Lecture 13: Comparing types of convergence

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Recap: Convergence in probability

Definition: A sequence of random variables $X_1, X_2, \ldots$ *converges in probability* to a random variable X if, for every $\varepsilon > 0$,

$$\lim_{n \to \infty} P(|X_n - X| \geq \varepsilon) = 0$$

We write $X_n \xrightarrow{p} X$.

Convergence in distribution

Definition: A sequence of random variables $X_1, X_2, ...$ *converges in distribution* to a random variable X if

$$\lim_{n \to \infty} F_{X_n}(x) = F_X(x)$$

at all points where $F_X(x)$ is continuous. We write $X_n \xrightarrow{d} X$.

Example

Suppose that $X \sim N(0,1)$, and let $X_n = -X$ for $n = 1, 2, 3, \ldots$.

Claim: $X_n \xrightarrow{d} X$, but X_n does *not* converge in probability to X

Relationships between types of convergence

(a) If $X_n \xrightarrow{d} c$, where c is a constant, then $X_n \xrightarrow{p} c$

(b) If $X_n \xrightarrow{p} X$, then $X_n \xrightarrow{d} X$