Lecture 31

Data: flipped classrooms?

- A *flipped classroom* involves students watching lectures at home, and doing activities during class time
- There is debate about the pros and cons of this teaching method
- Here we will look at simulated data from an experiment with flipped classrooms

Data: flipped classrooms?

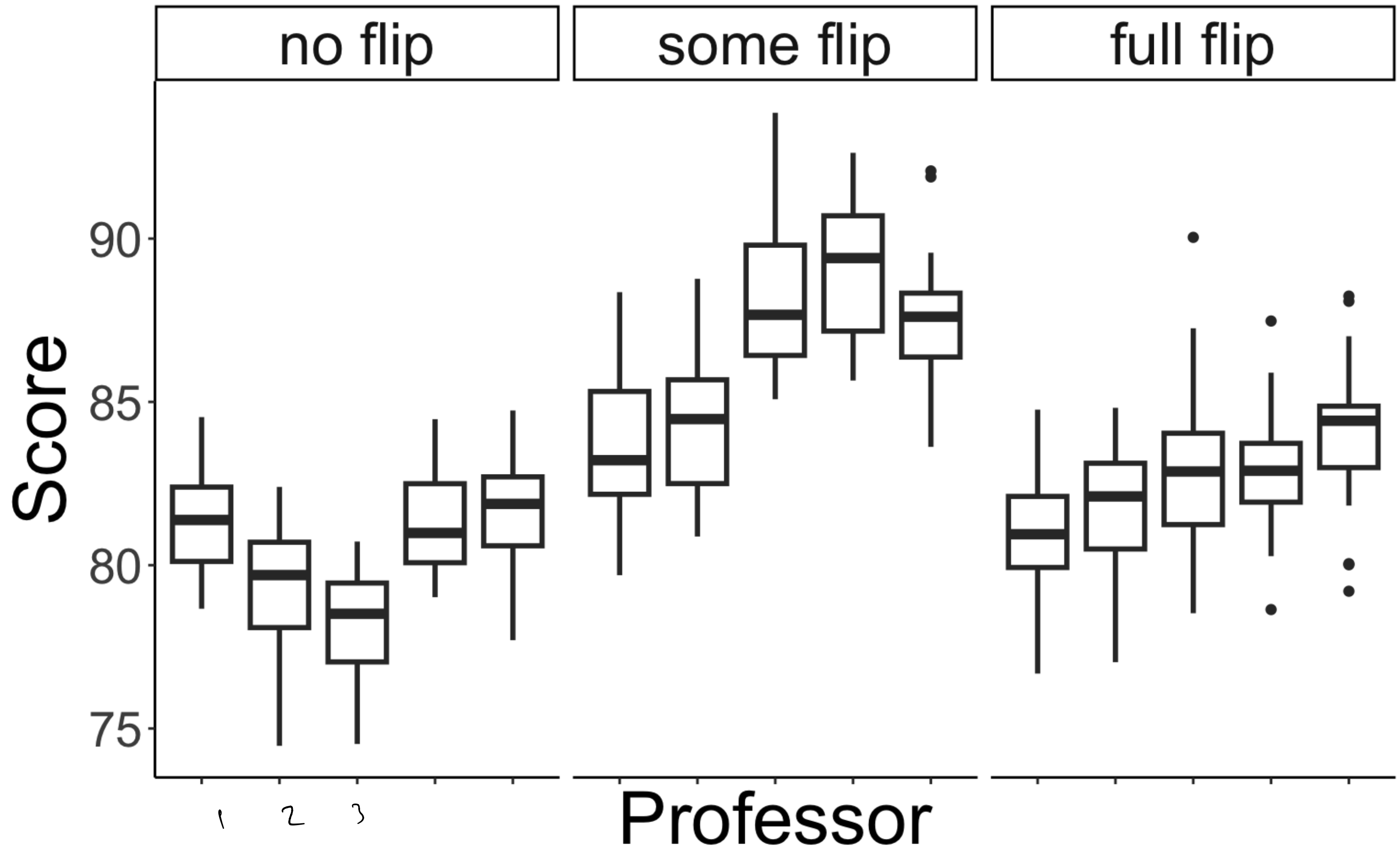
- 15 classes of introductory statistics
- 25 students in each class (so 375 students total)
- Each class taught by a different professor
- Each professor randomly assigned a teaching style: No flip, Some flip, and Fully flipped
- At the end of the semester, we give all the students in all the classes the same exam, and compare their results

Data: flipped classrooms?

