# STA 711 Homework 2

**Due:** Friday, January 30, 11:59pm on Canvas.

**Instructions:** Submit your work as a single PDF. You may choose to either hand-write your work and submit a PDF scan, or type your work using LaTeX and submit the resulting PDF. See the course website for a homework template file and instructions on getting started with LaTeX and Overleaf.

## Maximum likelihood estimation

1. Let $Y_1, ..., Y_n \overset{iid}{\sim} Poisson(\lambda)$, and let $\mathbf{y} = (Y_1, ..., Y_n)$ denote the combined sample.
   (a) Write down the likelihood $L(\lambda|\mathbf{y})$.
   (b) Find the maximum likelihood estimator $\widehat{\lambda}$ of $\lambda$.

2. Let $Y_1, ..., Y_n \overset{iid}{\sim} Exponential(\theta)$, so $f(y|\theta) = \theta e^{-\theta y}$.
   (a) Write down the likelihood $L(\theta|\mathbf{y})$.
   (b) Find the maximum likelihood estimator $\widehat{\theta}$ of $\theta$.
   (c) Show that $Y_{(1)} \sim Exponential(n\theta)$.

3. Let $Y_1, ..., Y_n$ be iid from a distribution with pdf
$$f(y|\theta) = \theta y^{-2} \mathbb{1}\{y \geq \theta\},$$
where $\theta > 0$. Find the maximum likelihood estimator of $\theta$.

4. Let $Y_1, ..., Y_n$ be iid with one of two pdfs. If $\theta = 0$, then
$$f(y|\theta) = \begin{cases} 1 & 0 < y < 1 \\ 0 & \text{else.} \end{cases}$$
If $\theta = 1$, then
$$f(y|\theta) = \begin{cases} \dfrac{1}{2\sqrt{y}} & 0 < y < 1 \\ 0 & \text{else.} \end{cases}$$
Find the maximum likelihood estimator of $\theta$.

5. Let $Y$ be a single observation from a normal distribution with mean $\theta$ and variance $\theta^2$, where $\theta > 0$. Find the maximum likelihood estimator of $\theta^2$.

6. Let $Y_1, ..., Y_n$ be iid from a distribution with pdf
$$f(y|\mu, \sigma) = \frac{1}{\sigma} \exp\left\{-\left(\frac{y-\mu}{\sigma}\right)\right\} \mathbb{1}\{y \geq \mu\},$$
where $-\infty < \mu < \infty$, and $\sigma > 0$.
   (a) Find the maximum likelihood estimators of $\mu$ and $\sigma$. *(Hint: find $\widehat{\mu}$ first)*
   (b) Let $\tau(\mu, \sigma) = \mathbb{P}_{\mu,\sigma}(Y_1 \geq t)$, where $t > \mu$, and $\mathbb{P}_{\mu,\sigma}$ denotes probability when $\mu, \sigma$ are the true parameters. Find the maximum likelihood estimator of $\tau(\mu, \sigma)$.