MSE for estimating Normal variance

Suppose $X_1, ..., X_n \overset{iid}{\sim} N(\mu, \sigma^2)$. Consider two estimates of σ^2:

$$\widehat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \overline{X})^2 \qquad s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})^2$$

Our goal is to find the MSE for these two estimators. We will use the following key facts:

- $\frac{1}{\sigma^2} \sum\limits_{i=1}^{n} (X_i - \overline{X})^2 \sim \chi^2_{n-1}$
- If $V \sim \chi^2_\nu$, then $\mathbb{E}[V] = \nu$ and $Var(V) = 2\nu$

Questions

1. Using the information above, show that $\mathbb{E}[s^2] = \sigma^2$ (that is, the sample variance s^2 is an unbiased estimator).

2. Now calculate $\mathbb{E}[\widehat{\sigma}^2]$. Does $\widehat{\sigma}^2$ tend to overestimate or underestimate σ^2?

3. Using the information above, compute $Var(s^2)$ and $Var(\widehat{\sigma}^2)$.

4. Which estimator has a lower MSE?