Data set has 375 rows (one per student), and the following variables:

- `professor`: which professor taught the class (1 – 15)
- `style`: which teaching style the professor used (no flip, some flip, fully flipped)
- `score`: the student's score on the final exam

Considering results

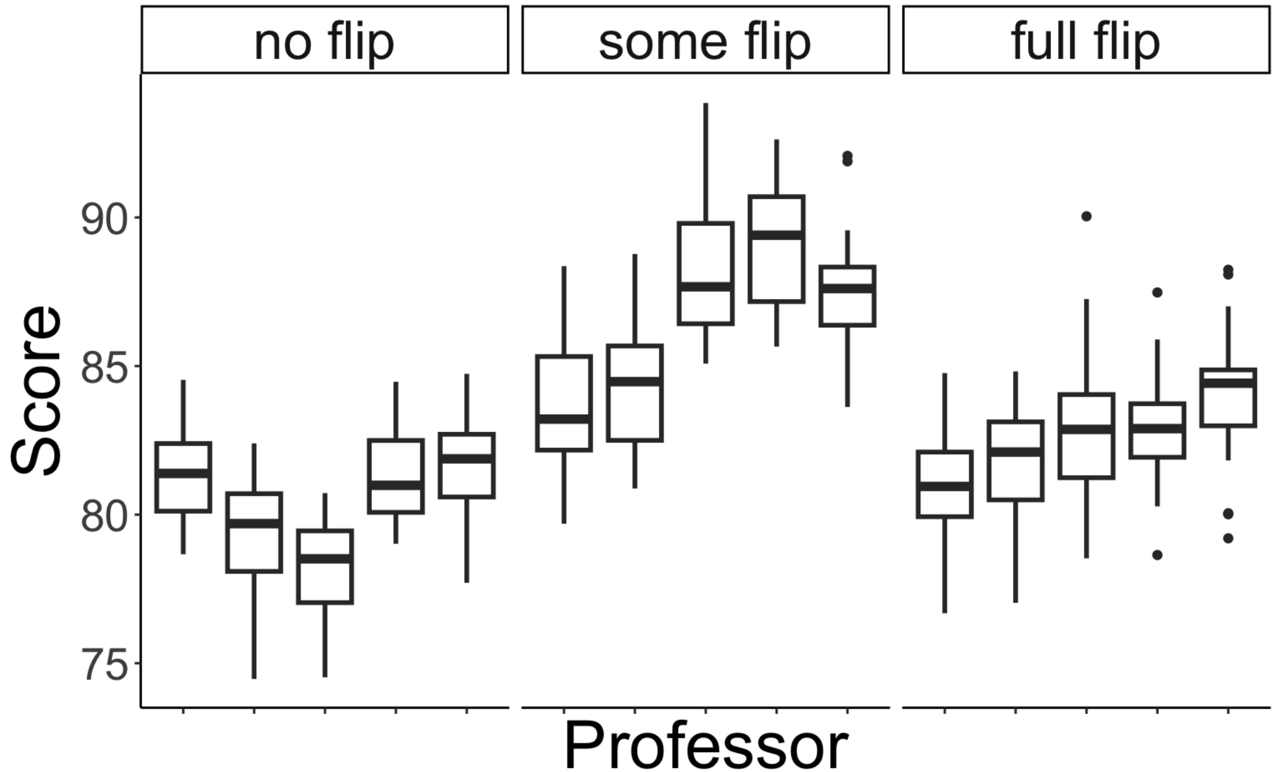


What do you notice about the scores?

Differences between teaching

Differences between professors / classes

Considering results



- There may be some differences between styles
- There may be some differences between professors

Considering results

Suppose we notice that, on average, students in the “Some Flipped” classes have higher scores than students in the “Fully Flipped” classes. What might explain this difference?

Considering results

Suppose we notice that, on average, students in the “Some Flipped” classes have higher scores than students in the “Fully Flipped” classes. What might explain this difference?

- The “Some Flipped” method may lead to higher test results.
- The professors assigned to teach “Some Flipped” may teach in such a way that their scores are higher than those in the “Fully Flipped” group (more experience, etc.).
- The students in the “Some Flipped” classes may have been stronger than those in the “Fully Flipped” group.

