Warmup: Maximum likelihood estimation

Maximum likelihood estimation

Let $Y_1, ..., Y_n \overset{iid}{\sim} N(\mu, \sigma^2)$, with both μ and σ^2 unknown. Let $\theta = \begin{bmatrix} \mu \\ \sigma^2 \end{bmatrix}$. We would like to estimate θ using the method of maximum likelihood. Recall that

$$f(Y_i|\theta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}(Y_i - \mu)^2\right\}$$

1. Let $\mathbf{y} = (Y_1, ..., Y_n)$. Find expressions for the likelihood $L(\theta|\mathbf{y})$ and log likelihood $\ell(\theta|\mathbf{y})$.

2. By taking the partial derivatives $\frac{\partial}{\partial \mu}\ell(\theta|\mathbf{y})$ and $\frac{\partial}{\partial \sigma^2}\ell(\theta|\mathbf{y})$, show that the MLEs for μ and σ^2 are

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

and

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{\mu})^2 = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \overline{Y})^2$$

(Don't worry about the second derivative for now, just set the partial derivatives equal to 0 and solve).