Wald tests and likelihood ratio tests

Wald tests for multiple parameters

Class activity

https://sta712-f22.github.io/class_activities/ca_lecture_11.html

- Wald tests for the dengue data

Likelihood ratio tests

```
m1 <- glm(Dengue ~ WBC + PLT, data = dengue,
          family = binomial)
summary(m1)
```

```
...
##                Estimate Std. Error z value Pr(>|z|)
## (Intercept)  2.6415063  0.1213233   21.77   <2e-16 ***
## WBC         -0.2892904  0.0134349  -21.53   <2e-16 ***
## PLT         -0.0065615  0.0005932  -11.06   <2e-16 ***
## ---
##     Null deviance: 6955.8  on 5719  degrees of freedom
## Residual deviance: 5399.7  on 5717  degrees of freedom
## AIC: 5405.7
...
```

What information replaces R^2 and R^2_{adj} in the GLM output?

Deviance

Definition: The *deviance* of a fitted model with parameter estimates $\widehat{\beta}$ is given by

$$2 \log \ell(\text{saturated model}) - 2 \log \ell(\widehat{\beta})$$

Comparing deviances

```
m1 <- glm(Dengue ~ WBC + PLT, data = dengue,
          family = binomial)
summary(m1)
```

```
...
##     Null deviance: 6955.8  on 5719  degrees of freedom
## Residual deviance: 5399.7  on 5717  degrees of freedom
## AIC: 5405.7
...
```

Comparing deviances

Full model: $\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 WBC_i + \beta_2 PLT_i$

Reduced model: $\log\left(\frac{p_i}{1-p_i}\right) = \beta_0$

$$G = 2\log \ell(\widehat{\beta}) - 2\log \ell(\widehat{\beta}^0)$$

Why is G always ≥ 0?

Comparing deviances

Full model: $\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 WBC_i + \beta_2 PLT_i$

Reduced model: $\log\left(\frac{p_i}{1-p_i}\right) = \beta_0$

$$G = 2\log \ell(\widehat{\beta}) - 2\log \ell(\widehat{\beta}^0) = 1556.1$$

> If the reduced model is correct, how unusual is $G = 1556.1$?

Likelihood ratio test