Different effects

- *Effect of interest* (*treatment effect*): The "Some Flipped" method may lead to higher test results; *the treatment imposed by the researchers has an effect on the outcome.*
- *Group effect*: The professors assigned to teach "Some Flipped" may have had an impact on the test scores; *the group the students are in has an effect on the outcome.*
- *Individual effect*: The students in the "Some Flipped" classes may have been stronger than those in the "Fully Flipped" group; *the individuals' characteristics or abilities have an effect on the outcome.*

Writing down a model

Score is a continuous response, so we can go back to linear models:

score for student i (i = 1, ..., 375)

$$\mathrm{Score_i} = \beta_0 + \underbrace{\beta_1 \mathrm{SomeFlipped_i} + \beta_2 \mathrm{FullyFlipped_i}}_{\text{Treatment effects}} + \varepsilon_i$$

individual effect

$$\varepsilon_i \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

Which effects does this model capture?

Assumptions

$$\text{Score}_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \varepsilon_i$$

$$\varepsilon_i \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

What does this model assume about group effects (differences between professors)?

Assuming that there is no effect due to the professor / no group effects

Assumptions

$$\text{Score}_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \varepsilon_i$$

$$\varepsilon_i \overset{\text{iid}}{\sim} N(0, \sigma_\varepsilon^2)$$

Students are all independent (no correlation within class, after accounting for teaching style)

What does this model assume about correlation within a class?

Writing down a model

$$\text{Score}_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \varepsilon_i$$

$$\varepsilon_i \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

How can I incorporate systematic differences between classes?

Writing down a model

$$\text{Score}_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \varepsilon_i$$

$$\varepsilon_i \overset{\text{iid}}{\sim} N(0, \sigma_\varepsilon^2)$$

Add a variable for the different professors:

$$\text{Score}_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \\ \beta_3 \text{Class2}_i + \cdots + \beta_{16} \text{Class15}_i + \varepsilon_i$$

$$\varepsilon_i \overset{\text{iid}}{\sim} N(0, \sigma_\varepsilon^2)$$

Writing down a model

$$\text{Score}_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \beta_3 \text{Class2}_i + \cdots + \beta_{16} \text{Class15}_i + \varepsilon_i$$

$$\varepsilon_i \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

How many parameters did we add to the model to capture class differences?

14 !

Writing down a model

$$\text{Score}_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \\ \beta_3 \text{Class2}_i + \cdots + \beta_{16} \text{Class15}_i + \varepsilon_i$$

$$\varepsilon_i \overset{\text{iid}}{\sim} N(0, \sigma_\varepsilon^2)$$

Do we want to do inference on $\beta_3, \ldots, \beta_{16}$?

Our first mixed effects model

Linear model:

score for student i (i = 1, ..., 375)

$$\text{Score}_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \beta_3 \text{Class2}_i + \cdots + \beta_{16} \text{Class15}_i + \varepsilon_i$$

fixed effects

$$\varepsilon_i \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

Linear mixed effects model: Let Score_{ij} be the score of student j in class i (i = 1, ..., 15) (j = 1, ..., 25)

$$\text{Score}_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

random effect

$$\varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2) \quad u_i \overset{iid}{\sim} N(0, \sigma_u^2)$$

Anatomy of the mixed effects model

Linear mixed effects model: Let Score_{ij} be the score of student j in class i

$$\text{Score}_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

random effect (group level)

noise term (individual level)

$$\varepsilon_{ij} \overset{\text{iid}}{\sim} N(0, \sigma_\varepsilon^2) \quad u_i \overset{\text{iid}}{\sim} N(0, \sigma_u^2)$$

ε_{ij} independent of u_i

For Professor i:

