Activity: Fisher information

Fisher information

Let $Y_1, ..., Y_n \overset{iid}{\sim} Poisson(\lambda)$. We have previously shown that the MLE is $\hat{\lambda} = \frac{1}{n} \sum\limits_{i=1}^{n} Y_i$.

1. Compute $Var(\hat{\lambda})$.

$$Var(\hat{\lambda}) = \frac{1}{n^2} \cdot n Var(Y_i) = \frac{\lambda}{n}$$

2. For the Poisson, the derivative of the log-likelihood for a single observation is $\frac{d}{d\lambda} \log f(Y|\lambda) = -1 + \frac{Y}{\lambda}$. Compute the Fisher information, and compare with your answer to question 1.

$$\mathcal{I}_1(\lambda) = Var\left(\frac{d}{d\lambda} \log f(Y|\lambda)\right) = Var\left(-1 + \frac{Y}{\lambda}\right)$$

$$= \frac{1}{\lambda^2} Var(Y) = \frac{1}{\lambda}$$

$$\boxed{\mathcal{I}_1(\lambda) = \frac{1}{\lambda}}$$

For iid data, $\mathcal{I}_n(\lambda) = n \mathcal{I}_1(\lambda) = \frac{n}{\lambda}$

$$Var(\hat{\lambda}) = \mathcal{I}_n^{-1}(\lambda)$$