Maximum likelihood estimation

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Warmup

Work on the warmup activity (handout) with your neighbors, then we will discuss as a class.

Warmup

$Y_1, ..., Y_n \overset{iid}{\sim} N(\mu, \sigma^2)$. Let $\boldsymbol{\theta} = \begin{bmatrix} \mu \\ \sigma^2 \end{bmatrix}$ and $\mathbf{y} = (Y_1, ..., Y_n)$.

$$L(\boldsymbol{\theta}|\mathbf{y}) =$$

Warmup

$$\ell(\boldsymbol{\theta}|\mathbf{y}) = -\frac{n}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(Y_i - \mu)^2$$

$$\frac{\partial}{\partial\mu}\ell(\boldsymbol{\theta}|\mathbf{y}) =$$

$$\frac{\partial}{\partial\sigma^2}\ell(\boldsymbol{\theta}|\mathbf{y}) =$$

Invariance of the MLE

For $Y_1, ..., Y_n \overset{iid}{\sim} N(\mu, \sigma^2)$, we saw that the MLE of the variance is $\widehat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (Y_i - \overline{Y})^2$. What if we instead want to estimate the *standard deviation*, σ?

Another example: *Uniform*$(0, \theta)$

Let $Y_1, ..., Y_n \overset{iid}{\sim}$ *Uniform*$(0, \theta)$, where $\theta > 0$. We want the maximum likelihood estimator of θ.

Maximum likelihood estimation and linear regression

Let $(\mathbf{x}_1, Y_1), ..., (\mathbf{x}_n, Y_n)$ be iid samples from the model

$$Y_i|\mathbf{x}_i \sim N(\mu_i, \sigma^2)$$
$$\mu_i = \mathbf{x}_i^T \boldsymbol{\beta},$$

$$L(\beta, \sigma^2|\mathbf{y}, \mathbf{X}) =$$