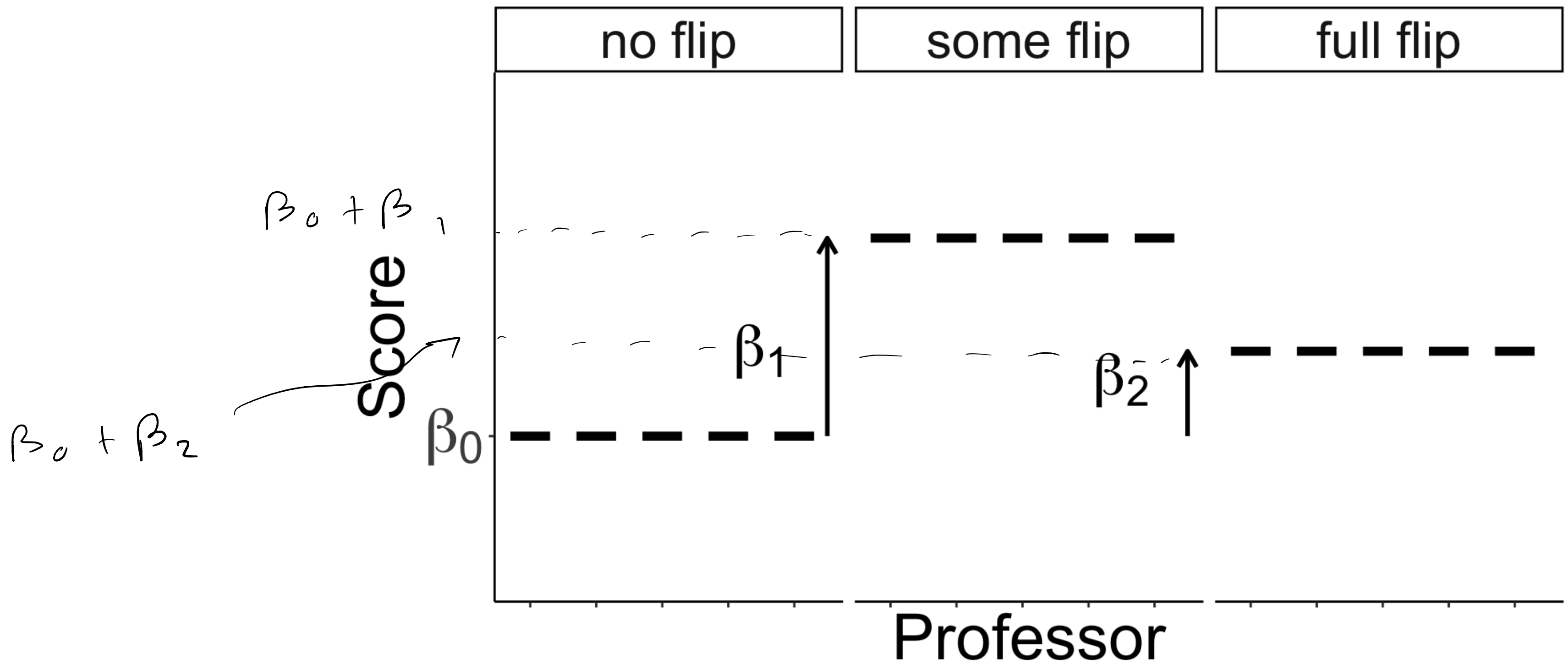
$$\text{Score}_{ij} = \underbrace{\beta_0 + u_i}_{\text{random intercept}} + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \varepsilon_{ij}$$

intercept for professor i: random variable w/ $N(\beta_0, \sigma_u^2)$

Anatomy of the mixed effects model

$$\text{Score}_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

Part 1: Fixed effects (treatment effects)

