

Activity: Fisher information

Fisher information

Let $Y_1, \dots, Y_n \stackrel{iid}{\sim} Poisson(\lambda)$. The log-likelihood is

$$\ell_n(\lambda) = -n\lambda + \log(\lambda) \sum_{i=1}^n Y_i - \log \left(\prod_{i=1}^n Y_i! \right)$$

and we have previously shown that the MLE is $\hat{\lambda} = \frac{1}{n} \sum_{i=1}^n Y_i$.

1. Compute $\text{Var}(\hat{\lambda})$.
 2. Compute $\text{Var}(\ell'_n(\lambda))$, and compare with your answer to question 1.
 3. Compute $-\mathbb{E}[\ell''_n(\lambda)]$, and compare with your answer to question 2.