Intro to mixed effects models

Recap: data and motivation

We have data from a 2010 study on performance anxiety in 37 undergraduate music majors. For each musician, data was collected on anxiety levels before different performances (between 2 and 15 performances were measured for each musician), with variables including:

- `id`: a unique identifier for the musician
- `na`: negative affect score (a measure of anxiety)
- `perform_type`: whether the musican was performing in a large ensemble, small ensemble, or solo

> How can we model the relationship between performance type and anxiety?

Recap: a mixed effects model

$$Anxiety_{ij} = \beta_0 + u_i + \beta_1 \, SmallEnsemble_{ij} + \beta_2 \, Solo_{ij} + \varepsilon_{ij}$$

$$u_i \overset{iid}{\sim} N(0, \sigma_u^2) \quad \varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

Fitting the model in R

```r
library(lme4)
m1 <- lmer(na ~ perform_type + (1|id),
           data = music)
summary(m1)
```

```
...
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  id       (Intercept)  5.56    2.358
##  Residual             21.75    4.664
## Number of obs: 497, groups:  id, 37
## Fixed effects:
##                              Estimate Std. Error t value
## (Intercept)                   14.9654     0.5920  25.278
## perform_typeSmall Ensemble     0.7709     0.7210   1.069
## perform_typeSolo               2.0142     0.5521   3.648
...
```

Interpretation

```
...
## Fixed effects:
##                            Estimate Std. Error t value
## (Intercept)                 14.9654     0.5920  25.278
## perform_typeSmall Ensemble   0.7709     0.7210   1.069
## perform_typeSolo             2.0142     0.5521   3.648
...
```

How would we interpret the estimated fixed effects?

Prediction

```
...
## Fixed effects:
##                            Estimate Std. Error t value
## (Intercept)                 14.9654     0.5920  25.278
## perform_typeSmall Ensemble   0.7709     0.7210   1.069
## perform_typeSolo             2.0142     0.5521   3.648
...
```

What is the estimated anxiety for Musician 1 before a solo performance?

Prediction

> What is the estimated anxiety for Musician 1 before a solo performance?

```
coef(m1)
```

```
...
## $id
##   (Intercept) perform_typeSmall Ensemble perform_typeSolo
## 1    11.61227                    0.7708706         2.014226
## 2    12.78968                    0.7708706         2.014226
## 3    12.85152                    0.7708706         2.014226
...
```

Prediction

```
...
## Fixed effects:
##                              Estimate Std. Error t value
## (Intercept)                   14.9654     0.5920  25.278
## perform_typeSmall Ensemble     0.7709     0.7210   1.069
## perform_typeSolo               2.0142     0.5521   3.648
...
```

What is the estimated anxiety for a *new* musician (not in the data) before a solo performance?

Assumptions

$$Anxiety_{ij} = \beta_0 + u_i + \beta_1\,SmallEnsemble_{ij} + \beta_2\,Solo_{ij} + \varepsilon_{ij}$$

$$u_i \overset{iid}{\sim} N(0, \sigma_u^2) \quad \varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$
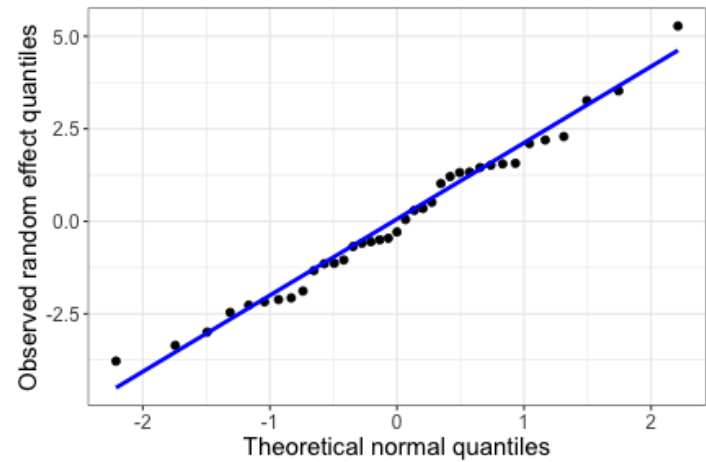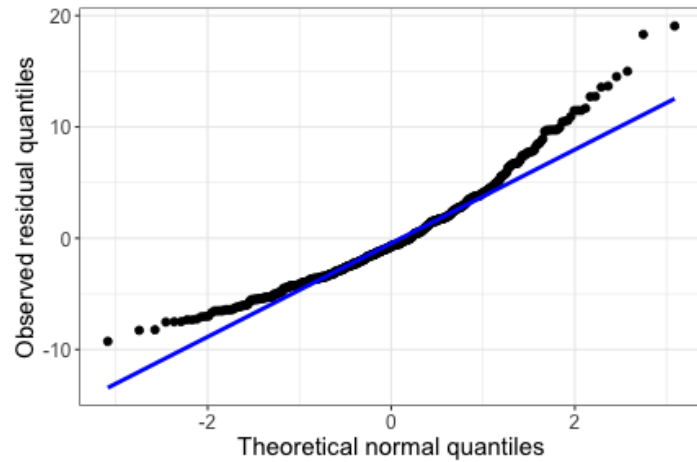
What assumptions does this mixed effects model make?

Assessing normality

$$Anxiety_{ij} = \beta_0 + u_i + \beta_1\, SmallEnsemble_{ij} + \beta_2\, Solo_{ij} + \varepsilon_{ij}$$

$$u_i \overset{iid}{\sim} N(0, \sigma_u^2) \quad \varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

How should we check the normality assumption?

QQ plots

Changing the model

$$Anxiety_{ij} = \beta_0 + u_i + \beta_1\, SmallEnsemble_{ij} + \beta_2\, Solo_{ij} + \varepsilon_{ij}$$

$$u_i \overset{iid}{\sim} N(0, \sigma_u^2) \quad \varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

How could we change the model to allow the effect of performance type to differ between musicians?

Fitting the model

```
m2 <- lmer(na ~ perform_type + (perform_type|id),
           data = music)
summary(m2)
```

```
...
## Random effects:
##  Groups   Name                         Variance Std.Dev. Corr
##  id       (Intercept)                     3.986   1.997
##           perform_typeSmall Ensemble      2.019   1.421     -0.43
##           perform_typeSolo                1.017   1.008      0.74  0.29
##  Residual                                21.288   4.614
## Number of obs: 497, groups:  id, 37
##
## Fixed effects:
##                             Estimate Std. Error t value
## (Intercept)                  15.0503     0.5436  27.685
## perform_typeSmall Ensemble    0.6996     0.7410   0.944
## perform_typeSolo              2.0134     0.5671   3.550
...
```

Prediction

> What is the estimated anxiety for Musician 1 before a solo performance?

```
coef(m2)
```

```
...
## $id
##   (Intercept) perform_typeSmall Ensemble perform_typeSolo
## 1    12.37560                   0.84623321        0.6590148
## 2    13.61693                   0.30915635        1.0413577
## 3    12.86707                   1.31366273        1.1674007
...
```