Lecture 12

GLMs so far

- Linear and logistic regression:
  - Estimation
  - Inference
  - Diagnostics
  - Prediction
- Poisson regression:
  - Estimation
  - Inference

Poisson regression model

$$Y_i \sim \text{Poisson}(\lambda_i)$$

$$\log(\lambda_i) = \beta^T X_i$$

Question: What assumptions does this model make?

- Shape: $\log(\lambda_i)$ is linear in X_i
- Distribution: $Y_i \sim \text{Poisson}(\lambda_i)$

  $\Rightarrow \mathbb{E}[Y_i] = \text{Var}(Y_i)$
- Independence: observations (X_i, Y_i) are independent

The importance of assumptions

$$Y_i \sim \text{Poisson}(\lambda_i)$$

$$\log(\lambda_i) = \beta^T X_i$$

Question: How could we assess the *importance* of the Poisson regression assumption? I.e., what is the impact if this assumption is wrong?

Simulation:
- Simulate data from a non-Poisson distribution
- fit Poisson model
  (may end up w/ SEs that don't match truth)
- calculate a confidence interval and check whether CI contains β_1
- Repeat many times & assess coverage

Simulation plan

$$Y_i \sim NB(r, p_i)$$

$$E[Y_i] = \mu_i = \frac{r p_i}{1 - p_i} \qquad Var(Y_i) = \mu_i + \frac{\mu_i^2}{r}$$

$$\log(\mu_i) = \beta_0 + \beta_1 X_i$$

① vary r

② For each r:

repeat many times to estimate coverage
- Simulate data
- Fit Poisson model
- Calculate a 95% CI for β_1
- Check if CI contains β_1

③ Plot coverage vs. r

ADEMP: A useful framework for simulation studies

- **Aims:** Why are we doing the study? assess importance of Poisson assumption
- **Data generation:** How are the data simulated? Different NB
- **Estimand/target:** What are we estimating for each simulated dataset? β_1
- **Methods:** What methods are we using for model fitting, estimation, etc? Poisson regression model
- **Performance measures:** How do we measure performance of our chosen methods?

Coverage of 95% CIs

ADEMP

For the Poisson simulation study:

- **Aims:**
- **Data generation:**
- **Estimand/target:**
- **Methods:**
- **Performance measures:**

Class activity

https://sta712-f23.github.io/class_activities/ca_lecture_12.html