Best unbiased estimators and the Cramer-Rao lower bound

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Best unbiased estimators

Suppose we restrict ourselves to **unbiased** estimators.

Definition (best unbiased estimator):

Cramer-Rao lower bound

Let $Y_1, ..., Y_n$ be a sample from a distribution with probability function $f(y|\theta)$, and let $\widehat{\theta}$ be an estimator of $\theta \in \mathbb{R}$. Under regularity conditions,

$$Var_\theta(\widehat{\theta}) \geq \frac{\left(\frac{d}{d\theta}\mathbb{E}_\theta(\widehat{\theta})\right)^2}{\mathbb{E}_\theta\left[\left(\frac{d}{d\theta}\log f(Y_1, ..., Y_n|\theta)\right)^2\right]}$$

Example

Suppose $Y_1, ..., Y_n \overset{iid}{\sim} Poisson(\lambda)$

Your assignment

Suppose $Y_1, ..., Y_n \overset{iid}{\sim} Bernoulli(p)$.

- Find the Cramer-Rao lower bound for an unbiased estimator of p
- The MLE $\widehat{p} = \overline{Y}$ is unbiased for p. Does the MLE attain the CRLB?