Comparing estimators

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Course so far

- Maximum likelihood estimation
- Tools for asymptotic results
- Asymptotic properties of MLEs

Questions:

- How else could we estimate parameters?
- What makes a good estimator? How do we compare estimators?
- Why is maximum likelihood estimation so widespread?
- What happens to MLEs if our model is *wrong*?

Example

Suppose $X_1, ..., X_n \overset{iid}{\sim} Uniform(0, \theta)$. How could I estimate θ?

Example

Suppose $X_1, ..., X_n \overset{iid}{\sim} Uniform(a, b)$. How could I estimate a and b?

Method of moments

Let $X_1, ..., X_n$ be a sample from a distribution with probability function $f(x|\theta_1, ..., \theta_k)$, with k parameters $\theta_1, ..., \theta_k$.

Example

Suppose $X_1, ..., X_n \overset{iid}{\sim} N(\mu, \sigma^2)$.

What makes a good estimator?

Suppose $X_1, ..., X_n \overset{iid}{\sim} Uniform(0, \theta)$. Two possible estimates:

$$\text{MLE: } \widehat{\theta} = X_{(n)} \qquad \text{MoM: } \widehat{\theta} = 2\overline{X}$$

Question: How would I choose between these estimators?

Bias, variance, and MSE

Example

Suppose $X_1, ..., X_n \overset{iid}{\sim} Uniform(0, \theta)$.