Convergence of random variables

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Warmup

Work on the warmup activity (handout), then we will discuss as a class.

Warmup

Let $X_1, X_2, \ldots$ be iid random variables with $\mathbb{E}[X_i] = \mu$ and $Var(X_i) = \sigma^2 < \infty$.

$$\mathbb{E}[\overline{X}_n] = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] =$$

$$Var(\overline{X}_n) = Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) =$$

$$P(|\overline{X}_n - \mu| \geq \varepsilon) \leq$$

Convergence in probability

Definition: A sequence of random variables $X_1, X_2, \ldots$ *converges in probability* to a random variable X if, for every $\varepsilon > 0$,

$$\lim_{n \to \infty} P(|X_n - X| \geq \varepsilon) = 0$$

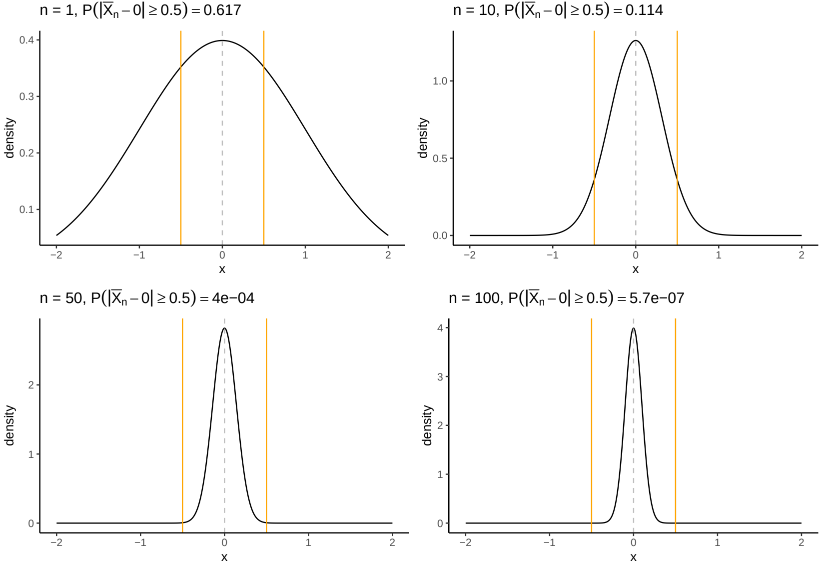
We write $X_n \xrightarrow{p} X$.

Weak Law of Large Numbers (WLLN): Let $X_1, X_2, \ldots$ be iid random variables with $\mathbb{E}[X_i] = \mu$ and $Var(X_i) = \sigma^2 < \infty$. Then

$$\overline{X}_n \xrightarrow{p} \mu$$

Example: WLLN

Suppose $X_1, X_2, \ldots \overset{iid}{\sim} N(0,1)$. Then $\overline{X}_n \sim N(0, \frac{1}{n})$. Let $\varepsilon = 0.5$.



Example

Let $U \sim \mathit{Uniform}(0,1)$, and let $X_n = \sqrt{n}\,\mathbb{I}\{U \leq 1/n\}$.

Then $X_n \xrightarrow{p} 0$.

Class activity, Part I

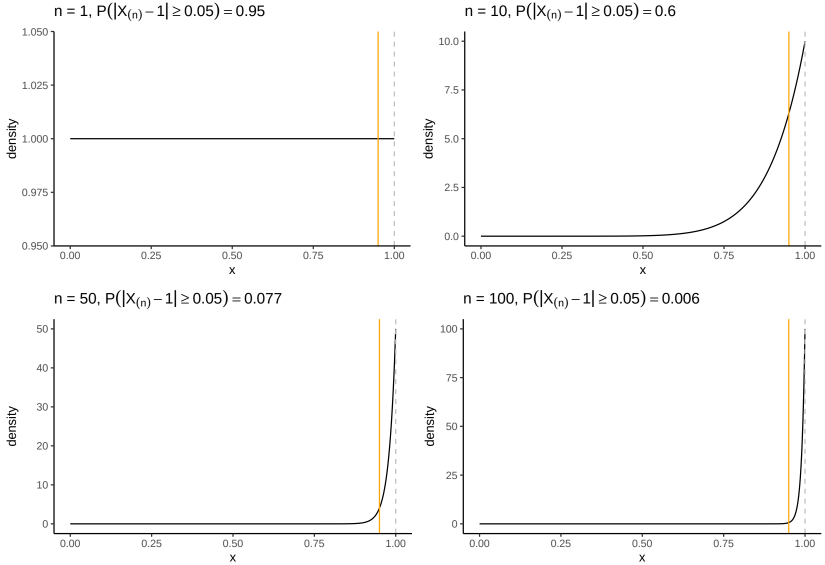
Work on the class activity (handout), then we will discuss as a class.

Class activity, Part I

Suppose that $X_1, X_2, ... \overset{iid}{\sim} Uniform(0, 1)$, and let $X_{(n)} = \max\{X_1, ..., X_n\}$. Then $X_{(n)} \xrightarrow{p} 1$.

Class activity, Part I

Suppose that $X_1, X_2, \ldots \overset{iid}{\sim} \mathit{Uniform}(0,1)$. Then $X_{(n)} \sim \mathit{Beta}(n,1)$



Class activity, Part II

Suppose that $X_1, X_2, ... \overset{iid}{\sim} Exp(1)$, and let $Y_n = \min\{X_1, ..., X_n\}$.
Then $Y_n \sim Exp(n)$, and $Y_n \xrightarrow{p} 0$.