Class overview and linear regression

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Welcome to STA 711!

Agenda:

- Brief course overview and syllabus highlights
- Sketch plan for first couple weeks
- Linear regression

Core course content

- Estimation
  - How do we estimate unknown parameters?
  - How do we assess uncertainty in our estimates?
  - What makes a "good" estimator?
- Asymptotics
  - What happens to our estimators as the sample size gets large?
- Hypothesis testing
  - How do we assess competing hypotheses about the data generating process?

Course motivation

- Regression models are a natural setting for many of the key topics in 711
- We are already familiar with the idea of regression, and these models provide a good motivation for much of our course content
- Regression models are naturally multivariable
- STA 712 (GLMs) builds on our 711 material; I view 711/712 as a two-course sequence
- So: Regression (particularly linear and logistic regression) will be used as motivation throughout

Course structure

- Class participation and seminar attendance
- Regular HW assignments (due most weeks)
- Homework presentations (3)
- Two midterm exams
- Final exam

Plan for the first couple weeks

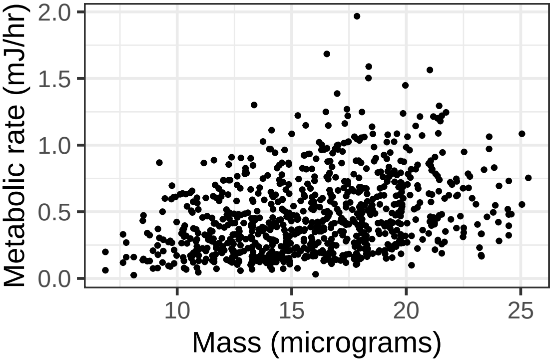
- Brief overview/review of linear regression
- Introduction to logistic regression and parameter estimation
- Maximum likelihood estimation

Motivating example: mass and metabolic rate

- Researchers are interested in the relationship between an organism's mass and metabolic rate (how much energy it consumes)
- To study this relationship, they collected data on 568 individuals from two species of marine bryozoan

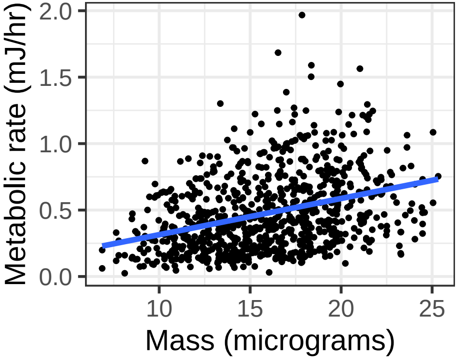
Motivating example: mass and metabolic rate

For each individual, the researchers recorded the mass and metabolic rate:



Question: What do you notice about this relationship?

Motivating example: mass and metabolic rate



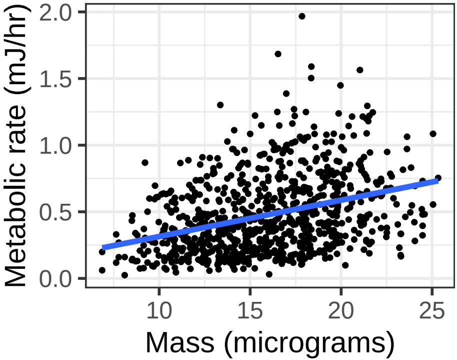
If I wanted to write down a model for the relationship between mass and metabolic rate, what would it look like?

Linear regression model

$$\text{Metabolic}_i = \beta_0 + \beta_1 \text{Mass}_i + \varepsilon_i$$

What do the parameters β_0 and β_1 represent?

Linear regression model



$$\text{Metabolic}_i = \beta_0 + \beta_1 \text{Mass}_i + \varepsilon_i$$

The true β_0 and β_1 are **unknown**. How do I estimate them from data?

Fitting a linear regression model

$$\text{Metabolic}_i = \beta_0 + \beta_1 \text{Mass}_i + \varepsilon_i$$

Choosen β_0, β_1 to minimize

$$SSE(\beta_0, \beta_1) = \sum_{i=1}^{n} (\text{Metabolic}_i - \beta_0 - \beta_1 \text{Mass}_i)^2$$

Question: How would I solve this minimization problem?

Fitting a linear regression model

Model:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_k X_{ik} + \varepsilon_i$$

Observed data: $(X_{11}, X_{12}, ..., X_{1k}, Y_1), ..., (X_{n1}, X_{n2}, ..., X_{nk}, Y_n)$

Estimation: choose $\beta_0, \beta_1, ..., \beta_k$ to minimize

$$SSE(\beta_0, ..., \beta_k) = \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_{i1} - \cdots - \beta_k X_{ik})^2$$

How would I solve this minimization problem?

Improving our life with linear algebra

$$
\begin{aligned}
Y_1 &= \beta_0 + \beta_1 X_{11} + \beta_2 X_{12} + \cdots + \beta_k X_{1k} + \varepsilon_1 \\
Y_2 &= \beta_0 + \beta_1 X_{21} + \beta_2 X_{22} + \cdots + \beta_k X_{2k} + \varepsilon_2 \\
&\vdots \\
Y_n &= \beta_0 + \beta_1 X_{n1} + \beta_2 X_{n2} + \cdots + \beta_k X_{nk} + \varepsilon_n
\end{aligned}
$$

Improving our life with linear algebra

$$
\begin{aligned}
Y_1 &= \beta_0 + \beta_1 X_{11} + \beta_2 X_{12} + \cdots + \beta_k X_{1k} + \varepsilon_1 \\
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&\vdots \\
Y_n &= \beta_0 + \beta_1 X_{n1} + \beta_2 X_{n2} + \cdots + \beta_k X_{nk} + \varepsilon_n
\end{aligned}
$$

$$
\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & X_{12} & \cdots & X_{1k} \\ 1 & X_{21} & X_{22} & \cdots & X_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{n1} & X_{n2} & \cdots & X_{nk} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \vdots \\ \beta_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}
$$

Rewriting the optimization problem

$$\boldsymbol{Y} = \mathbf{X}\beta + \boldsymbol{\varepsilon}$$

$$SSE(\beta) =$$

Solving the optimization problem