Lecture 35

Data: flipped classrooms?

Data set has 375 rows (one per student), and the following variables:

- `professor`: which professor taught the class (1 – 15)
- `style`: which teaching style the professor used (no flip, some flip, fully flipped)
- `score`: the student's score on the final exam

Inference with linear models

$$\text{Score}_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \varepsilon_i$$

Research question: Is there a relationship between teaching style and student score?

What are my null and alternative hypotheses, in terms of one or more model parameters?

$H_0: \beta_1 = \beta_2 = 0$

H_A: at least one of $\beta_1, \beta_2 \neq 0$

Inference with linear models

$$\text{Score}_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \varepsilon_i$$

Research question: Is there a relationship between teaching style and student score?

$H_0 : \beta_1 = \beta_2 = 0$

H_A : at least one of $\beta_1, \beta_2 \neq 0$

What test would I use to test these hypotheses?

nested F test

reduced model: $\text{Score}_i = \beta_0 + \varepsilon_i$

F tests

$$\text{Score}_i = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + \varepsilon_i$$

Research question: Is there a relationship between teaching style and student score?

$H_0 : \beta_1 = \beta_2 = 0$

H_A : at least one of $\beta_1, \beta_2 \neq 0$

What are my degrees of freedom for the F test?

numerator df = 2

denominator df = n - 3 (e.g. n = 375)

F tests for mixed effects models

$$\text{Score}_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

Research question: Is there a relationship between teaching style and student score?

What are my null and alternative hypotheses, in terms of one or more model parameters?

$H_0: \beta_1 = \beta_2 = 0$

H_A: at least one of $\beta_1, \beta_2 \neq 0$

F tests for mixed effects models

$$\text{Score}_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

Research question: Is there a relationship between teaching style and student score?

$$H_0 : \beta_1 = \beta_2 = 0$$

$$H_A : \text{at least one of } \beta_1, \beta_2 \neq 0$$

Test: We will use an F test again

- numerator df = number of parameters tested = 2
- denominator df = ??

What *are* degrees of freedom?

Intuition:

Suppose we observe $X_1, \dots, X_n$

Calculate $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

If I know $X_1, \dots, X_{n-1}$ and $\bar{X}$

$\Rightarrow$ also know X_n

If I fit $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$

get $\hat{\beta}_0, \hat{\beta}_1$

then if I know $Y_1, \dots, Y_{n-2}$ and $\hat{\beta}_0, \hat{\beta}_1,$ and $X_1, \dots, X_n$

then I can get Y_{n-1}, Y_n

In general: df = # of independent observations − # of parameters estimated

E.g. know $\bar{X} = 0$

$n = 4$

$X_1 = 3$, $X_2 = -1$, $X_3 = -1$

must have $X_4 = -1$

Turns out: intuitive idea of df coincides w/ parameterizing a distribution for test statistics

Mathematically, df is just the parameter for sampling ~~d~~ distribution of a test statistic (e.g. t statistic)

For a mixed model:

- calculate an F statistic
- denominator df is just the parameter st the F distribution approximates sampling distribution of the test statistic

Denominator degrees of freedom for mixed models

$$\text{Score}_{ij} = \beta_0 + \beta_1 \text{SomeFlipped}_i + \beta_2 \text{FullyFlipped}_i + u_i + \varepsilon_{ij}$$

$$H_0 : \beta_1 = \beta_2 = 0 \quad H_A : \text{at least one of } \beta_1, \beta_2 \neq 0$$

Test: We will use an F test again

- numerator df = number of parameters tested = 2
- denominator df =

$$\text{number of independent observations} - \text{number of parameter}$$

Are all observations independent?

Denominator degrees of freedom for mixed models

if $\sigma_u^2 = 0$ (no group effect)

indep. observations $= N$ (total # of obs in data)

if $\rho = 1$ $(\sigma_\varepsilon^2 = 0)$ (no individual variance within groups)

indep. obs $= m$ (# groups)

if $0 < \rho < 1$

indep. obs $\in (m, N)$

Approximating the degrees of freedom

```r
groups <- rep(1:30, each=10)
sigma_u <- 0.1
sigma_e <- 0.5

u <- rnorm(30, sd=sigma_u)
x1 <- rnorm(300)
y <- 1 + u[groups] + 0.5*x1 + rnorm(300, sd=sigma_e)

m1 <- lmer(y ~ x1 + (1|groups))
summary(m1)$coefficients
```

```
             Estimate Std. Error       df  t value     Pr(>|t|)
(Intercept) 0.9435989 0.03195537  28.9317 29.52865 3.736113e-23
x1          0.4560607 0.02533352 293.2377 18.00226 2.547871e-49
```

$m = 30$ $N = 300$

df between $30 - 2$ and $300 - 2$

$$\rho = \frac{0.1^2}{0.1^2 + 0.5^2}$$

avg low

Approximating the degrees of freedom

```r
groups <- rep(1:30, each=10)
sigma_u <- 1
sigma_e <- 0.5

u <- rnorm(30, sd=sigma_u)
x1 <- rnorm(300)
y <- 1 + u[groups] + 0.5*x1 + rnorm(300, sd=sigma_e)

m1 <- lmer(y ~ x1 + (1|groups))
summary(m1)$coefficients
```

```
             Estimate Std. Error        df   t value     Pr(>|t|)
(Intercept) 1.2545954 0.18763891  29.00394  6.686222 2.469481e-07
x1          0.5155989 0.03198145 270.54591 16.121812 1.920600e-41
```

$\rho = \frac{1}{1 + 0.25}$ high

≈ 0.8

Class activity

https://sta712-f23.github.io/class_activities/ca_lecture_35.html