Maximum likelihood estimation and linear regression

Maximum likelihood estimation

Let $(\mathbf{x}_1, Y_1), ..., (\mathbf{x}_n, Y_n)$ be iid samples from the model

$$Y_i|\mathbf{x}_i \sim N(\mu_i, \sigma^2)$$
$$\mu_i = \mathbf{x}_i^T\beta,$$

where the distribution of $\mathbf{x}_i$ does not depend on σ^2 or β.

1. Using the fact that $f(\mathbf{x}_i, Y_i|\beta, \sigma^2) = f(\mathbf{x}_i)f(Y_i|\mathbf{x}_i, \beta, \sigma^2)$, show that

$$L(\beta, \sigma^2|\mathbf{y}, \mathbf{X}) \propto (2\pi\sigma^2)^{-n/2} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(Y_i - \mathbf{x}_i^T\beta)^2\right\}.$$

2. From question 1, conclude that the maximum likelihood estimator of β is found by minimizing the sum of squared errors $\sum\limits_{i=1}^{n}(Y_i - \mathbf{x}_i^T\beta)^2$.