STA 712 Homework 3

Due: Friday, September 23, 12:00pm (noon) on Canvas.

Instructions: Submit your work as a single PDF. For this assignment, you may include written work by scanning it and incorporating it into the PDF. Include all R code needed to reproduce your results in your submission.

Multivariate normal distributions

The multivariate normal distribution appears frequently in 712, for example as the asymptotic distribution of our coefficient estimates $\widehat{\beta}$. The purpose of this section is to derive a basic property of the multivariate normal distribution that we use regularly, for example in constructing our Wald test statistic.

One way to define a multivariate normal distribution is with its *moment generating function* (MGF). Let $X \in \mathbb{R}^k$ be a random vector. The (multivariate) moment generating function $M_X(t)$ of X is defined by

$$M_X(t) = \mathbb{E}[e^{t^T X}],$$

where $t \in \mathbb{R}^k$. As with univariate MGFs, if $M_X(t) = M_Y(t)$ for all t, then the two random variables X and Y have the same distribution.

We say that the random vector $X \in \mathbb{R}^k$ follows a multivariate normal distribution with mean $\mu \in \mathbb{R}^k$ and variance matrix $\Sigma \in \mathbb{R}^{k \times k}$, and write $X \sim N(\mu, \Sigma)$, if

$$M_X(t) = e^{t^T \mu} e^{\frac{1}{2} t^T \Sigma t}.$$

1. An important property of multivariate normal random variables is that if $X \sim N(\mu, \Sigma)$, then

$$\boldsymbol{a} + \boldsymbol{B}X \sim N(\boldsymbol{a} + \boldsymbol{B}\mu, \boldsymbol{B}\Sigma\boldsymbol{B}^T),$$

where $\boldsymbol{a} \in \mathbb{R}^k$ and $\boldsymbol{B} \in \mathbb{R}^{m \times k}$. Our goal is to use MGFs to prove this property.

   (a) Show that for any random vector X in $\mathbb{R}^k$, the MGF of $Y = \boldsymbol{a} + \boldsymbol{B}X$ is given by

$$M_Y(t) = e^{t^T \boldsymbol{a}} M_X(\boldsymbol{B}^T t).$$

   (b) Using (a), show that if $X \sim N(\mu, \Sigma)$, then $\boldsymbol{a} + \boldsymbol{B}X \sim N(\boldsymbol{a} + \boldsymbol{B}\mu, \boldsymbol{B}\Sigma\boldsymbol{B}^T)$.

Multicollinearity and power

One of the potential issues with multicollinearity is that it

Data analysis