Lasso & Bayesian Lasso

Readings Chapter 15 Christensen

STA721 Linear Models Duke University

Merlise Clyde

October 23, 2019

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- Posterior mode

$$\max_{\boldsymbol{\beta}^s} -\frac{\phi}{2} \{\|\mathbf{Y}^c - \mathbf{X}^s \boldsymbol{\beta}^s\|^2 + \lambda^* \|\boldsymbol{\beta}^s\|_1\}$$

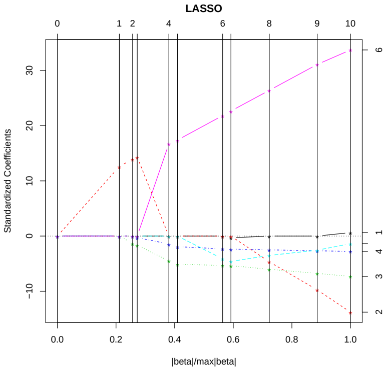
Picture

duke.eps

R Code

The entire path of solutions can be easily found using the "Least Angle Regression" Algorithm of Efron et al (Annals of Statistics 2004)

```
> library(lars)
> longley.lars = lars(as.matrix(longley[,-7]), longley[,7],
                      type="lasso")
> plot(longley.lars)
```



Solutions

```
> round(coef(longley.lars),5)
      GNP.deflator      GNP Unemployed Armed.Forces Population    Year
 [1,]      0.00000  0.00000    0.00000      0.00000    0.00000 0.00000
 [2,]      0.00000  0.03273    0.00000      0.00000    0.00000 0.00000
 [3,]      0.00000  0.03623   -0.00372      0.00000    0.00000 0.00000
 [4,]      0.00000  0.03717   -0.00459     -0.00099    0.00000 0.00000
 [5,]      0.00000  0.00000   -0.01242     -0.00539    0.00000 0.90681
 [6,]      0.00000  0.00000   -0.01412     -0.00713    0.00000 0.94375
 [7,]      0.00000  0.00000   -0.01471     -0.00861   -0.15337 1.18430
 [8,]     -0.00770  0.00000   -0.01481     -0.00873   -0.17076 1.22888
 [9,]      0.00000 -0.01212   -0.01663     -0.00927   -0.13029 1.43192
[10,]      0.00000 -0.02534   -0.01869     -0.00989   -0.09514 1.68655
[11,]      0.01506 -0.03582   -0.02020     -0.01033   -0.05110 1.82915
```

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Min $C_p = SSE_p/\hat{\sigma}_F^2 - n + 2p$

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   Df     Rss        Cp
0   1 185.009 1976.7120
1   2   6.642   59.4712
2   3   3.883   31.7832
3   4   3.468   29.3165
4   5   1.563   10.8183
5   4   1.339    6.4068
6   5   1.024    5.0186
7   6   0.998    6.7388
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Interval estimates?

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Scale Mixture of Normals (Andrews and Mallows 1974)

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Homework: Derive the full conditionals for $\boldsymbol{\beta}^s$, ϕ, $1/\tau^2$ see `http://www.stat.ufl.edu/~casella/Papers/Lasso.pdf`

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Half-Cauchy prior induces a Beta(1/2, 1/2) distribution on κ_i a priori

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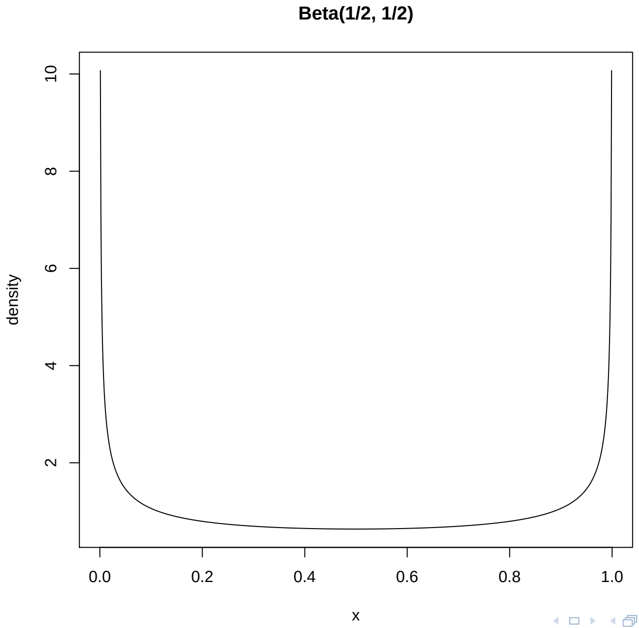
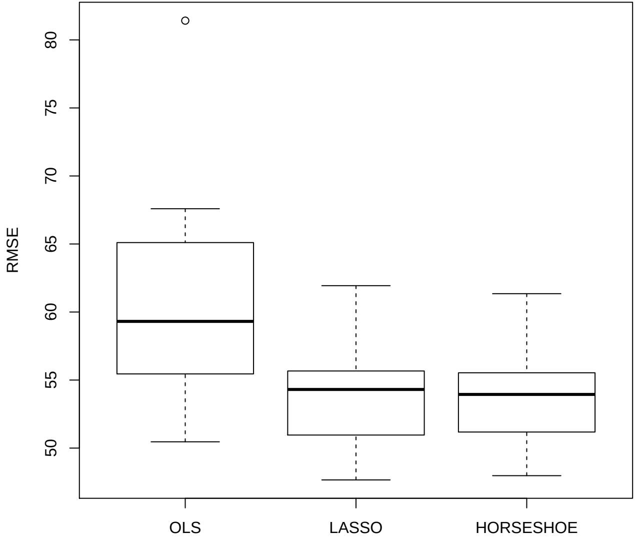


Simulation Study with Diabetes Data



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Choice of prior? Properties? Fan & Li (JASA 2001) discuss Variable selection via nonconcave penalties and oracle properties

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