{m}\alpha \leq \alpha$

Summary

- BH controls FDR at level $\frac{m_0}{m}\alpha$
- If $m_0 = m$, then controlling FDR is equivalent to controlling FWER
- If $m_0 < m$, then controlling FDR provides more power to reject H_0 when H_0 is false