Lecture 29: Delta method and variance stabilizing transformations

Ciaran Evans

Motivating example: Exponential confidence interval

$$X_1, ..., X_n \overset{iid}{\sim} Exponential(\theta)$$

Last time: Used a pivotal quantity to find $1 - \alpha$ CI for θ:

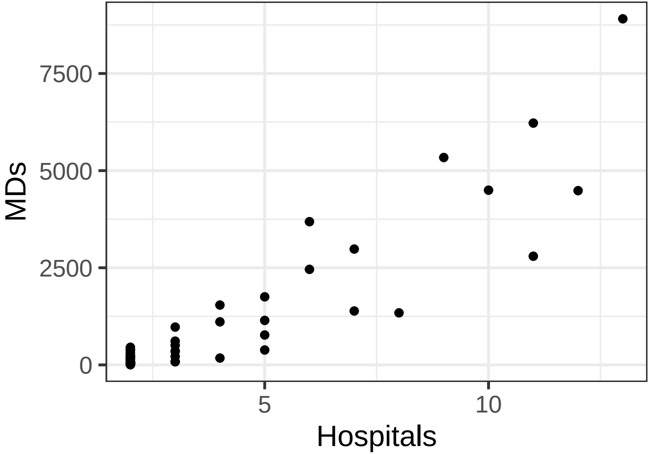
$$\left[\frac{a}{\sum_i X_i}, \frac{b}{\sum_i X_i}\right]$$

Alternative: Wald Confidence interval

- MLE: $\widehat{\theta} =$
- Asymptotic distribution: $\sqrt{n}(\widehat{\theta} - \theta) \approx$
- Wald CI:

Motivating example: non-constant variance

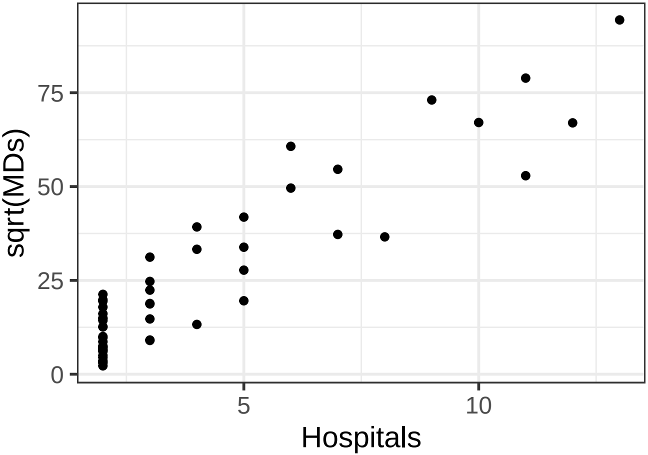
Example: Data on the number of hospitals and number of doctors (MDs) in US counties



Question: How do we adjust for non-constant variance in a linear model?

Motivating example: non-constant variance

Example: Data on the number of hospitals and number of doctors (MDs) in US counties



Goal: variance stabilizing transformation

Suppose $\widehat{\theta}$ is an estimator, and $Var(\widehat{\theta})$ depends on θ. Examples:

- Exponential: $\sqrt{n}(\widehat{\theta} - \theta) \approx N(0, \theta^2)$
- Poisson: $\sqrt{n}(\widehat{\lambda} - \lambda) \approx N(0, \lambda)$
- Bernoulli: $\sqrt{n}(\widehat{p} - p) \approx N(0, p(1-p))$

Goal: Find a transformation g such that $Var(g(\widehat{\theta}))$ does **not** depend on θ

Delta method

Delta method

Suppose that $\widehat{\theta}$ is an estimate of $\theta \in \mathbb{R}$, such that

$$\sqrt{n}(\widehat{\theta} - \theta) \xrightarrow{d} N(0, \sigma^2)$$

for some σ^2, and g is a continuously differentiable function with $g'(\theta) \neq 0$. Then

$$\sqrt{n}(g(\widehat{\theta}) - g(\theta)) \xrightarrow{d} N(0, \sigma^2[g'(\theta)]^2)$$

Example: Exponential

$$\sqrt{n}(\widehat{\theta} - \theta) \approx N(0, \theta^2)$$

Example: Poisson

$$\sqrt{n}(\widehat{\lambda} - \lambda) \approx N(0, \lambda)$$

Example: Bernoulli

$$\sqrt{n}(\widehat{p} - p) \approx N(0, p(1-p))$$