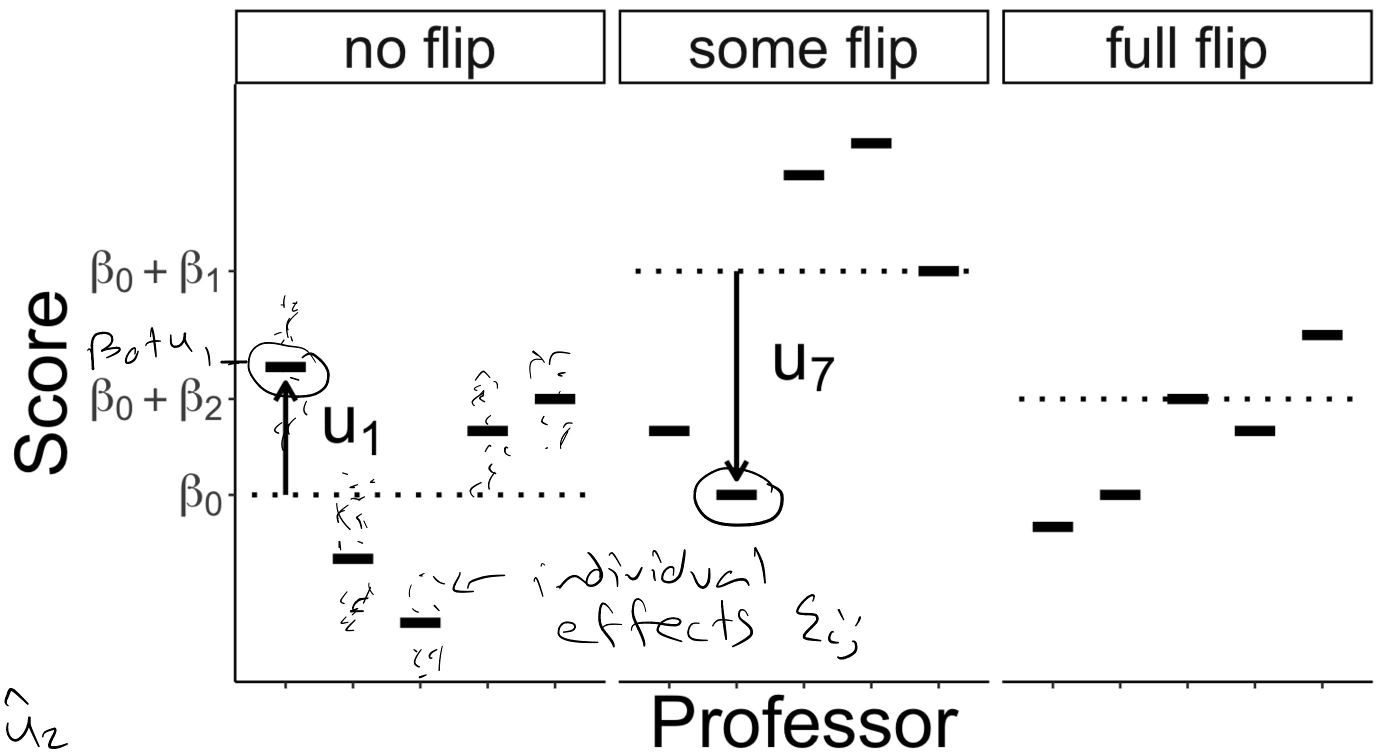


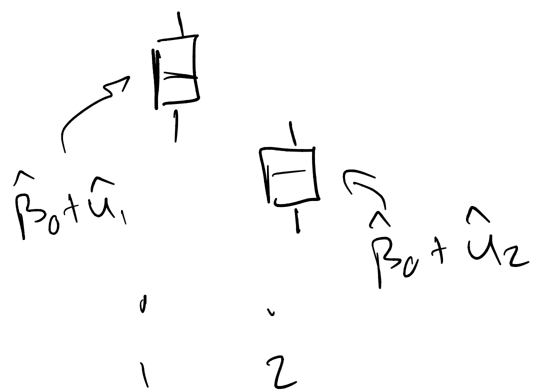
Anatomy of the mixed effects model

$$\text{Score}_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

Part 2: Random effects (group effects)

no flip





Linear mixed model

$$Y_{ij} = \beta_0 + u_i + \beta_1 x_{ij} + \varepsilon_{ij}$$

$$u_i \overset{iid}{\sim} N(0, \sigma_u^2)$$

$$\varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

$$u_i \perp\!\!\!\perp \varepsilon_{ij}$$

$$Y_{ij} \mid u_i \sim N(\beta_0 + u_i + \beta_1 x_{ij}, \sigma_\varepsilon^2)$$

matrix form:

$$\begin{bmatrix} Y_{11} \\ Y_{12} \\ \vdots \\ Y_{1n_1} \\ Y_{21} \\ \vdots \\ Y_{mn_m} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & x_{11} \\ 1 & x_{12} \\ \vdots & \vdots \\ 1 & x_{mn_m} \end{bmatrix}}_{\text{design matrix for fixed effects}} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & & 0 \\ \vdots \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & \\ \vdots \\ 0 & 0 & \cdots & & 1 \end{bmatrix}}_{\text{design matrix for random effects}} \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} + \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \vdots \\ \varepsilon_{mn_m} \end{bmatrix}$$

m = # groups

n_i = # obs in group i

$$N = \sum_i n_i$$

$$Y = X\beta + Zu + \varepsilon$$

$Y \in \mathbb{R}^N$, $\beta \in \mathbb{R}^p$, $X \in \mathbb{R}^{N \times p}$, $Z \in \mathbb{R}^{N \times m}$, $u \in \mathbb{R}^m$, $\varepsilon \in \mathbb{R}^N$

$$Y = X\beta + Zu + \varepsilon$$

$Y \in \mathbb{R}^N$, $\beta \in \mathbb{R}^p$, $X \in \mathbb{R}^{N \times p}$, $Z \in \mathbb{R}^{N \times m}$, $u \in \mathbb{R}^m$, $\varepsilon \in \mathbb{R}^N$

$u \sim N(0, \sigma_u^2 I_m)$ $\quad\quad$ $\varepsilon \sim N(0, \sigma_\varepsilon^2 I_N)$

$$\underset{\substack{\uparrow \\ \text{professor}}}{} \quad Score_{ijk} = \beta_0 + \underbrace{u_i + v_{ij}}_{?} + \beta_1 Same_{ij} + \beta_2 Full_{ij} + \varepsilon_{ijk}$$

i ↑ professor, j ↑ class, k ↑ student

β_1 ↑ random slope?

Anatomy of the mixed effects model

$$\text{Score}_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

Part 3: Noise (individual effects)