Convergence of random variables

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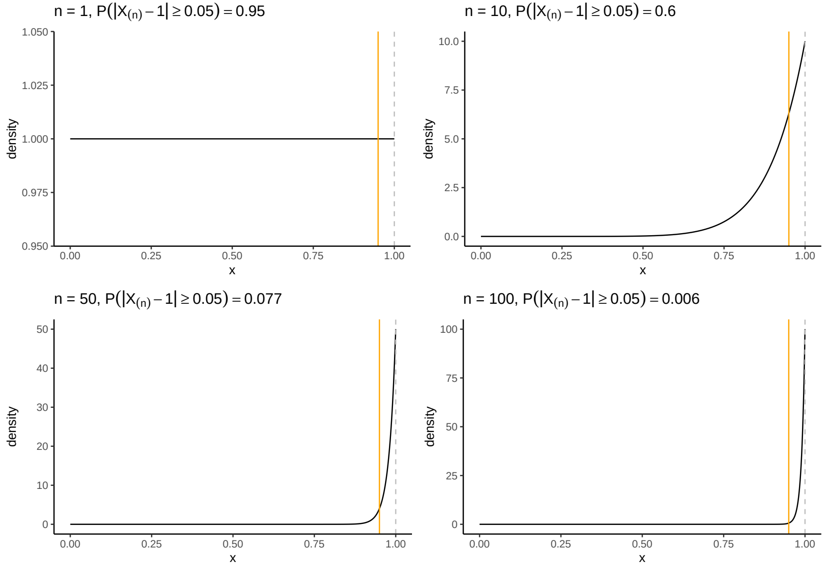
Last time: Class activity

Suppose that $X_1, X_2, \ldots \overset{iid}{\sim} Uniform(0,1)$, and let $X_{(n)} = \max\{X_1, \ldots, X_n\}$. Then $X_{(n)} \xrightarrow{p} 1$.

Last time: Class activity

Suppose that $X_1, X_2, \ldots \overset{iid}{\sim} Uniform(0,1)$. Then $X_{(n)} \sim Beta(n,1)$



Warmup

Work on the warmup activity (handout), then we will discuss as a class.

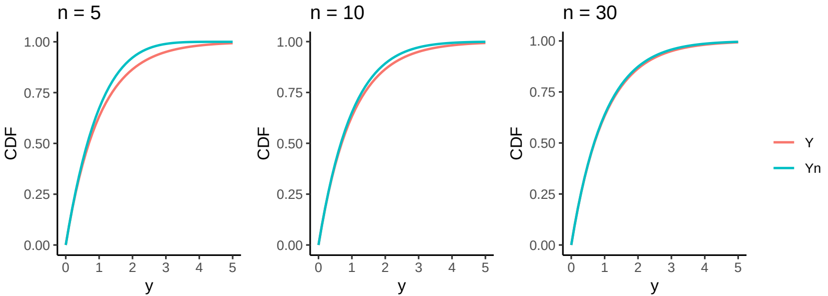
Warmup

Suppose that $X_1, X_2, ... \overset{iid}{\sim} Uniform(0, 1)$, $X_{(n)} = \max\{X_1, ..., X_n\}$, and let $Y_n = n(1 - X_{(n)})$.

$F_{Y_n}(t) = P(Y_n \leq t) =$

Warmup

$$F_{Y_n}(t) = 1 - \left(1 - \frac{t}{n}\right)^n \to F_Y(t) = 1 - e^{-t}$$



Convergence in distribution

Definition: A sequence of random variables $X_1, X_2, \ldots$ *converges in distribution* to a random variable X if

$$\lim_{n \to \infty} F_{X_n}(x) = F_X(x)$$

at all points where $F_X(x)$ is continuous. We write $X_n \xrightarrow{d} X$.

Class activity

Work on the class activity (handout), then we will discuss as a class.

Class activity

Suppose that $X_1, X_2, ...$ are iid random variables with cdf

$$F(x) = \begin{cases} 1 - \left(\frac{1}{x}\right)^{\alpha} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

Let $X_{(n)} = \max\{X_1, ..., X_n\}$, and $Y_n = n^{-1/\alpha} X_{(n)}$.

$F_{Y_n}(t) = P(n^{-1/\alpha} X_{(n)} \leq t) =$

Convergence in distribution: Central Limit Theorem

Let $X_1, X_2, \ldots$ be iid random variables, with $\mu = \mathbb{E}[X_i]$ and $\sigma^2 = Var(X_i) < \infty$. Then

$$\frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} \xrightarrow{d} Z$$

where $Z \sim N(0, 1)$.