Lecture 5: Maximum likelihood estimation

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Recap: maximum likelihood estimation

Definition: Let $\mathbf{Y} = (Y_1, ..., Y_n)$ be a sample of n observations, and let $f(\mathbf{y}|\theta)$ denote the joint pdf or pmf of $\mathbf{Y}$, with parameter(s) θ. The *likelihood function* is

$$L(\theta|\mathbf{Y}) = f(\mathbf{Y}|\theta)$$

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$$\widehat{\theta} = \text{ argmax}_\theta \; L(\theta|\mathbf{Y})$$

Example: $N(\mu, \sigma^2)$

Linear regression with normal errors

$$Y_i \sim N(\mu_i, \sigma^2)$$

$$\mu_i = \beta_0 + \beta_1 X_{i,1} + \cdots + \beta_k X_{i,k}$$

Suppose we observe independent samples $(X_1, Y_1), ..., (X_n, Y_n)$. Write down the likelihood function

$$L(\beta|\mathbf{X}, \mathbf{Y}) \propto \prod_{i=1}^{n} f(Y_i|\beta, X_i)$$

Logistic regression

$$Y_i \sim Bernoulli(p_i)$$

$$\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 X_{i,1} + \cdots + \beta_k X_{i,k}$$

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