

Maximum likelihood estimation

Ciaran Evans

Fitting a *logistic* regression model?

Linear regression: minimize $\sum_{i=1}^n (Y_i - \beta_0 - \beta_1 X_{i1} - \cdots - \beta_k X_{ik})^2$

Question: Should we minimize a similar sum of squares for a *logistic* regression model?

Motivation: likelihoods and estimation

Let $Y \sim \text{Bernoulli}(p)$ be a Bernoulli random variable, with $p \in [0, 1]$. We observe 5 independent samples from this distribution:

$$Y_1 = 1, Y_2 = 1, Y_3 = 0, Y_4 = 0, Y_5 = 1$$

The true value of p is unknown, so two friends propose different guesses for the value of p : 0.3 and 0.7. Which do you think is a “better” guess?

Likelihood

Definition: Let $\mathbf{y} = (Y_1, \dots, Y_n)$ be a sample of n observations, and let $f(\mathbf{y}|\theta)$ denote the joint pdf or pmf of \mathbf{y} , with parameter(s) θ . The *likelihood function* is

$$L(\theta|\mathbf{y}) = f(\mathbf{y}|\theta)$$

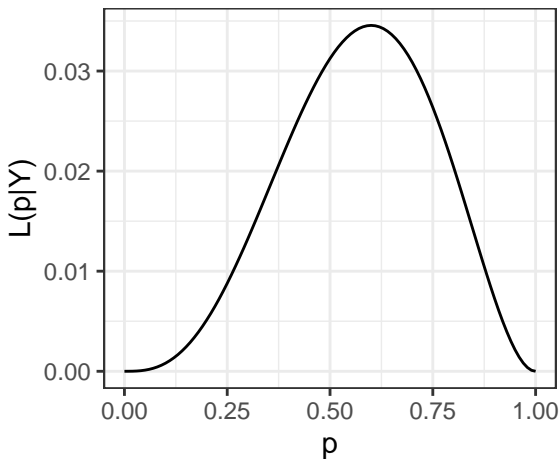
Example: Bernoulli data

Example: Bernoulli data

$Y_1, \dots, Y_5 \stackrel{iid}{\sim} \text{Bernoulli}(p)$, with observed data

$$Y_1 = 1, Y_2 = 1, Y_3 = 0, Y_4 = 0, Y_5 = 1$$

$$L(p|\mathbf{y}) = p^3(1-p)^2$$



Maximum likelihood estimator

Definition: Let $\mathbf{y} = (Y_1, \dots, Y_n)$ be a sample of n observations. The *maximum likelihood estimator* (MLE) is

$$\hat{\theta} = \operatorname{argmax}_{\theta} L(\theta|\mathbf{y})$$

Example: *Bernoulli*(p)

Example: $N(\theta, 1)$

Example: $Uniform(0, \theta)$

Let $Y_1, \dots, Y_n \stackrel{iid}{\sim} Uniform(0, \theta)$, where $\theta > 0$. We want the maximum likelihood estimator of θ .

Discuss with your neighbors what the MLE of θ might be. *Hint: focus on finding and sketching the likelihood function $L(\mathbf{y}|\theta)$*