Goodness of fit and overdispersion

Recap: EDMs and deviance

Saddlepoint approximation

Data

A concerned parent asks us to investigate crime rates on college campuses. We have access to data on 81 different colleges and universities in the US, including the following variables:

- `type`: college (C) or university (U)
- `nv`: the number of violent crimes for that institution in the given year
- `enroll1000`: the number of enrolled students, in thousands
- `region`: region of the US C = Central, MW = Midwest, NE = Northeast, SE = Southeast, SW = Southwest, and W = West)

Goodness of fit

Goodness of fit test: If the model is a good fit for the data, then the residual deviance follows a χ^2 distribution with the same degrees of freedom as the residual deviance

Residual deviance = 621.24, df = 75

```
pchisq(621.24, df=75, lower.tail=F)
```

```
## [1] 5.844298e-87
```

So our model might not be a very good fit to the data.

> Why might our model not be a good fit?

Offsets

We will account for school size by including an **offset** in the model:

$$\log(\lambda_i) = \beta_0 + \beta_1 MW_i + \beta_2 NE_i + \beta_3 SE_i + \beta_4 SW_i + \beta_5 W_i + \log(Enrollment_i)$$

Motivation for offsets

We can rewrite our regression model with the offset:

$$\log(\lambda_i) = \beta_0 + \beta_1 MW_i + \beta_2 NE_i + \beta_3 SE_i + \beta_4 SW_i + \beta_5 W_i + \log(Enrollment_i)$$

Fitting a model with an offset

```
m2 <- glm(nv ~ region, offset = log(enroll1000),
          data = crimes, family = poisson)
summary(m2)
```

```
...
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.30445    0.12403 -10.517  < 2e-16 ***
## regionMW     0.09754    0.17752   0.549  0.58270
## regionNE     0.76268    0.15292   4.987 6.12e-07 ***
## regionSE     0.87237    0.15313   5.697 1.22e-08 ***
## regionSW     0.50708    0.18507   2.740  0.00615 **
## regionW      0.20934    0.18605   1.125  0.26053
...
```

- The offset doesn't show up in the output (because we're not estimating a coefficient for it)

Fitting a model with an offset

$$\log(\widehat{\lambda}_i) = -1.30 + 0.10MW_i + 0.76NE_i + 0.87SE_i + 0.51SW_i + 0.21W_i + \log(Enrollment_i)$$

How would I interpret the intercept -1.30?

When to use offsets

Offsets are useful in Poisson regression when our counts come from groups of very different sizes (e.g., different numbers of students on a college campus). The offset lets us interpret model coefficients in terms of rates instead of raw counts.

> With your neighbor, brainstorm some other data scenarios where our response is a count variable, and an offset would be useful. What would our offset be?

Goodness of fit

```
m2 <- glm(nv ~ region, offset = log(enroll1000),
          data = crimes, family = poisson)
summary(m2)
```

```
...
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 491.00  on 80  degrees of freedom
## Residual deviance: 433.14  on 75  degrees of freedom
...
```

```
pchisq(433.14, df=75, lower.tail=F)
```

```
## [1] 8.33082e-52
```

Overdispersion

Overdispersion occurs when the response Y has higher variance than we would expect from the specified EDM

> Why is it a problem if Y has more variance than we account for in our model?

Estimating ϕ

Using $\hat{\phi}$

```
pearson_resids <- residuals(m2, type="pearson")
sum(pearson_resids^2)/df.residual(m2)
```

```
## [1] 7.58542
```

```
...
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.30445    0.12403 -10.517  < 2e-16 ***
## regionMW     0.09754    0.17752   0.549  0.58270
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...
```