Lecture 28: Pivotal quantities

Ciaran Evans

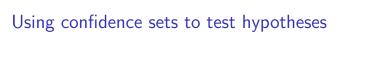
Recap

Confidence set: Let $\theta \in \Theta$ be a parameter of interest, and $X_1,...,X_n$ a sample. A set $C(X_1,...,X_n) \subseteq \Theta$ is a $1-\alpha$ **confidence set** for θ if

$$\inf_{\theta \in \Theta} P_{\theta}(\theta \in C(X_1, ..., X_n)) = 1 - \alpha$$

Inverting a test: Create a confidence set by inverting a test:

$$C(X_1,...,X_n) = \{\theta_0 : (X_1,...,X_n) \notin \mathcal{R}(\theta_0)\}$$



t interval

Suppose $X_1,...,X_n \stackrel{iid}{\sim} N(\mu,\sigma^2)$. We want to construct a $1-\alpha$ CI for μ .

Pivotal quantities

Let $X_1, ..., X_n$ be a sample and θ be an unknown parameter. A function $Q(X_1, ..., X_n, \theta)$ is called a **pivot** if the distribution of $Q(X_1, ..., X_n, \theta)$ does not depend on θ .

Example

Let $X_1,...,X_n \stackrel{iid}{\sim} Uniform(0,\theta)$. We want to construct a $1-\alpha$ CI for θ using a pivotal quantity.

Example

Let $X_1,...,X_n \stackrel{iid}{\sim} Exponential(\theta)$, with pdf $f(x|\theta) = \theta e^{-\theta x}$. Find a pivotal quantity $Q(X_1,...,X_n,\theta)$ and construct a $1-\alpha$ confidence interval for θ using that quantity.

- ▶ Begin with the MLE, $\widehat{\theta} = \frac{n}{\sum_{i=1}^{n} X_i}$
- ▶ If $X \sim Exponential(\theta)$, then $cX \sim Exponential\left(\frac{\theta}{c}\right)$