Lecture 26: Wald vs. LRT

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Comparing Wald and LRT statistics

Suppose we observe data $X_1, ..., X_n$ from some distribution with parameter $\theta \in \mathbb{R}^q$, and we wish to test

$$H_0 : \theta = \theta_0 \qquad H_A : \theta \neq \theta_0$$

Consider three possible scenarios:

- H_0 is true: $\theta =$
- H_A is true, **fixed** alternative: $\theta =$
- H_A is true, **local** alternative: $\theta =$

Comparing Wald and LRT statistics

Under H_0, or for a local alternative $\theta = \theta_0 + \frac{d}{\sqrt{n}}$, Wald and LRT are asymptotically equivalent as $n \to \infty$ (under certain regularity conditions).

Power under a local alternative

Recall asymptotic normality of the MLE: $\widehat{\theta} \approx N(\theta, \mathcal{I}^{-1}(\theta))$

Suppose we test $H_0 : \theta = \theta_0$ vs. $H_A : \theta \neq \theta_0$.

$$W =$$

Under H_0, $W \approx \chi^2_q$

Class activity

- Simulate data under a local alternative
- Verify that the Wald statistic follows a non-central χ^2 distribution