Warmup: Maximum likelihood estimation and linear regression

Maximum likelihood estimation

Suppose that we have independent observations $(\mathbf{x}_1, Y_1), ..., (\mathbf{x}_n, Y_n)$ from the model

$$Y_i | \mathbf{x}_i \sim N(\mathbf{x}_i^T \beta, \sigma_i^2).$$

Suppose that the variances $\sigma_1^2, ..., \sigma_n^2$ are known. Show that the maximum likelihood estimator of β minimizes the weighted sum of squares

$$WSS(\beta) = \sum_{i=1}^{n} w_i (Y_i - \mathbf{x}_i^T \beta)^2 = (\mathbf{y} - \mathbf{X}\beta)^T \mathbf{W} (\mathbf{y} - \mathbf{X}\beta)$$