Warmup: Convergence of the sample mean

Convergence of the sample mean

Let $X_1, X_2, ..., X_n$ be an iid sample from a distribution with $\mathbb{E}[X_i] = \mu$ and $Var(X_i) = \sigma^2 < \infty$. Let $\overline{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$ be the sample mean.

1. Show that $\mathbb{E}[\overline{X}_n] = \mu$ and $Var(\overline{X}_n) = \frac{\sigma^2}{n}$.

2. Using question 1 and a probability inequality from last time, show that for any $\varepsilon > 0$,

$$P(|\overline{X}_n - \mu| \geq \varepsilon) \to 0$$

as $n \to \infty$.