Activity: MSE

MSE

Let $X_1, ..., X_n \overset{iid}{\sim} Uniform(0, \theta)$. The method of moments estimator of θ is $\hat{\theta}_{\text{MoM}} = 2\overline{X}$, and the maximum likelihood estimator is $\hat{\theta}_{\text{MLE}} = X_{(n)}$.

We would like to compute and compare the MSE for both estimators. It will help to know that $\mathbb{E}[X_i] = \frac{\theta}{2}$, $Var(X_i) = \frac{\theta^2}{12}$, $\mathbb{E}[X_{(n)}] = \frac{n}{n+1}\theta$, and $Var(X_{(n)}) = \frac{n\theta^2}{(n+1)^2(n+2)}$.

1. Calculate $MSE(\hat{\theta}_{\text{MoM}})$ and $MSE(\hat{\theta}_{\text{MLE}})$. Which estimator has a smaller MSE?

2. The maximum likelihood estimator $\hat{\theta}_{\text{MLE}} = X_{(n)}$ is a *biased* estimator of θ. We may wonder if we can improve the MSE by *unbiasing* the MLE. Find a simple transformation $g(\hat{\theta}_{\text{MLE}})$ such that $\mathbb{E}[g(\hat{\theta}_{\text{MLE}})] = \theta$, then calculate $MSE(g(\hat{\theta}_{\text{MLE}}))$ and compare to $MSE(\hat{\theta}_{\text{MLE}})$.