

STA 711 Homework 2

Due: Friday, January 30, 11:59pm on Canvas.

Instructions: Submit your work as a single PDF. You may choose to either hand-write your work and submit a PDF scan, or type your work using LaTeX and submit the resulting PDF. See the course website for a homework template file and instructions on getting started with LaTeX and Overleaf.

Maximum likelihood estimation

1. Let $Y_1, \dots, Y_n \stackrel{iid}{\sim} \text{Poisson}(\lambda)$, and let $\mathbf{y} = (Y_1, \dots, Y_n)$ denote the combined sample.
 - (a) Write down the likelihood $L(\lambda|\mathbf{y})$.
 - (b) Find the maximum likelihood estimator $\hat{\lambda}$ of λ .
2. Let $Y_1, \dots, Y_n \stackrel{iid}{\sim} \text{Exponential}(\theta)$, so $f(y|\theta) = \theta e^{-\theta y}$.
 - (a) Write down the likelihood $L(\theta|\mathbf{y})$.
 - (b) Find the maximum likelihood estimator $\hat{\theta}$ of θ .
 - (c) Show that $Y_{(1)} \sim \text{Exponential}(n\theta)$.
3. Let Y_1, \dots, Y_n be iid from a distribution with pdf

$$f(y|\theta) = \theta y^{-2} \mathbb{1}\{y \geq \theta\},$$

where $\theta > 0$. Find the maximum likelihood estimator of θ .

4. Let Y_1, \dots, Y_n be iid with one of two pdfs. If $\theta = 0$, then

$$f(y|\theta) = \begin{cases} 1 & 0 < y < 1 \\ 0 & \text{else.} \end{cases}$$

If $\theta = 1$, then

$$f(y|\theta) = \begin{cases} \frac{1}{2\sqrt{y}} & 0 < y < 1 \\ 0 & \text{else.} \end{cases}$$

Find the maximum likelihood estimator of θ .

5. Let Y be a single observation from a normal distribution with mean θ and variance θ^2 , where $\theta > 0$. Find the maximum likelihood estimator of θ^2 .
6. Let Y_1, \dots, Y_n be a random sample from a distribution with pdf

$$f(y|\mu, \sigma) = \frac{1}{\sigma} \exp \left\{ - \left(\frac{y - \mu}{\sigma} \right)^2 \right\} \mathbb{1}\{y \geq \mu\},$$

where $-\infty < \mu < \infty$, and $\sigma > 0$.

- (a) Find the maximum likelihood estimators of μ and σ . (*Hint: find $\hat{\mu}$ first*)
- (b) Let $\tau(\mu, \sigma) = \mathbb{P}_{\mu, \sigma}(Y_1 \geq t)$, where $t > \mu$, and $\mathbb{P}_{\mu, \sigma}$ denotes probability when μ, σ are the true parameters. Find the maximum likelihood estimator of $\tau(\mu, \sigma)$.